February 13: Mathematics of the Electric Grid
March 13: Probability for People and Places
April 10: Social and Biological Networks
May 8: Mathematics of Redistricting
June 12: Number Theory: The Riemann Hypothesis
July 10: Topology
August 14: Algorithms for Threat Detection
September 11: Mathematical Analysis
October 9: Combinatorics
November 13: Why Machine Learning Works
December 11: Mathematics of Epidemics
MATHMATICAL FRONTIERS
Mathematics of the Electric Grid

Sean Meyn,
University of Florida

Steven Low,
Caltech

Mark Green,
UCLA (moderator)

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Networks of Power
Today and Tomorrow

Sean Meyn,
University of Florida

Professor and Robert C. Pittman Eminent Scholar Chair in the Department of Electrical and Computer Engineering
Networks of Power
Today and Tomorrow

Department of Electrical and Computer Engineering
University of Florida

Thanks to our sponsors: NSF, Google, DOE, ARPA-E

See also:

National Academies Workshop: https://vimeo.com/album/3275353
NREL AEG Workshop: https://www.nrel.gov/grid/autonomous-energy.html
Simons Center Bootcamp: https://simons.berkeley.edu/workshops/realtime2018-boot-camp
Networks of Power

Outline

1) Motivation
2) Network Today
3) Network Tomorrow: New Balancing Resources
4) Conclusions
5) References
Motivation: Grid in Transition
What’s the Big Deal?

• Revolution in energy and communication technologies
  ⇒ revolutionary thinking about how to manage the power grid

• Solar and wind energy bring variability and uncertainty
  ⇒ opportunities and challenges

• Research challenges in
  • networks, control & communication
  • creative modeling, such as PDE and mean-field models
  • statistics, optimization and “machine learning”
  • economics (new viewpoints are needed!)
  • and of course, all aspects of powersystems

These lectures focus on large-scale systems questions, leaving out
• New technologies: power electronics, solar cells, electric storage
• Cyber-security
• Detection and response to cascading failures, ...
The ISO grid

The ISO manages the flow of electricity for about 80 percent of California and a small part of Nevada, which encompasses all of the investor-owned utility territories and some municipal utility service areas. There are some pockets where local public power companies manage their own transmission systems.

The ISO is the largest of about 38 balancing authorities in the western interconnection, handling an estimated 35 percent of the electric load in the West. A balancing authority is responsible for operating a transmission control area. It matches generation with load and maintains consistent electric frequency of the grid, even during extreme weather conditions or natural disasters.
Network Today: Balancing Energy, Frequency, and Phase
View of the Balancing Authority

Balancing Frequency

Frequency deviation of 0.1 Hz ⇒ Panic!

NERC report 2002

Rockport Incident - 23 April 2002
Initial Trigger 14:50:20 EST, 13:50:20 CST
Frequency Change 95 mHz
Generation Loss 2800 MW

Frequency recorded at Rochester, N.Y.

L/O 765 kV transmission Rockport — Jefferson along with
Rockport Bus 1 was followed by L/O Rockport Units 1 and 2 and
765 kV transmission Rockport — Sullivan

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Network Today: Balancing Energy, Frequency, and Phase
View of the Balancing Authority

Balancing Frequency

Frequency is continuous across interconnected regions

FNET/GridEye Web Display

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Network Today: Balancing Energy, Frequency, and Phase
View of the Balancing Authority

Balancing Frequency

Phase angle is also continuous

FNET/GridEye Web Display

January 23rd 2018, 1:44:19 am
Frequency floats more freely in other regions of the globe

A disturbance in Agra appears to spread instantly to Mumbai and Calcutta.
Network Today: Balancing Energy, Frequency, and Phase
View of the Balancing Authority

*Ducks, Peaks, Ramps, Voltage, Power, Energy ...*

Dreaded Duck Curve in the South West

**Ramp limitations cause price-spikes**

- Price spike due to high net-load ramping need when solar production ramped out
- Negative prices due to high mid-day solar production

Ramps in *net-load* stress equipment and markets

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Network Today: Balancing Energy, Frequency, and Phase

View of the Balancing Authority

Ducks, Peaks, Ramps, Voltage, Power, Energy ...

Based on 5-min readings from the BPA SCADA system for points 45583, 79687, 79682, and 79685
Balancing Authority Load in Red, Wind Gen. in Green, Hydro Gen. in Blue, and Thermal Gen. in Brown
Click chart for installed capacity info
BPA Technical Operations (TOT-OpInfo@bpa.gov)
Generators and other resources ramp up and down power output

Analogy: ailerons on an airplane
New Balancing Resources

Balancing Authority

Ancillary Services

Grid

Measurements:
Voltage
Frequency
Phase

Brains

Brawn

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Where do they find Ancillary Services to provide needed actuation?

Many generalized storage solutions. If we are stuck with generators, then gas-combustion or hydro generation are best in terms of responsiveness.

Also, compressed air, flywheels, molten salt, trains pulled up a hill, ...

https://en.wikipedia.org/wiki/Grid_energy_storage
Network Tomorrow: New Balancing Resources
Secondary Control
Balancing Authority: In need of Balancing Services

Where do they find Ancillary Services to provide needed actuation?

