Sterilisation limits for sample return planetary protection measures

TN 19 Description of the sterilisation statistical model

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CHANGE RECORDS

ISSUE	DATE	§ CHANGE RECORDS	AUTHOR
Draft A	3/2/16	Document Initiated	DJS
Draft B	25/2/16	Updated Mars Ejecta to a rotating frame	DJS
Draft C	24/8/17	Draft sent for review by the team	DJS
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		Added to section 3.10 and 4.9 radiation inactivation on how time duration on the surface is generated	
		Updates through section 4.6 to accommodate confidence levels on heat inactivation.	
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1. EXECUTIVE SUMMARY

This document gives the derivation of the model used for mapping the sterilization measurements performed as part of the Sterlim project, onto the martian process where material is transferred from the planet to its two moons, Phobos and Deimos.

The model is built up of a Monte Carlo simulation, where the distributions for the various processes are modelled. By building a Monte Carlo the variations in processes are integrated over, and so the simulation is sensitive to those variations.

The work has been performed in two phases:

- The original simulation, which considered material ejected from Mars at a rate averaged over the last 10 Million Years, and so gives the averaged behaviour.
- An extension to the simulation, under a CCN, which covered discrete ejections from Mars, based on the known crater rate on Mars.

In both phases the derivation of the simulation has started from known references, used to define theoretical models of the various processes. These models have been transformed into requirements against which the numerical model can be compared. The requirements have been used to build a C code model of the process, and each requirement tested; both that the requirement has been coded, and that the resulting model matches the theoretical distribution on which it is based.

This brings high confidence that the numerical model derived is an accurate representation of the references fitted.

The processes fitted in each phase:

- Phase 1 – averaged ejections from Mars
  - Ejection from Mars by a power law, both velocity and mass
  - Orbital transfer to Phobos/Demios using Newtonian laws
  - Probability of collision with Phobos/Demios given by volume of phase space, depending on the velocity at the moon
  - Collision with moon, associated sterilization due to heat, and the possibility of it being ejected
  - When ejected, chance of it staying in orbit about Mars, and eventual re-collision with the moon
  - Eventually settling on the moon, deposited at depth
  - The radiation environment at depth causes sterilization to the current time
- Phase 2 - discrete ejection from the creation of Martian Craters
  - The density of craters against time on Mars from martian isochrones, and so the frequency of ejection against size
  - The typical speed of impact causing the crater, assuming impactor originates from the asteroid belt
  - The size of the impactor, dependent on the size of crater and velocity of impact
  - The distribution of mass ejection against velocity for a crater
  - The density of mass in phase space at the orbit of the moon
  - The mass that collides with the moon

Thanks

This document has been assembled with thanks to the following people

Person	Institution	Area
Matt Balme	Open University	Mars ejecta References
Bill Hartmann	PSI	Mars crater isochrones
Jay Melosh	Purdue University	Mass ejection modelling

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2. INTRODUCTION

2.1 Scope

This document describes the development of the sterilisation statistical model to evaluate the probability that an unsterilized Martian material naturally transferred to Phobos is accessible to a Phobos Sample Return mission.

The concepts behind the model were explained in Phase 1 of the project [RD1], needing modelling of the process through which material is naturally transferred from Mars to the Martian Moons, as well as how the process sterilizes material transferred:

- The Hyper Velocity impact of material on Martian Moons giving rise to flash heating, and sterilization.
- The radiation environment of the Martian Moon, and the extended period transferred material spends on the moon pre collection.

This document develops the model:

- the algorithms needing implementation,
- the requirements that define the model,
- and the verification of model

It does not document the results of the model, which are detailed in TN21.

This work has been performed in ESA Contract number: 4000112742/14/NL/HB.

2.2 Applicable Documents

[AD1] “Sterilisation limits for sample return planetary protection measures – Statement of Work” ESA-SRE-F-ESTEC-SOW-2015-00 Issue 1
[AD2] Material Transfer from the Surface of Mars to Phobos and Deimos, Final Report: NNX10AU88G, H. J. Melosh, Purdue University, 2011.
[AD5] Mars impact ejecta in the regolith of Phobos:Bulk concentration and distribution. Kenneth R. Ramsley , James W. Head III. Planetary and Space Science 87(2013)115–129.

2.3 Reference Documents

[RD1] “Statistical Analysis Issue 2 Rev 2” David Summers SterLim-Ph1-TAS-TN-08 Issue 2-2
[RD2] “TN06 - Test and simulation plan” The SterLim Team, SterLim-OU-TN-06-TestPlan_Iss0_3
[RD3] “Hypervelocity impacts on dry and wet sandstone: Observations of ejecta dynamics and crater growth” Tobias HOERTH, Frank SCHAFER, Klaus THOMA, Thomas KENKMANN, Michael H. POELCHAU, Bernd LEXOW, and Alexander DEUTSCH, Meteoritics & Planetary Science 48, Nr 1, 23–32 (2013).
[RD4] “SterLim Project: Radiation Simulation Analysis Results (WP 5400)” Issue 0.c/Rev0, Pete Truscott, SterLim-PH2-KC-TN-0016 & KALLISTO/TN/16019.
[RD5] “TN15 - Test report on the irradiation inactivation tests results”, SterLim-OU-TN-15.
[RD6] “TN18 – Hypervelocity Impact Modelling” David Evans. SterLim-PH2-FGE-TN-0018.
[RD7] “Dynamical erosion of the asteroid belt and implications for large impacts in the inner Solar System”, David A. Minton & Renu Malhotra, Icarus 207 (2010) 744.

[RD8] “Exogenic Dynamics, Cratering and Surface Ages”, B. A. Ivanov & W. K. Hartmann.
[RD9] “Cratering saturation and equilibrium: A new model looks at an old problem” James E. Richardson, Icarus 204 (2009) 697–715.
[RD10] “Global Surface Modification Of Asteroid 4 Vesta Following The Rheasilvia Impact” Timothy J Bowling, PhD Thesis, Purdue University.
[RD11] “Martian cratering 8: Isochron refinement and the chronology of Mars” William K. Hartmann, Icarus 174 (2005) 294–320.
[RD12] Martian cratering 11. Utilizing decameter scale crater populations to study Martian history” William K. Hartmann & I.J. DauBar. Meteoritics & Planetary Science 1–18 (2016).
[RD13] Bill Hartmann, Private Communication.

3. ARCHITECTURE

3.1 Intro

The basic architecture was derived in [RD1]. Specifically it closely follows the process through which material is transferred from Mars to a moon, and the sterilization that happens during those processes.

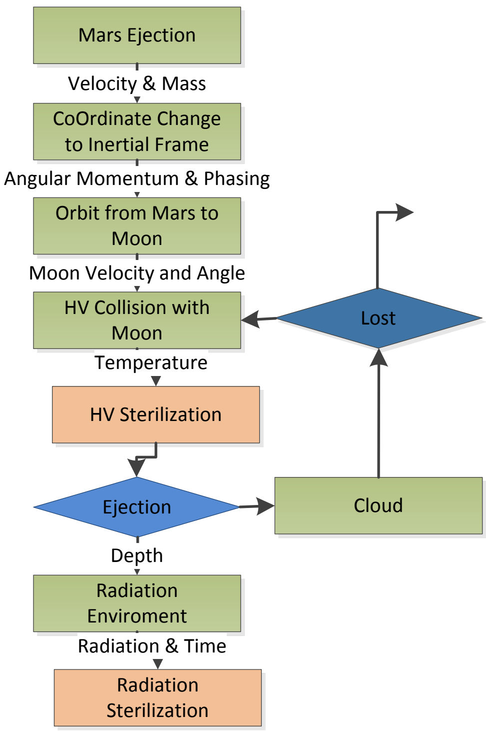


Figure 3-1. The process through which material is transferred from Mars to Its Moons, and the sterilization caused. The model closely follows the same architecture.

3.2 Process Modelling – The Monte Carlo Technique

Phase 1 [RD1], established Monte Carlo Integration as the method of probing the multi parameter phase space of the problem of how Martian eject ends up in the Phobos Regolith.

This give several issues:

- How do we trace material in modelling in its transfer from Mars to Phobos, and its time on Phobos. This is performed by tracing mass transfer.
- For rigour, the Monte Carlo is implemented as an integration. This means that all properties described need to be as distributions which are integrated over.
- The sterilization of a specific organism needs to be followed

These are discussed in following sections.

3.3 Mass

To follow progress of material through the process of transition from Mars to Phobos, mass is followed. Specifically each loop through the Monte Carlo is envisaged as following the progress of a portion of the total mass transferred. Each iteration will take that portion of the mass, and aim to take it randomly through the full parameter space of the variable under consideration. As an example, consider a hyper velocity collision with Phobos, and the resulting distribution of ejecta speeds.

Firstly of the mass that impacts Phobos, only a certain fraction is ejected. This fraction is modelled via the Monte Carlo – so specifically in some of the Monte Carlo iterations, mass is ejected, in other iterations that mass is deposited on Phobos. This is performed exactly with the fraction fitted from the model (so with no approximation). Now for the ejected material, to model the ejected velocity (v)– firstly the modelling needs to produce the velocity distribution of the ejected mass, e.g. $\frac{dm}{dv}$. Now the total mass ejected can then be found from the integral:

$$m = \int_0^{v_{max}} \frac{dm}{dv} dv$$

And this is the Integral over which the Monte Carlo is performed. Key to this integral is the form of:

$$\frac{dm}{dv} = F(v, \theta, v_{impact}, \theta_{impact})$$

This introduces variables, v and θ, the velocity and angle of the ejected mass. In general these variables will be coupled, so can write:

$$\frac{d^2m}{dvd\theta} = G(v, \theta, v_{impact}, \theta_{impact}) \neq H(v, v_{impact}, \theta_{impact})K(\theta, v_{impact}, \theta_{impact})$$

Now having coupled variables such as this doesn’t stop the Monte Carlo integral method:

$$m = \int_0^{v_{max}} \int_0^{\pi/2} \frac{d^2m}{dvd\theta} dvd\theta$$

However it will typically not be possible to make changes of variables that exactly flatten this integral. Each routine will typically not produce the total mass of impact on Phobos, instead it follows the evolution of mass flow. Hence each routine is normalised to the total mass. E.g.:

$$1 = \frac{1}{m} \int_0^{v_{max}} \int_0^{\pi/2} \frac{d^2m}{dvd\theta} dvd\theta$$

This means that for each Monte Carlo event, the event weight will give the weighting by which the weight evolves. Where the change in variable exactly flattens the variable, the normalisation is such that the weight will be unity.

3.4 Selection vs Normalisation

With several process, e.g.:

- Ejection of Martial from Mars
- Impact with Phobos/Deimos

There is a hard cut off in one of the parameters because:

- Material ejected from Mars must have sufficient velocity to reach Phobos/Deimos
- Impact with Phobos/Deimos requires the correct phasing of the moon orbit and the ejection site

There are two ways to simulate this:

- Monte Carlo which produces events that cover the boundary, each event is tested for if it meets the required condition, where it does not its weight is set to zero
- Monte Carlo is configured to only produce events where events will meet the required condition, this decreases the size of the phase space, which means each event has lower weight

The first method has a sharp cut off at the boundary, and this typically increases Monte Carlo errors. However the second requires knowledge of the volume of phase space (effectively the total integral), and this is not always available. When using the first method it is desirable to ensure the Monte Carlo is designed such that the majority of samples lie inside any cut off, this increases the efficiency of the Monte Carlo.

For the second example, the phasing of the ejection and the phase of the orbit of the moon, under reasonable assumptions is totally random. Hence it can be assumed in simulation that the all events collide with the moon, if the volume of the moon in phase space can be related to the total volume of phase space. Each event is then weighted by the ratio of the moons phase space to the total phase space.

3.5 Sterilization

3.5.1 Thermal

At the end of Phase 1, sterilization during the HV impact was expected to be dominated via thermal effect. Thermal sterilization [RD5] was planned to be modelled by the formula:

$$\frac{1}{N}\frac{dN}{dt} = -k\big(T(t)\big) = -A\exp\left(-\frac{b}{T(t)}\right)$$

Where “A” gives the logarithmic kill rate at high temperature, and “b” which has units of temperature, gives the temperature where the kill rate turns off at low temperature.

This can be integrated to give:

$$\ln(\text{Life}) = -A\int dt\,\exp\big(-b/T(t)\big)$$

where the integral is over the process through which the organism proceeds, specifically the history of the organism, in this case the temperature. Life in this equation means the fraction of the original number of organism that survive exposure to the thermal shock. Now in our case we wish to apply this to a process, consider specifically a HV impact on Phobos from a Mars Ejecta. For the example there are two variables of interest:

- v_I – the velocity of the impactor
- v_E –the velocity of the ejected material

Key also is dm, the mass that we are following. Now where the organism has a fixed mass, as will be assumed in this study, following the mass becomes equivalent to following the organism.

Now for the Monte Carlo method the distribution that will be used is:

$$\frac{dm}{dv_E}(v_I, v_E)$$

This is the velocity distribution of mass with respect to the ejected velocity as a function of the impactor velocity (and the ejected velocity). Now how to fold sterilization into the process? Sterilization depending on temperature depends on much more than just the velocities of the impacted and ejected material. This means that sterilization builds on variables that the simulation does not model in the first place, as it does not affect the flow of material.

So for example the hyper velocity modelling will have to integrate over the volume of the impactor, the fate of mass on the front face being different from the rear face. In addition the temperature history of the different elements of the impactor will depend on many elements:

- Impact velocity
- Position in impactor
- Mass of Impactor
- Physical properties of the impactor
- Physical properties of the regolith
- Etc

Some of these variables will be known to the Monte Carlo, others just of interest to the Hyper Velocity Modelling; so how to account for the variables not of interest to the Monte Carlo. These variables are labelled θ in the equation below.

So to model sterilization of a particle through a full hyper velocity impact, with the sterilization applied to the flow of mass, which is proportional to the flow of organism:

$$\frac{1}{m}\frac{dm}{dv_E}\ln(\text{Life}) = -\frac{1}{m}\int d\theta dt \frac{d^2m}{dvd\theta} A\exp\left(-\frac{b}{T(t,\theta,v_E,v_I)}\right)$$

This corresponds to integrating the hyper velocity impact, over the variables relevant to the impact. The sterilization is calculated at each point of the process, but is weighted by the mass flow at that point (equivalent to the organism flow) – this mass flow is then scaled by the mass. Hence in both sides of the equation mass is scaled out, and so just used as a differential weighting for the sterilization. The process is left differential in the ejection velocity – this is the parameter which will flow into the rest of the Monte Carlo – and so is the responsibility for the Monte Carlo to integrate over. The internal variables of the hyper velocity collision are directly integrated (or averaged) over, this integral will typically be performed as part of the hypervelocity modelling, however if this proves not practical analytically or approximately – the Monte Carlo can perform the integral numerically.

Now once the integral has been performed over the internal properties of the hyper velocity collision, “A” and “b” may no longer be ideal variables through which to describe how the collision affects sterilization of an organism. Hence the Monte Carlo parameters used to describe sterilization can only be decided in combination with the Hyper Velocity Modelling.

3.5.2 Radiation

3.5.2.1 Organisms and Radiation

During the radiation test [RD2] the four organisms are tested with 3 radiation types:

- Gamma radiation
- Proton radiation
- Heavy Ions (Helium)

The absorbed dose of radiation is measured in Grays, characterising the sterilization of each of the organisms against dose in Grays.

The radiation inactivation modelling will fit a model to these measurements, as a simple illustration of the type of modelling, consider a similar kill model to the heat inactivation:

$$\ln(N) = -k_{OR} D_R$$

Where k is a constant that depends on both organism and radiation type. The dose (D) absorbed energy per mass, which as mass is the property followed in Monte Carlo gives the measure – in particular is proportional to the number of organism contained. With such a model k would need to be measured for each organism, and each radiation type. The dose is given by the modelling of moon environment.

3.5.2.2 The Phobos/Demos environment

The modelling if the moon environment provides the radiation dose. Specifically this is expected to be modelled as:

$$D_R(x) = \int_{t_0}^{\text{today}} dt \frac{dD_R}{dt}(x,t)$$

Where $D_R(x)$ is the dose of type radiation R, measured at depth of x. dD_R/dt is the rate at which the dose accumulates, this is a function of time, and has the potential to have varied in the past; hence the model integrates the rate from the time of arrival t_0 to the present day.

4. PROCESS REVIEW AND ALGORITHMS

4.1 Mars ejecta

4.1.1 Mars Ejecta modelling

Several parameters are relevant to the ejection of material from Mars:

- What is the total mass ejected
- What is the mass in each ejecta
- What is the velocity of ejection
- The angle at which the object is ejected

These are needed becuase:

- Total mass, gives the scale of material that impacts Phobos. So this parameter is of direct relevance to the amount of Martian material deposited on a moon.
- The mass of each ejecta, gives the mass of the Phobos impactor. So where the properties of the hyper velocity impact depend on mass of the impactor, this knowledge is needed.

- The velocity of ejection is primarily of importance for if the ejecta reaches the orbit of Phobos. It also has the effect that when the velocity is sufficient to reach Phobos, but below the escape velocity for Mars, the ejecta will cross the Phobos orbit twice, once on the way out, and a second time on the return to the Martian surface.
- The angle of ejection affect how the ejection velocity is split between radial and angular velocity – this has a secondary effect in any collision with Phobos. Hence is of lower priority

4.1.2 Total Mass and Velocity modelling

The total mass of ejection, in relation to velocity, is controlled by a power law. The guiding formula is taken from [AD2]

$$F_m(v) = m_{tot}\left(1 - \left[\frac{v}{v_{co}}\right]^{-\gamma}\right) \tag{V.11}$$

This can be interpreted as the total mass ejected between the velocities v_{co} and v; where m_{tot} is the mass ejected with velocity greater than v_{co}. Now for the modelling this is rewritten as the mass ejected with velocity greater than minimum velocity needed to reach Phobos:

$$\int_{v_{min}}^{\infty} \frac{dF_m}{dv} dv = m_{tot}\left[\frac{v_{min}}{v_{co}}\right]^{-\gamma} \equiv m_{min} \quad [1]$$

This is useful, as it shows that the "co" cut off point is just used to define the point at which m_{tot} is defined, but that it can be redefined for a physical point – such as the ejection speed necessary to reach Phobos. This is necessary, as the model needs to be extended to cover Deimos – and in particular it relates how the total mass ejected from Mars varies with its ability to reach Phobos or Deimos. If we now consider the mass fraction:

$$\frac{1}{m_{min}}\int_{v_{min}}^{\infty} \frac{dF_m}{dv} dv = \int_{v_{min}}^{\infty} \frac{m_{tot}}{m_{min}} \gamma \frac{v^{-\gamma-1}}{v_{co}^{-\gamma}} dv = \int_{v_{min}}^{\infty} \gamma \frac{v^{-\gamma-1}}{v_{min}^{-\gamma}} dv = \int_0^1 d\left(\left(\frac{v}{v_{min}}\right)^{-\gamma}\right) \quad [2]$$

This last equation is a completely flattened integral between 0 and 1 and is exactly what is required for an exact Monte Carlo. Specifically if X is a flat random variable between 0 and 1 we set:

$$\left(\frac{v}{v_{min}}\right)^{-\gamma} = X \quad [3]$$

$$v = \frac{v_{min}}{X^{1/\gamma}} \quad [4]$$

And the total mass ejected in this range is given by equation [1]. Ref [AD2] uses γ=1.5 – which is chosen here also.

Now this still leaves how to establish m_{tot} at the v_{co} velocity. As the main object of study is the ejecta material bound for Phobos, a sensible minimum velocity is the minimum velocity to reach Phobos. This is approximately v=3.8km/s, so this is chosen:

$$v_{co} = 3{,}800\ m/s \quad [5]$$

Leaving only m_{tot} to be established, this should be established from normalization. Specifically ref [AD2] gives the total mass impacting Phobos in 10Myears as 1.1217e6kg. Hence m_{tot} will be tuned to give this value, which requires turning over several models (Mars ejection, transit to Phobos, Impact with Phobos).

4.1.3 Ejecta Mass Modelling

4.1.3.1 Mass distribution

Modelling the Ejecta Mass distribution is the most uncertain aspect of Mars Ejecta modelling. To quote from [AD2]:

> However, estimation of the size of a particle is possibly the most uncertain parameter… In the current understanding of ejecta size distribution from observation of lunar boulders [Ba], the cumulative count of particles as a function of the particle size follows an inverse power law with an exponent varying around a value of 4. Defining a cut-off for the power law, which corresponds to a lower bound for the particle size distribution, $d_{min}=1\times10^{-6}$m, such a law can be exploited to generate a random sample of particles. However, because of the very steep character of the law, $\gamma\approx4$, the mean particle size as computed from a random sample consistent with this power law, is approximately equal to the minimum particle size.

The Ejecta size, becomes the impactor size on the Martian Moons. The size of the Impactor in turn affects the size of the crater created by the impact. Typically the larger craters are the deeper craters, and this in turn has the potential to despot material at greater depth. The depth of material in turn affects the exposure to radiation during the material's stay on the moon.

Hence the potential for the sterility of Martian material is sensitive to Mars Ejecta Mass distribution, which in turn is uncertain.

So whilst the consensus is that the mass distribution is a steep falling power law, with exponent about 4 this has problems:

- As noted above, this puts nearly all material at the minimal size
- The minimal size is not well defined, presumably the physicals of the hypervelocity collision, however violent, will still generate a minimal size
- The finer the material, the greater the drag when passing through the Martian atmosphere, this will term to remove the finer materials from the transfer

How is best to resolve this issue is not clear, however it is not the main focus of this study – where the attention is mainly on the process on the Martian Moon.

Hence what is proposed is modelling with the strong power law, however taking a range of minimal cut offs, varying this over a wide range. The model will be run for each of these, to test sensitivity of sterilization to ejecta mass.

Now [AD2] described the ejecta size in terms of the power law in diameter, here we model size as the mass of the ejecta. These are related via:

$$m = \frac{\pi\rho}{6}D^3$$

The differential form of this equation becomes:

$$\frac{dD}{D^4} = \frac{\pi\rho dm}{18m^2}$$

So a γ=4 power law in diameter, becomes a γ=2 power law in mass.

Now the power law will be from some minimum mass, m_{min} and normalised:

$$m_{min} \int_{m_{min}}^{\infty} \frac{dm}{m^2} = \int_{m_{min}}^{\infty} d\left(\frac{-m_{min}}{m}\right) = {}_{m_{min}}^{\infty}\left[\frac{-m_{min}}{m}\right] = 1 = \int_{0}^{1} dX$$

Where:

$$X = \frac{m_{min}}{m}$$

This is modelled in a Monte Carlo via:

$$m = \frac{m_{min}}{X}$$

Where X is a uniform random variable in the range [0:1].

4.1.3.2 Ejection Cone

Hyper Velocity impacts tend to ejecta material in a cone [RD3]. The cone angle varies dependent on the properties of the materials (e.g. porosity), and typically evolves during the hyper velocity collision [RD3].

In the context of the Mars ejecta, this means that although most impacts will have a cone, the exact angle of the cone, and the evolution of the cone, is unclear.

Now the angle of ejection affects the dynamics of the trajectory of emitted material. Steep cone angles create material with little angular momentum (about Mars), and such particles can take a direct route to Phobos. Shallow cones on the other hand have much more angular momentum about Mars, and this means higher ejection velocities are required to reach the Martian Moons.