Many generalized storage solutions. If we are stuck with generators, then gas-combustion or hydro generation are best in terms of responsiveness.

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[Links to grid energy storage]

California believes the answer is massive batteries
Network Tomorrow: New Balancing Resources
Demand Dispatch & Virtual Energy Storage

Demand Dispatch:
battery services from inherent flexibility of loads

View webinar videos and learn more about BMSA at www.nas.edu/MathFrontiers
**Demand Dispatch:** battery services from inherent flexibility of loads

**Example:** Tracking balancing reserves with 100,000 water heaters

Three cases, distinguished by the reference signal

(3rd is too big!)

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**Demand Dispatch:** battery services from inherent flexibility of loads

**Example:** Tracking with $10^4$ residential swimming pools

Simulation using 10,000 pool pumps that consume on average 5MW

Range of services provided by the one million residential pools in California

From Yue Chen’s thesis [3]
Network Tomorrow: New Balancing Resources
Demand Dispatch & Virtual Energy Storage

Point of view at UF/Inria

Local Intelligence at each load

Mean-field model of large population of loads
Aggregate dynamics: passive, predictable input-output system
Network Tomorrow: New Balancing Resources
Demand Dispatch & Virtual Energy Storage
DER Flexibility Assessment & Valuation

Ongoing GMLC project – PNNL/ORNL/UF

Virtual Battery-Based Characterization and Control of Flexible Building Loads Using VOLTTRON

<table>
<thead>
<tr>
<th></th>
<th>Energy Arbitrage $/year</th>
<th>Regulation Up $/year</th>
<th>Regulation Down $/year</th>
<th>Spinning Reserve $/year</th>
<th>Total $/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siskiyou</td>
<td>10,983</td>
<td>150,501</td>
<td>25,651</td>
<td>2,559</td>
<td>189,696</td>
</tr>
<tr>
<td>San Diego</td>
<td>1,534</td>
<td>11,764</td>
<td>42,447</td>
<td>0</td>
<td>55,746</td>
</tr>
</tbody>
</table>

Value in Siskiyou vs San Diego
Conclusions

Today: managing the grid is an enormous distributed control problem

The future: new communication and control architectures are required

Questions:

- Will frequency remain the global information signal?
- What is the impact of further increased decentralized resources? (power and storage)
- How will markets evolve to provide incentives for zero marginal-cost resources?
Thank You
References

Control Techniques for Complex Networks
Sean Meyn

Markov Chains and Stochastic Stability
S. P. Meyn and R. L. Tweedie
Selected References I


Selected References II


Selected References III


Frank J. Gilloon Professor of Computing & Mathematical Sciences and Electrical Engineering

Autonomous Grid

Steven Low, Caltech
Autonomous Grid

Steven Low

Caltech
Watershed moment

Energy network will undergo similar architectural transformation that phone network went through in the last two decades to become the world’s largest and most complex IoT.

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1876</td>
<td>Bell: telephone</td>
</tr>
<tr>
<td>1888</td>
<td>Tesla: multi-phase AC</td>
</tr>
<tr>
<td>1969</td>
<td>DARPAnet</td>
</tr>
<tr>
<td>1980-90s</td>
<td>deregulation started</td>
</tr>
<tr>
<td>1980-90s</td>
<td>convergence to Internet</td>
</tr>
</tbody>
</table>

Both started as natural monopolies, both provided a single commodity, both grew rapidly through two WWs.
**Risk:** active DERs introduce rapid random fluctuations in supply, demand, power quality increasing risk of blackouts

**Opportunity:** active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency

**Caltech research: distributed control of networked DERs**

- Foundational theory, practical algorithms, concrete applications
- Integrate engineering and economics
- Active collaboration with industry

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Autonomous Energy Grids

optimized for secure, resilient and economic operations

Ben Kroposki
NREL workshop 2017

https://www.nrel.gov/grid/autonomous-energy.html

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Autonomous grid

Computational challenge
- nonlinear models, nonconvex optimization

Increased volatility
- in supply, demand, voltage, frequency

Scalability challenge
- billions of intelligent DERs

Limited sensing and control
- design of/constraint from cyber topology

Incomplete or unreliable data
- local state estimation & closed-loop system identification

Data-driven modeling and control
- real-time learning at scale

many other important problems, inc. economic, regulatory, social, ...
Autonomous grid

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many other important problems, inc. economic, regulatory, social, ...
Autonomous energy grid

Two examples as illustration
- dealing with nonconvexity
- dealing with volatility

... close with a research challenge in each
Optimal power flow (OPF)

OPF is solved routinely for

- state estimation, stability analysis, topology reconfiguration
- generator commitment and dispatch
- pricing electric services
- at timescales of mins, hours, days, ...

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (linear program)
- Also: Newton-Raphson, interior point, ...
Relaxations of OPF
dealing with nonconvexity

Bose (UIUC)  Chandy  Farivar (Google)  Gan (FB)  Lavaei (UCB)  Li (Harvard)

many others at & outside Caltech ...