Hence for this study it is decided to perform a sensitivity analysis on the angle of ejection. Specifically several fixed angles are taken, and a couple of ranges:

- 30°
- 45°
- 60°
- Cone centred on 45° with a normal distribution in angle with standard deviation of 15°
- Cone centred on 60° with a normal distribution in angle with standard deviation of 15°

For the latter two when angles are generated outside the physical range [0:90°] the algorithm will be rerun, which cuts the distribution off at the physical limit.

4.2 Mars Rotation

The ejection of material from Mars will be in the frame where Mars is at rest, however Mars rotates with a period of 1.025957 Earth Day (= 88525.07s). This rotational speed will add to speed with which ejecta is expelled. The rotational speed, at a latitude of θ is given approximately by:

$$v = \frac{2\pi r_M}{T_M} \cos(\theta)$$

This adds in quadrature to the tangential speed of the ejecta in a random fashion ($0 \leq \varphi \leq 2\pi$):

$$v_{tangential,orbital} = v_{tangential,ejection} + \frac{2\pi r_M}{T_M} \cos(\theta) \sin(\varphi)$$

The radius of Mars is taken as a constant r_M=3389.5km, this gives a maximum speed increase of 240m/s.

4.3 Transfer to Phobos/Deimos

After ejection from Mars, the first question is does the ejecta reach the orbit of Phobos/Deimos.

This is most easily considered in a frame rotating at the same speed as Phobos about Mars, and centred on their centre of gravity.

The Hamiltonian for a object (ejecta) moving in this frame is given by:

$$H = m(\dot{x}^2 + \dot{y}^2)/2 - m(x^2 + y^2)\dot{\theta}^2/2 - Gm(m_M/d_M + m_P/d_P) \quad [6]$$

Is a conserved quantity. It has the usual kinetic term, and the potential term:

$$H_V = -m(x^2 + y^2)\dot{\theta}^2/2 - Gm(m_M/d_M + m_P/d_P) \quad [6]$$

Where the rate of rotation is given by

$$\dot{\theta}^2 = G\,(m_M + m_P)/d^3 \quad [6]$$

Plotting the equipotential of the Hamiltonian, for Mars Phobos system about Phobos is shown in Figure 4-1.

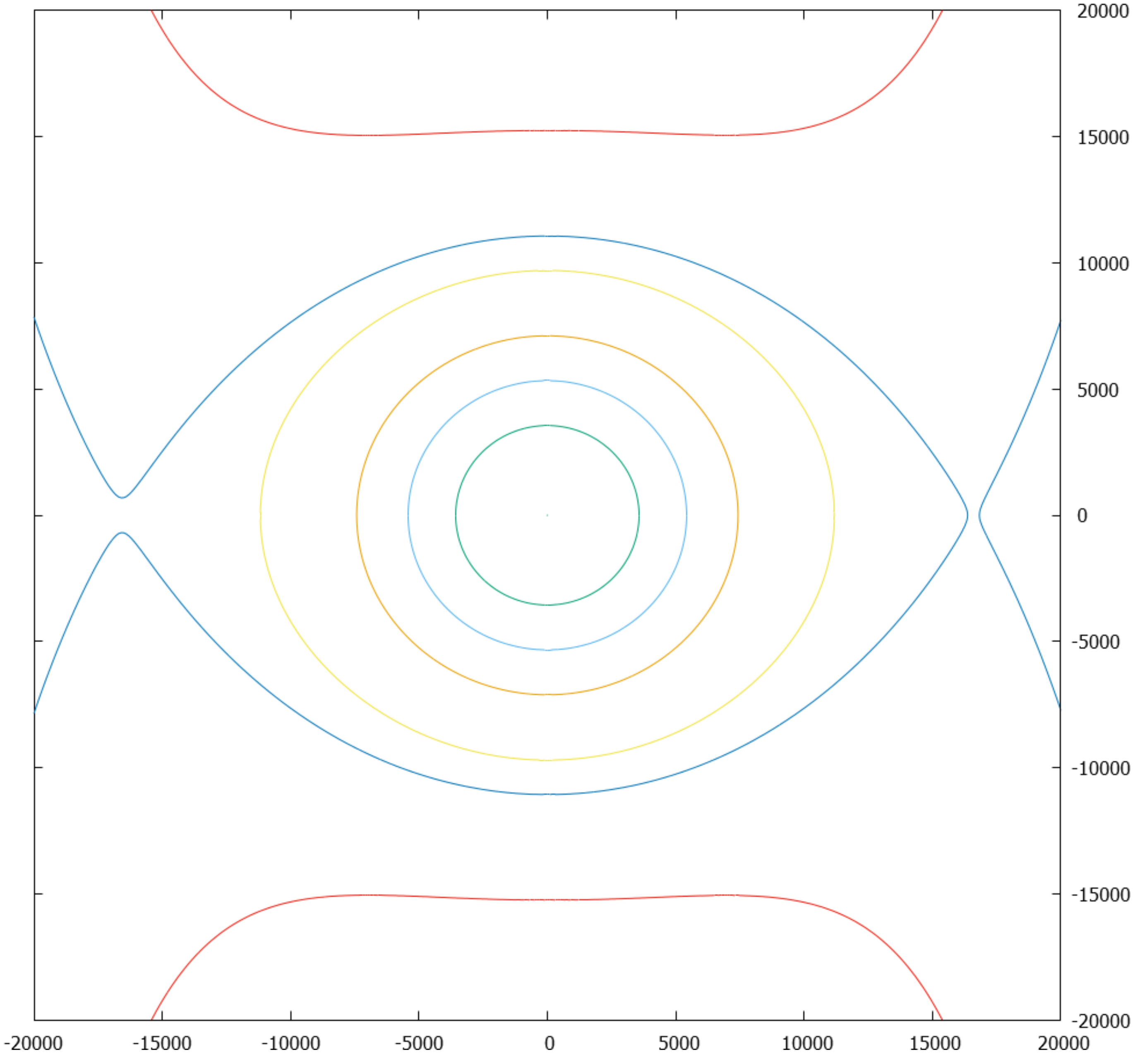


Figure 4-1. Equipotentials of the Mars-Phobos system about Phobos. Mars is 9376km to the left. The L1 Lagrange point can be seen to the left (~-17km), and the L2 to the right (~17km). Phobos occupies approximately the yellow ellipse.

The extent of space where Phobos gravity dominates, is known as the Hill Sphere, and it's outer extent is given by the Largange points. For a system where one mass dominates the other theradius of the hill sphere is approximated by:

$$r = d\left(\sqrt[3]{\frac{m_p}{3m_M}}\right) \qquad [6]$$

Which for the Mars Phobos system corresponds to r=16587m, which clearly correlates well with position of the Lagrange points.

Now for Mars ejecta to reach Phobos, it clearly has to pass over the L1 Lagrange point, that is the low point of the potential between Mars and Phobos. This gives a minimal velocity for an ejecta to reach Phobos. Now as Mars surface is approximately at an equipotential, this gives a lower limit for and ejecta to reach Phobos, reasonably independent of the launch position. For Phobos this gives 3488m/s, which is however the velocity in the rotating frame. In this frame the surface of Mars is rotating at 772m/s relative to a stationary Mars, so this escape velocity needs to be taken in context.

Next consider typical ejecta orbits from Mars to Phobos.

Consider two limiting cases to begin with, shown in Figure 4-2:

- The ejecta is launched vertically with no angular momentum
- The ejecta is launched horizontally with maximal angular momentum

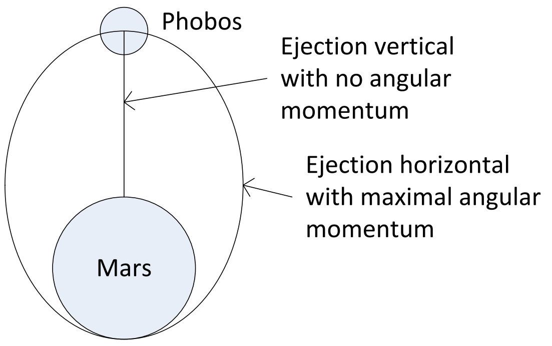


Figure 4-2. Ejection can happen at various angles, this affects angular momentum. How much effect does it have on the velocity?

Now as shown in Figure 4-2, in both cases it is possible to reach Phobos, but how does the required velocity vary. Specifically when there is no angular momentum it needs to be sufficient to reach Phobos:

$$GM\left(\frac{1}{r_{Mars}} - \frac{1}{r_{Phobos}}\right) = \frac{v^2}{2}$$

Whereas when launched tangentially:

$$\frac{GM}{r_{Mars}} = \frac{v^2}{2}\left(\frac{r_{Mars} + r_{Phobos}}{r_{Phobos}}\right)$$

This gives the various speeds needed to each Phobos/Deimos/Escape as:

	Radial velocity (m/s)	Tangential velocity (m/s)	@45° (m/s)
Phobos	4016.9[1]	4308.3	4154.9
Deimos	4649.8	4699.1	4674.2

[1] Note that this value is higher than found by Melosh in [AD2]. Difference not clear.

Escape	5027.0	5027.0	5027.0

This shows that the angle at which the ejecta is emitted has decreasing effect at increasing radius (and escape velocity does not depend on angle of emission). For Phobos the minimal ejection velocity varies by ~300m/s. This is a low level of variation, however with a power law for the ejection velocity 300m/s probably has a small, but measureable effect.

The implications are that a model for the angle of ejection is required, but the results are expected to be only mildly dependent. Hence a general expression is required for the dependence the minimal velocity to reach Phobos/Deimos this is given by:

$$2GM\left(\frac{1}{r_{Mars}}-\frac{1}{r_{Phobos}}\right)=v^2\left(\cos^2(\theta)+\sin^2(\theta)\left(1-\left(\frac{r_{Mars}}{r_{Phobos}}\right)^2\right)\right)$$

Where θ is the angle from vertical.
No similar the radial and tangential velocities at ejection and arrival at Phobos and Deimos need to be related. These are given by:

	Radial Velocity	Tangential velocity
Mars (ejection)	$v\cos(\theta)$	$v\sin(\theta)$
Phobos Orbit	$\sqrt{v^2-v^2\sin^2(\theta)\left(\frac{r_M}{r_P}\right)^2-2GM\left(\frac{1}{r_M}-\frac{1}{r_P}\right)}$	$v\sin(\theta)\ (r_{Mars}/r_{Phobos})$

This covers the propagation of orbital velocities from ejection to the Phobos/Deimos orbit.

4.4 Impact with Phobos/Deimos

4.4.1 Introduction

Consider the impact of a Mars ejecta with Phobos(Deimos). Firstly as the ejecta originates from Mars only three possibilities exist:

- The ejecta does not have sufficient velocity to reach Phobos/Deimos
- The ejecta has velocity to reach Phobos/Deimos, however does not have escape velocity. In this case it will fall back to Mars. Such ejecta twice passes through the orbit of Phobos/Deimos.
- The ejecta has velocity higher than the Mars ejection velocity, cross the moons orbit once, and then leaves the Martian system

And the Martian Ejecta cannot enter orbit about Mars.

So the three cases give different numbers of crossings of the Phobos/Deimos orbit:

- 0
- 2
- 1

And these will multiply the probability of collision with the moon.

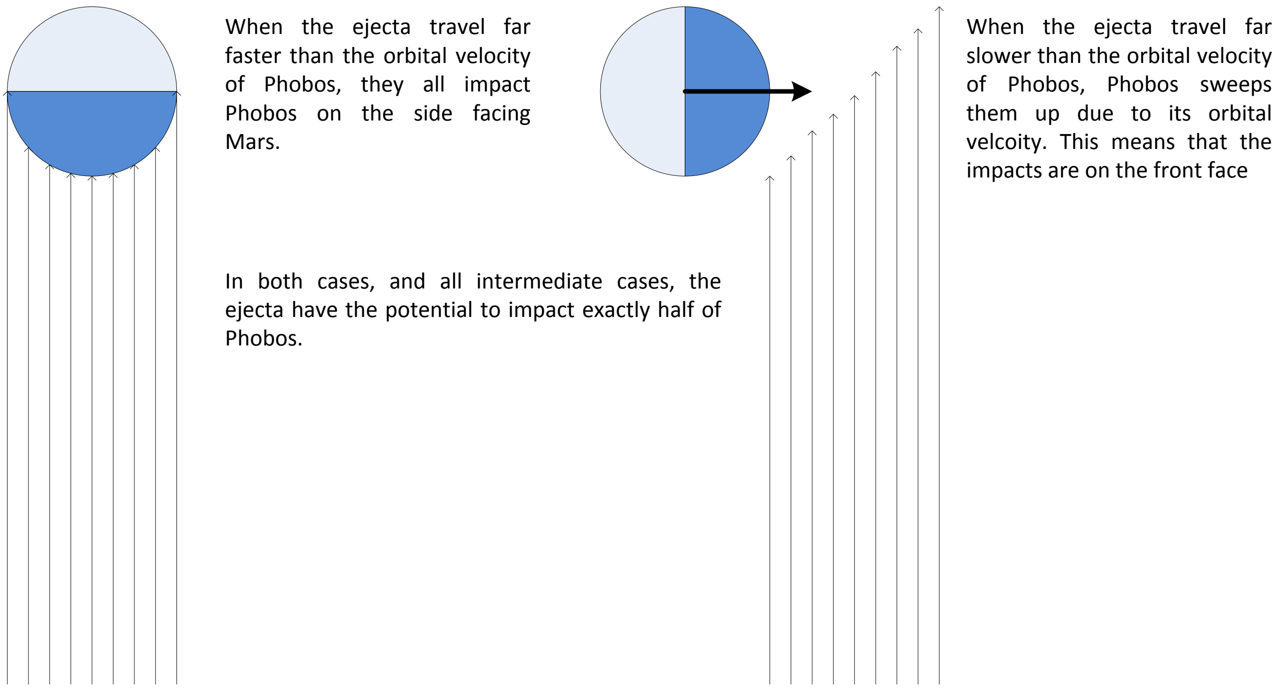


Figure 4-3. Depending on the velocity of ejection difference faces of Phobos receive the impact. However independent of the velocity, only 50% of the surface of Phobos can be impacted.

Consider next the cross section for collision with the moon, and how the velocity of the ejection affects the cross-section. Firstly from the position of Phobos/Deimos, as shown in Figure 4-3 – any velocity ejecta can hit exactly 50% of the Moon's surface, and this is independent of velocity. Fast ejecta flash past the moon, and can only collide with the side of the moon facing Mars. On the other hand ejecta with velocity just sufficient to reach Phobos reach the orbit with fairly slow speed, the orbital speed of Phobos then side swipes the ejecta, and the impact is mainly on the side face. This at this level suggests that the velocity of impact does not affect the area of collision (for a spherical moon), but a second interpretation is that fast ejecta cross the orbit of Phobos in a short period, whilst those on a slower velocity spend longer traversing the orbit of the moon, does this increase the probability of impact.

The approach taken here, is if we assume that ejecta are isotropically ejected from Mars, over what percentage of the Martian surface do the ejecta have trajectories that intersect Phobos. This is most easily considered in a rotating frame with Mars at the origin, and rotating with the period of rotation of the moon. In such a frame both Mars and the moon are stationary, but ejecta trajectories follow curved trajectories.

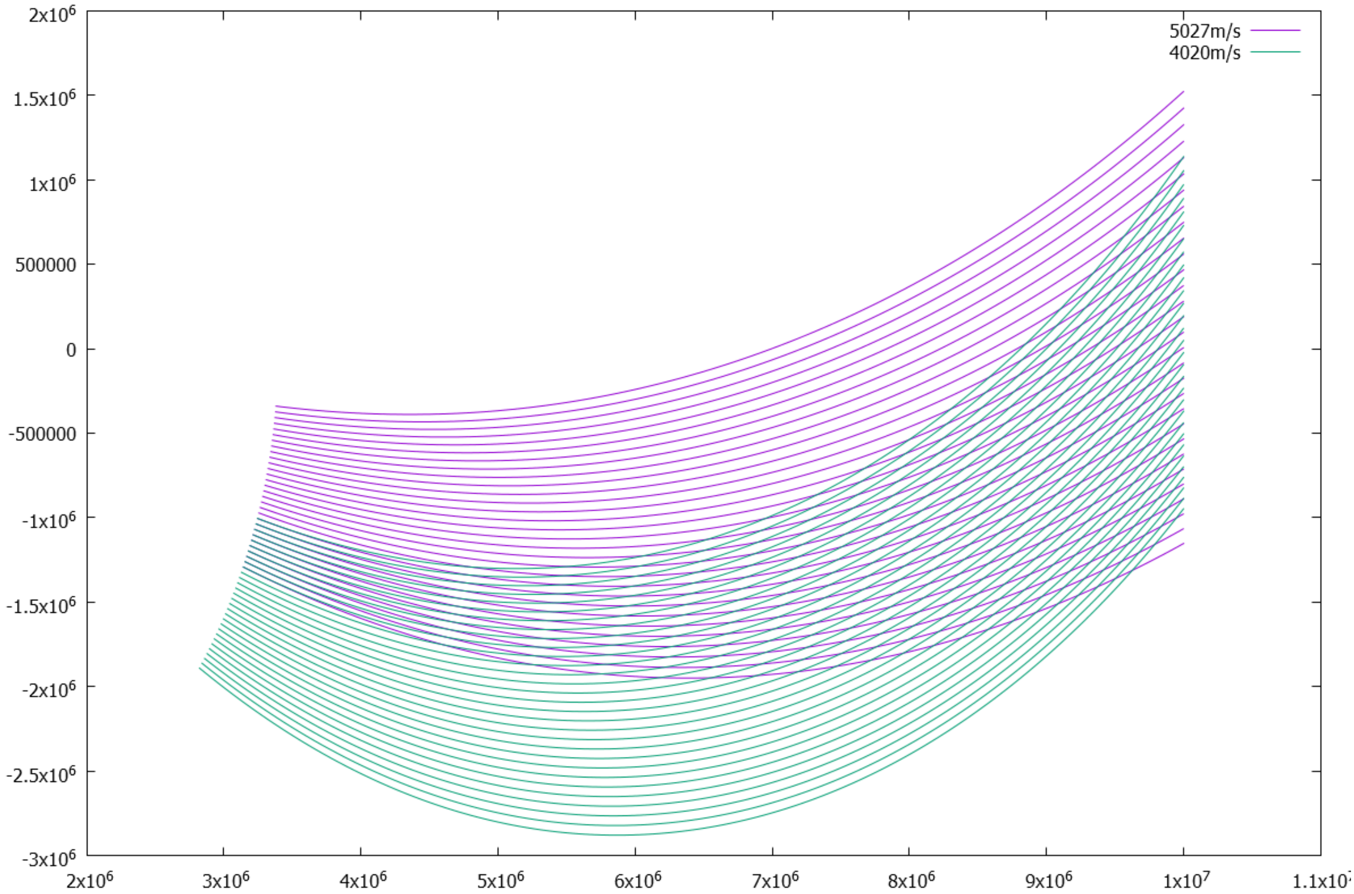


Figure 4-4. Two ejecta velocities, 4020m/s with just enough speed to reach Phobos, and 5027m/s the Martian escape velocity, plotted position x&y in a rotating frame (with Mars and Phobos stationary) both axis measured in meters.
The Mars-Phobos c.o.g. is at (0,0), Mars on the –ve x axis, and Phobos on the +ve x axis (at about 9235km). Plotted for a range of launch positions on Mars, separated by 0.01radians. The launch positions trace out the surface of Mars

This is shown in Figure 4-4, for two launch velocities, and a range of launch positions (all launches are vertical). The launch positions are separated by 0.01 radians, now Phobos is small enough that it does not significantly deviate the trajectories – and this means that at the Phobos orbit, the trajectories are again separated by 0.01 radians. However as the 4020m/s has been slowed, it has a large tangential velocity in relation to its radial velocity – this means that the trajectories as they impact Phobos are compressed – which leads to increased cross section.

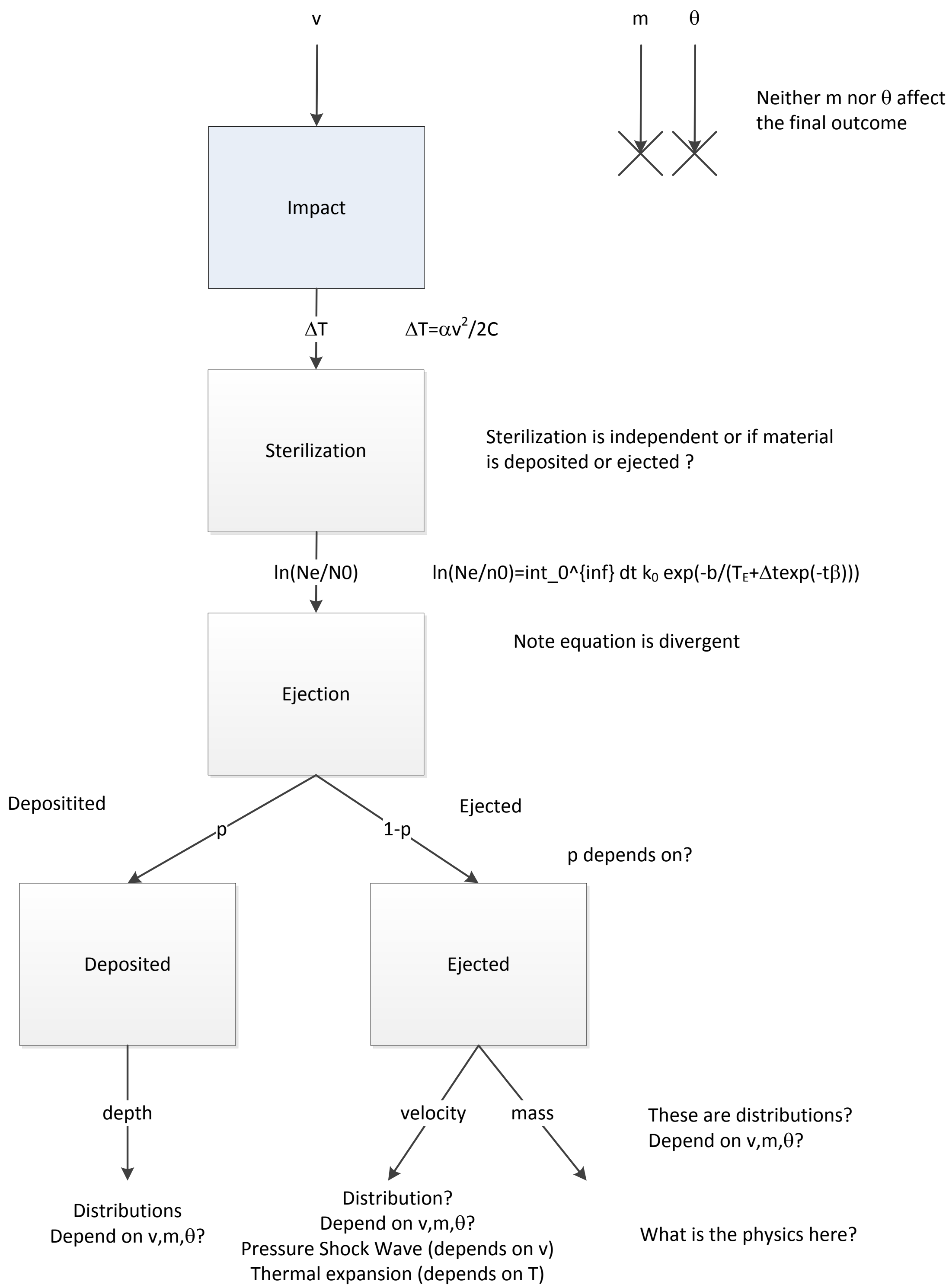
So how is best to model the compression of phase space, firstly assume that Phobos itself has little effect on the orbit, which given its low gravity and proximity of the surface to the Hill Sphere is reasonable.