Low, Convex relaxation of OPF, 2014
http://netlab.caltech.edu

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Optimal power flow

\[
\begin{align*}
\text{min} & \quad \text{tr} \left( CVV^H \right) \\
\text{over} & \quad (V, s, l) \\
\text{subject to} & \quad s_j = \text{tr} \left( Y_j^H VV^H \right) \\
& \quad l_{jk} = \text{tr} \left( B_{jk}^H VV^H \right) \\
& \quad s_j \leq s_j \leq \bar{s}_j \\
& \quad l_{jk} \leq l_{jk} \leq \bar{l}_{jk} \\
& \quad V_j \leq |V_j| \leq \bar{V}_j
\end{align*}
\]

gen cost, power loss
power flow equation
line flow
injection limits
line limits
voltage limits

Challenges
1. Nonconvexity: Kirchhoff’s laws (\( Y_j^H \) not positive semidefinite)
2. Volatility: time-varying optimization
Optimal power flow

Ian Hiskens, Michigan

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Dealing with nonconvexity

Linearization

– DC approximation

Convex relaxations

– Semidefinite relaxation (Lasserre hierarchy, ....)
– QC relaxation (van Hentenryck, Michigan)
– Strong SOCP (Sun, GATech)

Realtime OPF

– Online algorithm, as opposed to offline
– Also tracks time-varying OPF
Semidefinite relaxation

OPF: \[ \min_{x \in X} f(x) \]

relaxation: \[ \min_{\hat{x} \in X^+} f(\hat{x}) \]

If optimal solution \( \hat{x}^* \) satisfies easily checkable conditions, then optimal solution \( \hat{x}^* \) of OPF can be recovered.
Is OPF really hard?

For tree networks, sufficient conditions on

- power injections bounds, or
- voltage upper bounds, or
- phase angle bounds

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Is OPF really hard?

For **tree** networks, **sufficient** conditions on

- power injections bounds, or
- voltage upper bounds, or
- phase angle bounds

For **mesh** networks: observations

- no guarantee for general mesh networks (complexity: NP-hard)
- yet, relaxations often exact for practical networks
- ... and local algorithms often produce global solutions
- Do practical networks have special structure that make OPF easy?
Realtime OPF
dealing with volatility

Gan (FB)  Tang (Caltech)  Dvijotham (DeepMind)

Gan & L, JSAC 2016
Tang et al, TSG 2017

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Realtime OPF

\[
\min_{x \in X_i} f_i(x, y(x); \mu_i)
\]

Quasi-Newton algorithm:

\[
x(t+1) = \left[ x(t) - \eta (H(t))^{-1} \frac{\partial f_i}{\partial x}(x(t)) \right]_{X_i}
\]

\[
y(t) = y(x(t))
\]

Realtime OPF

$$\text{error} := \frac{1}{T} \sum_{t=1}^{T} \left\| x^{\text{online}}(t) - x^*(t) \right\|$$

**Theorem:** tracking performance

$$\text{error} \leq \frac{\epsilon}{\sqrt{\lambda_m / \lambda_M}} \cdot \frac{1}{T} \sum_{t=1}^{T} \left( \left\| x^*(t) - x^*(t-1) \right\| + \Delta_t \right)$$

- rate of OPF drifting
- approximation of Hessian
- conditioning of Hessian

Tang, Dj, & Low, TSG 2017
Learning + control?

New challenges:
Strategic agents (human, organizations) in the loop hard to model
Learning + control?

Closed-loop ID+state est+control

\[(u^t, y^t) \mapsto (\hat{f}_t, \hat{g}_t, \hat{x}(t), u(t))\]

Control \(u(t)\)

Measurement \(y(t)\)

Network model

\[x(t+1) = f(x(t), u(t), w(t))\]

\[y(t) = g(x(t), u(t), w(t))\]

New challenges:
Strategic agents (human, organizations) in the loop hard to model

Minimize

\[\frac{1}{T} \sum_{t=1}^{T} ||x^{\text{online}}(t) - x^*(t)||\]

Classical joint identification and control

Astrom & Wittenmark 1971; Gevers & Ljung 1986; Gu & Khargonekar 1992; ...
Learning + control?

**Network model**

\[
\begin{align*}
x(t+1) &= f(x(t), u(t), w(t)) \\
y(t) &= g(x(t), u(t), w(t))
\end{align*}
\]

- Closed-loop ID + state est + control
  \[(u^t, y^t) \mapsto (\hat{f}_t, \hat{g}_t, \hat{x}(t), u(t))\]

**Optimization**

\[
\min \frac{1}{T} \sum_{t=1}^{T} \left\| x^{\text{online}}(t) - x^*(t) \right\|
\]

**How to integrate new tools?**

- Statistical learning theory; advances in ML
- Learning high-dim data
- Diversity of data
- Algorithms & computing power

**Classical joint identification and control**

- Astrom & Wittenmark 1971; Gevers & Ljung 1986; Gu & Khargonekar 1992; ...

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Mathematics of the Electric Grid

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