Phobos is an approximate triaxial ellipsoid in shape with dimensions 27 km × 22 km × 18 km. It will orientate itself in the Martian gravitational field with its long axis in a radial direction (this being the most gravitationally stable orientation. The orientation in the other two axis is not clear, hence simplest is to model as a prolate spheroid with dimensions 27 km × 20 km × 20 km.

This spheroid can be mapped into the projection onto a sphere around Mars with the direction of the velocity vector of the impactor. The projection is an ellipse, and the size of the ellipse on the sphere around Mars gives the volume of the phase space that the moon sweeps out for that particular ejector, and this in turn gives the probability of impact.

4.5 Phobos Hypervelocity impact and ejector

The hyper velocity impact algorithm follows the basic process shown below.



Specifically the process splits into two parts [RD6]:

- The hypervelocity collision where kinetic energy is converted into thermal energy
- The thermal energy then sterilizes the lifeform. The temperature decays till no more sterilization occurs

Now of the information that feeds into the collision:

- Velocity, gives the kinetic energy per mass, $\frac{1}{2}v^2$
- The mass, to the first approximation, is not expected to have an effect
- The angle of impact,

Consider each in turn.

4.5.1 Kinetic Energy

The parameter of interest is the velocity which gives the kinetic energy:

$$\frac{K.E.}{m} = \frac{1}{2}v^2$$

The transfer of this kinetic energy to thermal energy is very dependent on the hyper velocity impact [RD6]:

- It depends on the velocity; above a certain velocity the impactor is fragmented into dust – and behaves like a liquid during the collision
- The position in the impactor; the front, and in particular the sides are heated more strongly than the internals of the impactor
- The structure of the impactor, material is accelerated into voids during the collision before filling the void – and being heated

This differential heating has been modelled in detail in [RD6]. It depends on two parameters:

- 'x' – how far towards the front of the impactor the point is, with larger values being towards the point of impact
- 'y' – how far the point is from the central line of the impactor

The azimuthal dependence about the central line, is not expected to affect the temperature.

Now the kinetic energy converted to thermal energy is not a fixed fraction, but varies at each point. In [RD6]§5.2 this is measured though the standard deviation – which is most simply modelled through a normal distribution:

$$\frac{dP}{dF} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(F - \langle F \rangle)^2}{2\sigma^2}\right)$$

Where F is the fraction of kinetic energy converted to thermal energy.

Whilst one would expect F to lie between 0 and 1 (e.g. no energy transfer, and total energy transfer, to heat), [RD6] indicates that F can have a value of over than 1 – as kinetic energy from distributed areas can heat in more localised area. Values for F below zero (which correspond to cooling) are not expected to happen. Hence if the normal distribution gives a value for F below zero, F will be moved to zero (so no heating occurs).

The velocity at which the impactor fragments into dust is given in [RD6] as 1250m/s and this is used as a simple binary switch for the table to read, where [RD6] splits the data.

'x' and 'y' distribution, is less simple. Organisms are assumed to be distributed by volume, now as 'x' and 'y' have to be rotated about the azimuthal direction the volume is given by:

$$dV = dx\, dy\;\; y\, d\theta = \; dx\, d(y^2)\; 0.5\; d\theta$$

Which is a flattened integral. Hence for uniform distribution in volume, 'x' and 'y^2' need to be generated uniformly, which in a Monte Carlo will uniformly integrate over volume. 'x' and 'y' are largely defined on a grid, but with occasional values that fell outside the impactor [RD6]. When this happens (rarely) that algorithm will just be rerun. This keeps the uniform distribution, which maintains the efficiency of the Monte Carlo; as it does not change the mass of the impactor, it does not change the weight of the Monte Carlo step – so this operation is safe.

The fraction of energy, F, converted to heat is generated in [RD6] on a grid in the two variables, the grid is Cartesian, a simple solution to interpolate x and y across this grid is bilinear interpolation. This will be performed for both the mean and the standard deviation. This will be used to form the initial temperature to which an organism is heated.

4.5.2 The angle of impact and ejection

[RD6] indicates that for angles of incidence between normal and 45° that all the impact material is deposited on the surface. At 45° some impact material is ejected, which increases with increased angle, till at 0° impact the whole impactor is ejected.

The simulations in [RD6] suggest that the ejected material undergoes little heating and so will not be sterilized.

The velocity of the ejected material is related to impact velocity. The vertical component of velocity is about 40% of the impact vertical velocity, and reversed in sign. The tangential component is about 76% of the impact tangential at 45° impact, increasing up to 100% at 0° incidence.

For a simple model this suggests a linear fit:

- For ejection/deposited probability:
    - 100% deposited at angles of incident greater than 45°
    - Rate of ejection increase linearly from 0% at 45° to 100% at 0°
- For ejected material
    - Vertical velocity is reversed and 40% of impact vertical velocity
    - Tangental increasing linearly from 76% at 45° to 100% at 0°

The model will need to know the orientation of impact about the moon, for breaking the velocity down into vertical and tangential. This can be performed by assuming a uniform random distribution – as the orientation depends sensitively on phasing, which when integrating over the ejection site on Mars will randomize.

4.5.3 Ejection Mass

The mass of ejection is not predicted by [RD6] however this mass mainly affects the chance of ejection when in the cloud about Mars. Hence two models will be used:

- M=0; corresponding to the material fragmenting to very small objects. This will mean that the material will be ejected from the Mars system in the cloud, and lost.
- M=%age change of ejection times the impactor mass. This corresponds to the ejected %age being ejected as a single object. This will lead to the object eventually setting on the moon, and deposited at greatest depth where radiation will be minimized.

So these two options cover the extremes of what could happen.

4.5.4 Depth Deposited

When deposited [RD6] indicates the most likely resting place is directly on the surface of the moon. Specifically this was observed during testing, no cratering was seen, suggesting the regolith "bounced back" after the impact. This is consistent with the HV modelling, although depends critically on the regolith properties – so the observation of no cratering takes priority. Now material deposited on the surface will be exposed to the full radiation environment – which is expected to sterilize the material.

Hence a second model is derived. During the HV modelling [RD6], the maximal depth the (remains) of the impactor reached was a few radii of the impactor, this was before bounce back. Now if the moon regolith does not bounce back, this would place material a few radii down. Over time the walls of the crater would collapse and cover the remains.

So a conservative model in that direction is to probe a depth of a variable number of radii up to a few of the impactor – this will test the sensitivity to this.

Specifically the model will allow setting how many radii the impactor is deposited at, and the testing will probe:

- 0 radii, the material is on the surface
- 1
- 2
- 3

To calculate the radii of the impactor, the formulas of section 4.1.3.1 will; be used.

4.6 Heat inactivation

Once an organism is heated, its temperature will decay back to ambient [RD6]:

$$T(t) = T_a + \Delta T e^{-\beta t}$$

Where the rate of cooling β depends on the organism. The temperature gives sterilization, which when integrated over time gives [RD6]:

$$S = \ln\left(\frac{N_e}{N_0}\right) = \int_0^{t_e} -k_0\, e^{-b/T(\tau)} d\tau$$

These two formulas can be combined to give the sterilization from an initial temperature:

$$S = \ln\left(\frac{N_e}{N_0}\right) = -\frac{k_0}{\beta}\int_{T_0}^{T_I} dT\, \frac{\exp\left(-\frac{b}{T}\right)}{T - Ta}$$

Where b, and k_0 are properties of the organism. Sterilization is assumed to stop when the organism reaches 50°C [RD6] – this cut off is needed, as otherwise total sterilization will occur, as the integral diverges at T=Ta – the ambient temperature. However at Mars orbit Ta, the mean ambient temperature is -63°C – which would mean the temperature needs to be raised by 113°C above the mean ambient until there is any sterilization. This is a conservative approach, it assumes that Martian life will survive as well as Earth models at 50°C.

The equation for sterilization diverges at T→Ta and as T→∞. These divergences arise due to the "T-Ta" term in the denominator. The Ta divergence is avoid by having a cut off at 50°C, whilst the ∞ divergence is physical – meaning that with enough temperature all life is sterilized.

Although both divergences are avoided, it is best to flatten these terms for numerical integration. Note that the numerator "exp(-b/T)" tends smoothly from 0 at T=0 to 1 at T=∞, and so does not have any pathologies. The numerator singularity can be flattened by making the substitution:

$$x = \ln(T - Ta)$$

So:

$$S = \ln\left(\frac{N_e}{N_0}\right) = -\frac{k_0}{\beta}\int_{\ln(T_0-Ta)}^{\ln(T_I-Ta)} dx \exp\left(-\frac{b}{Ta+e^x}\right)$$

The divergent at ∞ can be analytically calculated by subtracting the asymptotic value of the integral:

$$S = \ln\left(\frac{N_e}{N_0}\right) = \frac{k_0}{\beta}\int_{\ln(T_0-Ta)}^{\ln(T_I-Ta)} dx\left(1-\exp\left(-\frac{b}{Ta+e^x}\right)\right) - \frac{k_0}{\beta}\ln\left(\frac{T_I-Ta}{T_0-Ta}\right)$$

The remaining integral is most easily performed numerically, however performing this for each step of a Monte Carlo would be slow. Now for a chosen organism the integral only depends on T_I and so is suitable to be done via a look up table, which is calculated once at initialisation. A lookup table can only cover a finite range of temperatures, the lower value is tribally set to T_0=50°C where sterilization stops. The upper limit, could be set to a maximum temperature – however as Mars Ejecta have no upper speed, their energy per mass has no limit, and hence temperature can't be limited. Instead compare the size of the integral to the analytic logarithm, the first term of the integral at large T can be expanded to give:

$$\frac{b}{Ta+e^x} = \frac{b}{T}$$

Which is to be compared against 1. Now b is given by the organism, setting the maximal T to 1000 times higher will give an error below 0.1%.

4.7 Heat Inactivation and 99%

The equation above that combines the cooling and sterilisation:

$$S = \ln\left(\frac{N_e}{N_0}\right) = -\frac{k_0}{\beta}\int_{T_0}^{T_I} dT \frac{\exp\left(-\frac{b}{T}\right)}{T-Ta}$$

Factors into two terms:

$$-\frac{k_0}{\beta} \quad : \quad \int_{T_0}^{T_I} dT \frac{\exp\left(-\frac{b}{T}\right)}{T-Ta}$$

The second integral term gives the temperature dependent shape, this term is fitted from the heat sterilisation tests documented in TN18 [RD6].

The first term is a combination of the sterilisation rate k_0, and the cooling rate β, both of which are just an overall normalisation constant. Now the effect on sterilisation cannot be separated between these two, for example if twice the logarithmic sterilisation is seen at a set temperature, this could be due either to a doubling in the sterilisation rate k_0, or a halving of the cooling rate β, or any mixture of the two – and from the sterilisation alone there would be no difference.

Now the cooling rate needs refitting during the hyper velocity tests, as organisms on rock (fragments) cool very differently from organisms on metal foil, as used in the heat tests. This means that for the model used in the hyper velocity model the normalisation factor should be taken from the hypervelocity tests. This means that the thermal sterilisation model gets parameters from two sources:

- Shape from Heat Sterilisation
- Normalisation from HV impact testing

Turning to 99% levels of confidence, when fitting parameters three values are of interest:

- Best Fit
- Upper 99% confidence level
- Lower 99% confidence level

Req-30 [AD1] requires conservative values for inactivation of biological systems (e.g. maximal survival). Fitting the shape to 99% confidence level by varying the "b" parameter has been described in TN18 [RD6], due to strong cross correlation with k_0/β, to probe the full "b" space k_0/β has been optimised for each b to give the best fit. Hence the best, lower and upper 99% fit on "b" each have an associated "k_0/β":

Organism	Parameter	Lower 99%	Best Fit	Upper 99%
D. radiodurans	b (K)	1911	7639.5	9748.5
	k_0/β	2.3668e2	5.0916996e7	6.934636484e9
B. atrophaeus	b (K)	2706	3364.5	4004
	k_0/β	586.26	1991.38	5877.83
B. diminuta	b (K)	3202.5	6811.5	8623
	k_0/β	5.772e3	1.9067530e7	1.661639850e9
MS2	b (K)	2316.5	3474.5	4787
	k_0/β	7.2126e2	1.161629e4	1.9385347e5

Now in heat tests (TN13), the cooling parameter "β" has been fitted:

Organism	Mean β_{HT} (s^{-1})	Sd (s^{-1})
D. radiodurans	60.50456	18.273
B. atrophaeus	54.70921	16.44129
B. diminuta	60.37431	10.23999
MS2	63.24664	18.22549

When the Hyper Velocity tests were performed different cooling rates were needed for the fit:

Organism	Mean β_{HV} (s^{-1})
D. radiodurans	35
B. atrophaeus	6

B. diminuta	18
MS2	32

Now as the 99% confidence level on sterilization needs application to the HV modelling, the β values need modifying, giving:

Organism	Confidence	k_0/β_{HT}	β_{HT} (s^{-1})	β_{HV} (s^{-1})	k_0/β_{HV}
D. radiodurans	Lower 99%	2.3668e2	61	35	2.4e2*61/35
	Best fit	5.0916996e7			5.1e7*61/35
	Upper 99%	6.934636484e9			6.9e9*61/35
B. atrophaeus	Lower 99%	586.26	55	6	5.9e2*55/6
	Best fit	1991.38			2.0e3*55/6
	Upper 99%	5877.83			5.9e3*55/6
B. diminuta	Lower 99%	5.772e3	60	18	5.8e3*60/18
	Best fit	1.9067530e7			1.9e7*60/18
	Upper 99%	1.661639850e9			1.7e9*60/18
MS2	Lower 99%	7.2126e2	63	32	7.2e2*63/32
	Best fit	1.161629e4			1.2e4*63/32
	Upper 99%	1.9385347e5			1.9e5*63/32
Super Bug	Lower 99%	586.26	55	35	5.9e2*55/35
	Best fit	1991.38			2.0e3*55/35
	Upper 99%	5877.83			5.9e3*55/35

Where the values have been rounded to 2dp, which with the error on the known values of β is reasonable.

The Super bug uses the sterilisation values of B.atrophaeus with the cooling parameter of D.radiodurans.

This still leaves the question of which model to use in the Martian moon simulation, under Req-60 a conservative choice with respect to inactivation. Now as “b” is varied under 99% confidence, and “b” controls the shape – which of the 3 curves is conservative isn’t clear. This is illustrated by a fit to B. atrophaeus in the flash heat tests [RD6]:

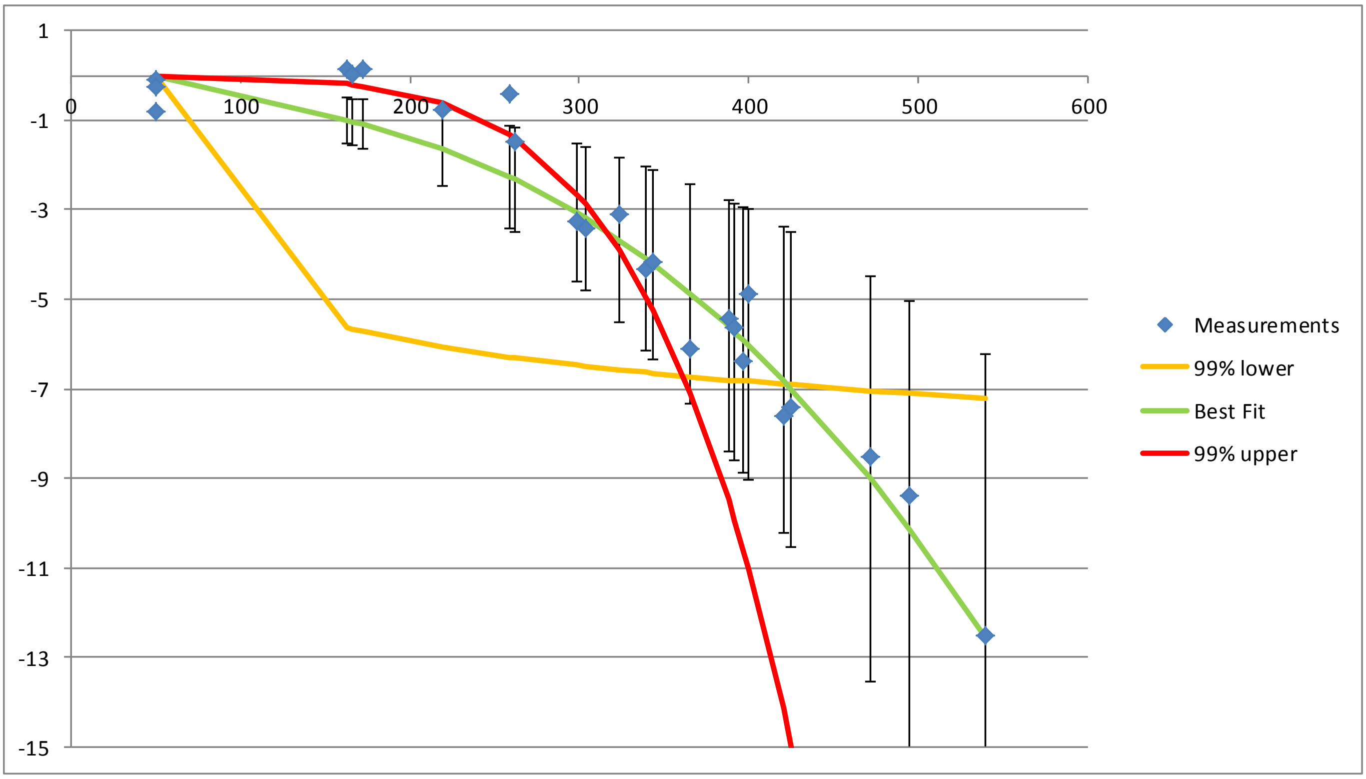


Figure 4-5. Example best fit, and 99%, shape fits for B. atrophaeus where an error proportional to sterilisation is used. Note that this is not the result from TN18 [RD6], and so not to be used – it is given just as an example.

As "b" defines shape, there is not a single value of "b" that gives the conservative (maximal survival), in various temperature ranges one or other value gives least sterilisation. Hence to be conservative, the model needs to calculate sterilisation for all three "b" values:

- Best Fit
- Upper 99%
- Lower 99%

This will give three possible sterilisations, of which the most conservative is used.

4.8 Martian cloud

When material is ejected from a Martian Moon, this enters orbit about Mars.

Firstly the velocity of ejection needs to have the velocity of the Moon added, to get the velocity about Mars. Now the position of the ejection on the moon is not recorded, neither is the ejection angle. This decision has been made for various reasons:

- Averaging over the Moon's surface greatly increases the efficiency of the Monte Carlo
- Once the point of impact is averaged over, the angle of ejection when taken into the Mars frame becomes isotropic which increases Monte Carlo efficiency
- The processes in the cloud which remove material don't have strong dependence on direction

This means that when adding the Moons velocity onto velocity of ejection from the moon, this is performed with a random angle between the two, as the orientation is random in 3 space, the angle lies over a sphere, where the measure of the sphere is:

$$d\varphi d(\cos\theta)$$

Phi is an azimuthal angle, so cos(θ) is generated uniformly between -1 and 1. The two velocities then add.

$$v_{Mars}^2 = v_{Moon}^2 + v_{Ejecta}^2 + 2v_{Moon}v_{Ejecta}\cos\theta$$

Which gives the magnitude of the velocity of the particle in the cloud in an inertial frame around Mars.

This velocity is important, where this velocity exceeds the escape velocity for Mars, the particle will escape Mars and so not enter the cloud, this escape velocity is independent of angle of emission, and so again the angle of emission does not need to be created.

That an ejector collides with Mars is a bit more involved. Firstly the velocity needs writing as a radial and tangential component (relative to Mars):

$$v_{radial}^{Moon} = v_{Ejecta}\sin\theta\sin\varphi$$

$$v_{tangental}^{Moon} = \sqrt{v_{Mars}^2 - v_{radial}^{Moon,2}}$$

Where φ is flat in [0:2π]. Translating these vectors to the surface of mars:

$$v_{tangental}^{Mars} = \frac{r_{Moon}}{r_{Mars}} v_{tangental}^{Moon}$$

$$v_{radial}^{Mars,2} = v_{Mars}^2 - v_{tangental}^{Mars,2} + 2Gm_{Mars}\left(\frac{1}{r_{Mars}} - \frac{1}{r_{Moon}}\right)$$

From conservation of angular momentum, and energy – now when $v_{radial}^{Mars,2}$ is negative, it is not possible for the orbit to impact Mars, as the ejecta has too much angular momentum about Mars. This is the criterion to check that the ejecta in the cloud does not impact Mars.

Next the size of the ejected particle is important where this is below a cut off, perturbations to the orbit (e.g. solar wind) will tend to be ejected [AD5]. The size of particles can be calculated from there mass:

$$m = \frac{4}{3}\pi\rho r^3$$

Taken from the volume of a sphere, the mass of the ejected object being a parameter that is modelled in the hyper velocity collision.

The time period in the cloud, expected to be up to centuries, is small in comparison to the ~10Myear considered in the study. So this is assumed to have little effect on any life in the ejecta.

Finally the re-collision with the Moon, as the Martian Moons orbit in an approximately circular orbit, they are always at approximately the same height in the gravitational potential of Mars. This means that any ejecta in the cloud will re-impact the Moon with a velocity similar to ejection. The angle of impact though depends critically on the phasing of the collision, and the position of impact. So this is modelled as for the original hyper velocity impact as a sin(2θ) distribution [RD1].

4.9 Radiation environment

On Phobos (Deimos) the radiation environment originates mainly from the solar system. Many types of high energy particles are present:

- Gamma
- Beta
- Alpha
- Heavier Elements

During modelling [RD4] as energy deposited depends on energy of radiation, it was decided to model radiation as a function of linear energy transfer (LET) [RD4]. Specifically:

$$D = \int dt\, d(LET) \frac{d^2 D}{dt\, d(LET)}$$

Where the time integrated over is the period over which material is on the surface of Phobos.

Now the radiation modelling [RD4] the average dose is considered over time. This makes the integral over time to be simple:

$$D = t \int d(LET) \frac{d^2 D}{dt\, d(LET)}$$

Where $d^2D/dt/d(LET)$ has no dependence on t.

Sterilization of an organism depends on both dose and LET. The dependence on LET is expected to be mild, however as ionisation increases for higher LET and ionization causes free radicals which correlate with organism inactivation. This gives the expectation.

$$S(\mathrm{LET1}) \geq S(\mathrm{LET2}) \quad \text{where } \mathrm{LET1} \geq \mathrm{LET2}$$

Turning to the sterilization dependence on dose, if we consider an organism exposed to two doses, D_1 and D_2, then the sterilization caused is given by:

$$S(D_1 + D_2) = S(D_1) \times S(D_2)$$

E.g. the sterilization is multiplicative. Now this complicates the fit to the total dose, as that comes from an integration – which sums over all the doses. Hence it is easier to follow the logarithm of sterilization:

$$\ln\big(S(D_1 + D_2)\big) = \ln\big(S(D_1)\big) + \ln(S(D_2))$$

Where the additive nature adapts well to the integration. Hence the sterilization is given by:

$$\ln\big(S(D)\big) = \ln\left(S\left(t \int d(LET) \frac{d^2 D}{dt\, d(LET)}\right)\right)$$

Consider now splitting the LET integral up into separate ranges:

$$\ln(S(D)) = \ln\left(S\left(t\left(\int_0^{69\mathrm{Mevcm^2/g}} d(LET)\frac{d^2D}{dt\,d(LET)} + \int_{69\mathrm{Mevcm^2/g}}^{534\mathrm{Mevcm^2/g}} d(LET)\ln\left(S\left(\frac{d^2D}{dt\,d(LET)}\right)\right)\right.\right.\right.$$

$$\left.\left.\left.+\int_{534\mathrm{Mevcm^2/g}}^{\infty} d(LET)\ln\left(S\left(\frac{d^2D}{dt\,d(LET)}\right)\right)\right)\right)\right)$$

$$= \ln\left(S\left(t\int_0^{69\mathrm{Mevcm^2/g}} d(LET)\frac{d^2D}{dt\,d(LET)}\right)\right)$$

$$+\ln\left(S\left(t\int_{69\mathrm{Mevcm^2/g}}^{534\mathrm{Mevcm^2/g}} d(LET)\frac{d^2D}{dt\,d(LET)}\right)\right)$$

$$+\ \ln\left(S\left(t\int_{534\mathrm{Mevcm^2/g}}^{\infty} d(LET)\frac{d^2D}{dt\,d(LET)}\right)\right)$$

$$> \ln\left(S\left(t\int_0^{69\mathrm{Mevcm^2/g}} d(LET)\frac{d^2D}{dt\,d(LET)}, LET = 0\right)\right)$$

$$+\ln\left(S\left(t\int_{69\mathrm{Mevcm^2/g}}^{534\mathrm{Mevcm^2/g}} d(LET)\frac{d^2D}{dt\,d(LET)}, LET = 69\mathrm{Mevcm^2/g}\right)\right)$$

$$+\ \ln\left(S\left(t\int_{534\mathrm{Mevcm^2/g}}^{\infty} d(LET)\frac{d^2D}{dt\,d(LET)}, LET = 534\mathrm{Mevcm^2/g}\right)\right)$$

Where in the last line, the sterization with dose has been sated at a fixed LET, that is the minimal in the integral and gives the conservative sterilization. The integrals are most easily defined via the integrated dose, that gives the dose greater than a value:

$$D_x = \int_x^{\infty} d(LET)\frac{d^2D}{dt\,d(LET)}$$

Which translates the sterilization equation into:

$$\ln\big(S(D)\big) = \ln\big(S(t(D_0 - D_{69}), LET = 0)\big) + \ln\big(S(t(D_{69} - D_{524}), LET = 69)\big) + \ln\big(S(tD_{524}, LET = 534)\big)$$

Or:

$$S(D) = S(t(D_0 - D_{69}), LET = 0) \times S(t(D_{69} - D_{524}), LET = 69) \times S(tD_{524}, LET = 534)$$

The radiation modelling [RD4] models these as a function of depth averaged over the surface of Phobos (Deimos). The radiation is calculated at fixed depths, with greater fidelity towards the surface, where samples are more likely to be taken:

- 0,1,2,3,4,5,6,7,8,9,10cm
- 20,30,40,50,60,70,80,90,100 cm
- 2,3,4,5,6,7,8,9,10 m

These are at discrete depths to aid the modelling, however the variation with depth is expected to be smooth.

Now the hypervelocity collision will deposit the Martian material at a certain depth, this depth will typically not be at exactly one of the depths; so the modelling will typically not have a value for the radiation at that depth. However with a smooth distribution, the radiation can be extrapolated from the calculated depths on either side, with minimal error.

A simple form of smooth extrapolation, that passes smoothly through measured points, is a spline; and a cubic spline can be calculated easily. In particular as the radiation vs depth is known at the start of the simulation; the spline parameters can be pre calculated – this means that the calculation of the cubic spline during the run can be very fast.

The rate of radiation absorption is assumed to be constant over the period the material is on the moon; hence to obtain the total dose the radiation rate needs multiplying by the elapsed time. Hence the data in the spline should be the radiation rate.

Modelling of the SEP has a minor issue. Data provided by Kallisto is shown in Figure 4-6.

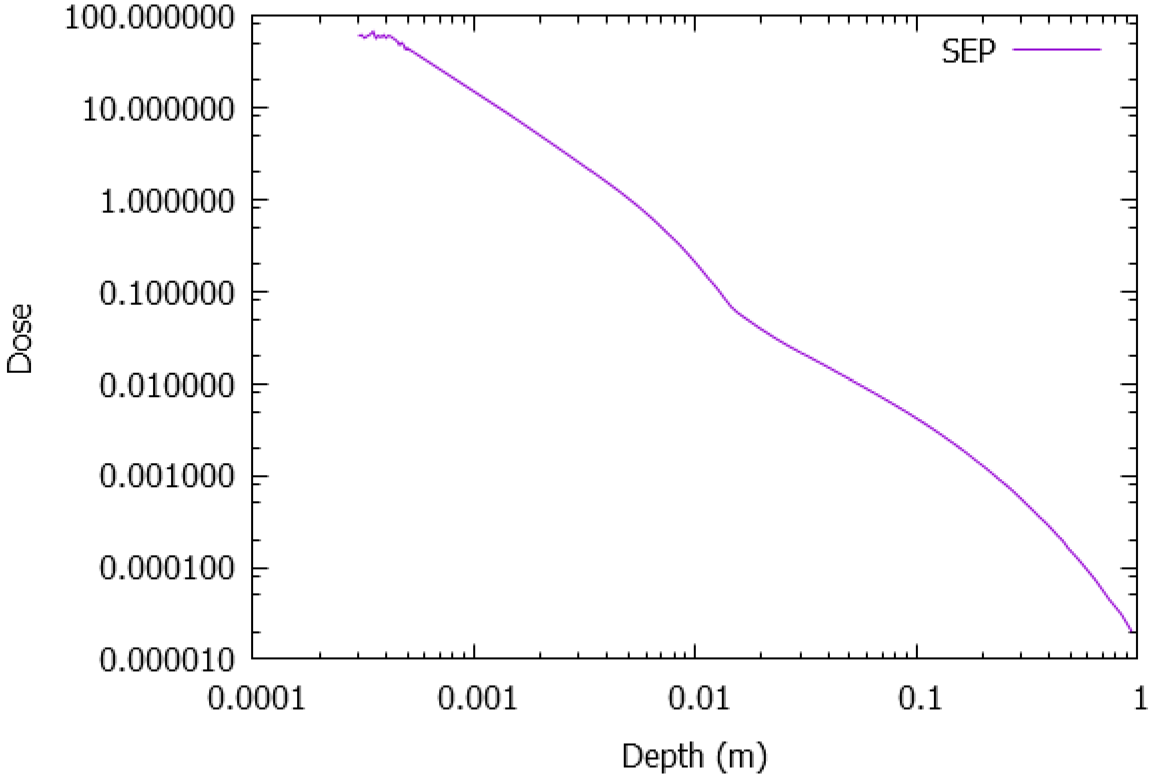


Figure 4-6. Dose vs depth for Solar Energetic Particles

This distribution is well modelled on a log-log distribution, however it grows to small depth, and on a log x distribution this would give unphysical radiation at low depths. At depths below 1mm the graph can be seen to turn over. This is shown on a linear graph in Figure 4-7.

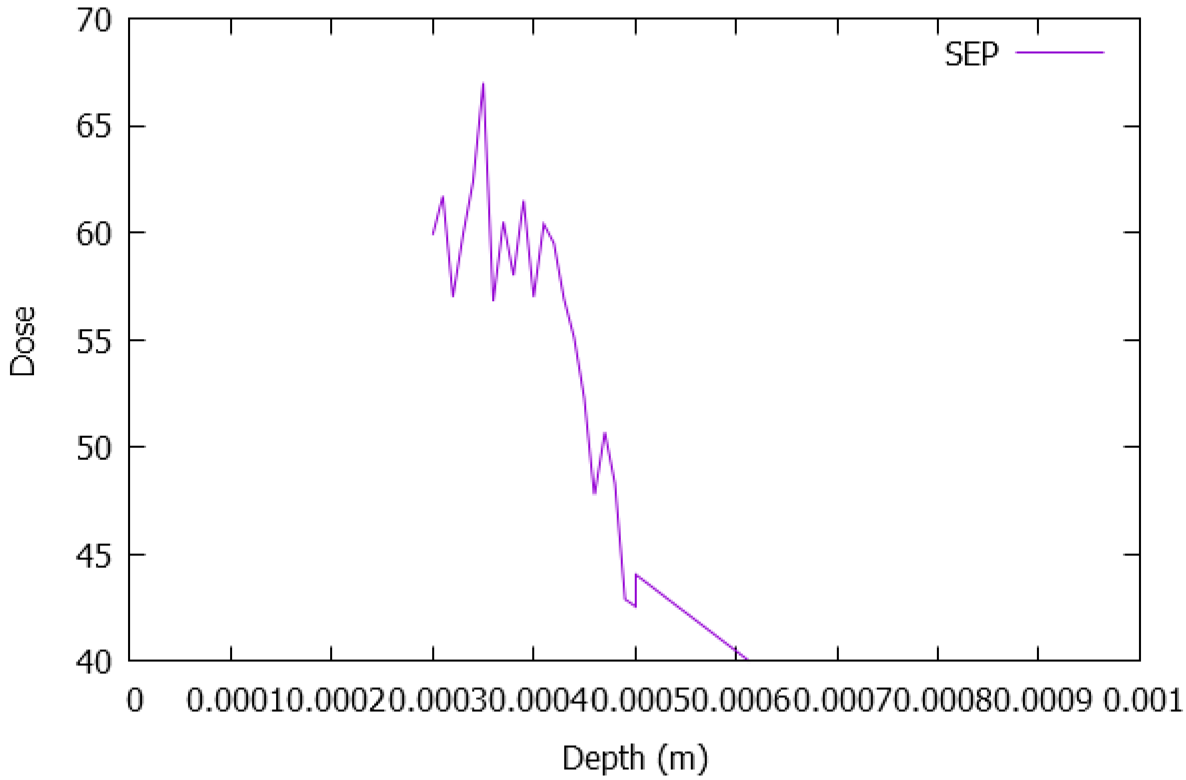


Figure 4-7. The SEP dose at small depths below 1mm.

What can be seen is that at depths below ~0.4mm that the radiation flattens. This is a very quick turn off.

As most of the SEP radiation is best modelled on a log-log distribution, this suggests just cutting the radiation off at 0.4mm, with a dose of around 60Gy/yr.

4.10 Radiation inactivation

The radiation inactivation is reported in [RD5]. In this the inactivation is fitted using the model:

$$\ln\left(\frac{N}{N_0}\right) = \ln(S) = \lambda D$$

Two rounds of testing were performed. The first round of gamma testing measured [RD5]:

organism	λ [kGy^{-1}] (log$_e$ reduction)	D$_{10}$ [kGy]	R	Comments
D. radiodurans	**-0.79 ± 0.10**	**2.90 ± 0.37**	0.848	
	-0.765 ± 0.035	3.01 ± 0.14	0.982	Excluding two results at 3 kGy with log10 reductions of -4.7
B. atrophaeus	**-1.330 ± 0.026**	**1.731 ± 0.033**	0.997	

B.Dim and MS2 gave inconsistent results.

The second round of testing gave [RD5]

Micro-organism	λ [kGY^{-1}]	D$_{10}$ [kGy]	Correlation.	Comments

	(log_e reduction)		**Coefficient**	
D. radiodurans	**-0.438 ± 0.027**	**5.26 ± 0.33**	0.959	
	-0.440 ± 0.028	5.23 ± 0.33	0.969	Excluding three results at 1 kGy where *Ni*/*N*0 > 1.0
MS2 coliphage	**-0.248 ± 0.009**	**9.27 ± 0.33**	0.982	
B. diminuta	-10.4	0.22	(N/A)	
	-9.56	**0.24**	(N/A)	Recommended worst case ("worst of 3 samples")

Consistent results were only obtained for the gamma test with LET=0. Now sterilization is expected to increase with increasing LET [RD5], hence a conservative approach is to use the gamma data for all LET values. Consider the effect this has on the sterilization at depth in the regolith:

$$\ln\big(S(D)\big) = \ln\big(S(t(D_0 - D_{69}), LET \geq 0)\big) + \ln\big(S(t(D_{69} - D_{524}), LET \geq 69)\big)$$
$$+ \ln\big(S(tD_{524}, LET \geq 534)\big)$$
$$= \lambda t\left(\frac{dD_0}{dt} - \frac{dD_{69}}{dt}\right) + \lambda t\left(\frac{dD_{69}}{dt} - \frac{dD_{524}}{dt}\right) + \lambda t\frac{dD_{524}}{dt}$$
$$= \lambda t\frac{dD_0}{dt}$$

So this gives the expected result when sterilization is not dependent on LET, that sterilization is just give by the total dose.

Now for the results to use in the modelling:

- B.atrophaeus was only measured in the first round of testing.
- MS2 and B.diminuta were only reliably measured in the second round of testing
- The D.radiodurans in the second round of testing is believed more reliable [RD5], and has the more conservative value for lambda
- The errors given, are 1sd. To get 99% confidence, this suggests taking the 3sd limit (nominally gives 99.7% confidence for normal distributions, however is chosen here as distribution may not be normal).
- B.diminuta had only a single measurement of any life at any dose, so the above value is already an lower limit (e.g. conservative)

So this suggests the values to use in modelling:

Micro-organism	**λ [kGY^{-1}] (log_e reduction) used**	**λ [kGY^{-1}] at 3sd**
D. radiodurans	**-0.438 ± 0.027**	**-0.357**
MS2 coliphage	**-0.248 ± 0.009**	**-0.221**
B. diminuta	**-9.56**	**-9.56**
B. atrophaeus	**-1.330 ± 0.026**	**-1.252**

And these values are used for all LET. Note that the Radiation Environment (on Phobos) will keep separate the dose for the range LET >0, >69MeVcm^2/g, >524MeVcm^2/g – for if the sterilization model is changed in future.

Calculating the duration, D, has two different modes. When looking at mass transferred, the transfer of mass from Mars is assumed to be uniform in time – so time is generated linearly. The total mass constructed:

$$M = \int dt \frac{dm}{dt} = \frac{dm}{dt} T_{Total}$$

The integral over time of the arrival rate of mass.

Now when looking an unsterilized mass, with respect to radiation sterilisation:

$$M_{US} = M\exp(\lambda D) = M\exp\left(\lambda \frac{dD}{dt} t\right)$$

And this decays with duration, t. Now λ and dD/dt depend on organism, and the depth of the deposited material – however both of these are known when the duration is decided.

Hence when calculating the unsterilized mass, this is best done:

$$M_{US} = \int dt \frac{dm_{US}}{dt} = \int dt \frac{dm}{dt} \exp\left(\lambda \frac{dD}{dt} t\right) = \int d\left(\exp\left(\lambda \frac{dD}{dt} t\right)\right) \frac{1}{\lambda \frac{dD}{dt}} \frac{dm}{dt}$$

Where this flattens the integral. And so the exponential is generated uniformly, and the mass scaled.

5. REQUIREMENTS AND VERIFICATION RESULTS

5.1 Introduction

In the section originally the requirements of the model are derived. Whilst some requirements need verification against an independent test, many can only verified that they have been coded by inspection. For all requirements though, and area of code meets the requirement. When coded this section of code is commented with the requirement it meets. Hence for verification of the requirement being in the code is performed automatically by searching code for each requirement.

Rather than documenting this verification independent of the requirement, instead the requirement is documented with where it is met in the code. This is recorded both as the file which contains the code, and also the line number where the code occurs. This is taken as the verification that the requirement has been coded. The notation of the code line is "*source_code*:*line_number*".

This is detailed in the following section.

5.2 Mars Ejecta

5.2.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-ME-In-01	The inputs shall be: • ConeMode • LogEjectaMinMass	I	Cone Mode sets the ejection cone angle as defined in SE-ME-Alg-07. LogEjectaMinMass sets the logarithm to base 10 of the minimum ejection mass as defined in SE-ME-Alg-10.	MarsEjecta.c:8
SL-ME-Out-01	The outputs shall be: • TotalMass	I	TotalMass is the differential amount of mass ejected in this event. So specifically summing	MarsEjecta.c:8

	• EjectaMass • EjectaVelocity • EjectaAngle		over TotalMass over events will give the Total Mass Ejected over a 10MY period EjectaMass gives the mass of the Ejecta EjectaVelocity gives the velocity of ejection of the differential mass EjectaAngle, gives the angle of ejection of the differential mass.	

5.2.2 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-ME-Alg-01	The EjectaVelocity shall be modelled as: $v = \frac{v_{min}}{X^{1/\gamma}}$	I	v_{min} is the minimal ejecta velocity modelled. It is set as an external parameter. γ is the power in the power law X is a uniform random number [0:1]	MarsEjecta.c:18
SL-ME-Alg-02	The default value of γ shall be 1.5	I	Taken from [AD2]	sterlim.h:19
SL-ME-Alg-03	The Total Mass shall be modelled as: $m_{min} = m_{ref}\left[\frac{v_{min}}{v_{ref}}\right]^{-\gamma}$	I/V	This scales the total mass ejected with the minimum velocity compatible with the velocity scaling law. In means that once the ejecta mass has been fitted for a reference minimum velocity, how this will scale with a change in minimum velocity.	MarsEjecta.c:27

SL-ME-Alg-04	v_{ref} shall be chosen to be 3800m/s.	I	From [AD2] this value is approximately the minimum ejecta velocity to reach Phobos, and so is a convenient normalisation point.	sterlim.h:33
SL-ME-Alg-05	m_{ref} shall be scaled to give normalised mass consistent with [AD2]	I/V	E.g. [AD2] calculation of the mass transferred to Phobos in 10MY shall be used as input into this model. Note: parameter set to m_{ref} = 2.9e12 kg with a cut off speed of 3.8km/s	sterlim.h:31
SL-ME-Alg-06	The ejection angle shall be modelled as an ejection cone.	I		MarsEjecta.c:31
SL-ME-Alg-07	Several ejection Cone angles shall be modelled: • 1: 30° • 2: 45° • 3: 60° • 4: Cone centred on 45° with a normal distribution in angle with standard deviation of 15° • 5: Cone centred	I/V		MarsEjecta.c:34 MarsEjecta.c:37 MarsEjecta.c:40 MarsEjecta.c:46 MarsEjecta.c:53

	on 60° with a normal distribution in angle with standard deviation of 15°			
SL-ME-Alg-08	Where the ejection angle is modelled as greater than $\pi/2$ or less than 0 radians, algorithm SL-ME-Alg-07 shall be repeated	I	This ensures that no unphysical angle is generated.	MarsEjecta.c:47 MarsEjecta.c:54
SL-ME-Alg-09	The mass of the ejecta shall be modelled as $$m = \frac{m_{min}}{X}$$ Where X is a uniform random number in range [0:1]	I/V	This value corresponds to the mass of the Ejecta, so is obtained from the distribution of ejecta masses from Mars. It is different from the mass simulated in each step of the Monte Carlo, that is represented by Total Mass.	MarsEjecta.c:61 sterlim.h:23
SL-ME-Alg-10	The value of m_{min} in SL-ME-Alg-09 shall be varied between: [1e-6,1e-5,1e-4,1e-3] kg	I	This will provide a sensitivity analysis	sterlim.h:24

5.3 Mars Rotation

5.3.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-MR-In-01	The inputs shall be: • EjectaVelocity • EjectaAngle	I	These are the output of Mars Ejecta	MarsToInertial.c:7
SL-MR-Out-01	The outputs shall be: • Velocity – radial • Velcoity – EW • Velocity – NS	I	These correspond to velocities in an inertial frame	MarsToInertial.c:7

5.3.2 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-MR-Alg-01	The latitude of ejection from Mars will be modelled	I		MarsToInertial.c:13
SL-MR-Alg-02	Mars shall be modelled as a sphere of radius 3389.5e3 m	I		sterlim.h:39
SL-MR-Alg-03	Mars rotation period shall be modelled as 1.025957*86400 s	I	Equals 1.025957 Earth days	sterlim.h:41
SL-MR-Alg-04	The sine of the latitude shall be distributed uniformly in the range [-1:1]	I	Uniform distribution on a sphere	MarsToInertial.c:13
SL-MR-Alg-05	The cosine of the latitude shall be used from: $\cos(lat) = \sqrt{1 - \sin^2(lat)}$	I	Standard transform	MarsToInertial.c:14
SL-MR-Alg-06	The radius of rotation shall be calculated from:	I	Large circle of rotation at	MarsToInertial.c:16

	$r = r_{Mars}\cos(lat)$		the martian equator	
SL-MR-Alg-07	The speed of rotation of the ejection point shall be calculated as: $$v_{rot} = \frac{2\pi r}{P}$$	I	Where r is the radius of rotation from SL_MR-Alg-06. P is the period of Mars rotation, taken from SK-MR-Alg-03.	MarsToInertial.c:19
SL-MR-Alg-08	The orientation of the ejection in relation to EW-NS axis ($\theta_{EjectaRotation}$) will be modelled as a flat distribution in angle in the range [0:2π].	I	This orientation is not modelled by Mars Ejecta code, however is important for the Mars rotation, as Mars rotation can either add or subtract velocity.	MarsToInertial.c:23
SL-MR-Alg-09	The velocities in an inertial frame will be calculated as: $$v_{radial} = v_{Ejecta}\cos(\theta_{Ejecta})$$ $$v_{EW} = v_{Ejecta}\sin(\theta_{Ejecta})\cos(\theta_{ER}) + v_{rot}$$ $$v_{NS} = v_{Ejecta}\sin(\theta_{Ejecta})\sin(\theta_{ER})$$	I	Standard projection formulas, adding in the rotational velocity of Mars.	MarsToInertial.c:21

5.4 Orbit Propagation Mars to Moon

5.4.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-OP-In-01	The inputs shall be: • Velocity – radial • Velcoity – EW • Velocity – NS	I	Velocity Outputs from Mars Rotation. Differential Mass is used to follow the progress of matter through processes.	MarsToMoon.c:25

	• Differential Mass			
SL-OP-Out-01	The Outputs shall be: • Differential Mass • Moon Impact Velocity • Moon Impact Angle	I	Differential Mass is updated to include the probability of impact. The impact parameters are the projected parameters from the orbit propagation.	MarsToMoon.c:25

5.4.2 Initialisation

Number	Requirement	Verification	Comment	Code
SL-OP-Init-01	The rotation speed of the moon shall be calculated at initialisation with value: $$\dot{\theta}_{Moon} = \sqrt{G\frac{m_{Mars} + m_{Moon}}{r^3}}$$	I	Standard Formula. R is the distance from Mars to the moon	MarsToMoon.c:12
SL-OP-Init-02	The Escape Velocity from Mars shall be calculated at initialisation with value: $$v^2{}_{Escape} = \frac{2Gm_{Mars}}{r_{Mars}}$$	I	The escape velocity from Mars at a radius of r_{Mars}	MarsToMoon.c:13

5.4.3 Algorithmic Form

Number	Requirement	Verification	Comment	Code

SL-OP-Alg-01	The EW velocity from Mars surface to the Moon will be calculated from conservation of angular momentum: $$v_{EW,Moon} = v_{EW,Mars}\frac{r_{Mars}}{r}$$	I	r is the Mars Moon distance	MarsToMoon.c:31
SL-OP-Alg-02	The NS velocity from Mars surface to the Moon will be calculated from conservation of angular momentum: $$v_{NS,Moon} = v_{NS,Mars}\frac{r_{Mars}}{r}$$	I	r is the Mars Moon distance	MarsToMoon.c:32
SL-OP-Alg-03	The radial velocity from Mars surface to the Moon will be calculated from conservation of energy: $$v_{Rad,Moon}^2 = v_{Rad,Mars}^2 + \left(v_{EW,Mars}^2 + v_{NS,Mars}^2\right) \times \left(1 - \left(\frac{r_{Mars}}{r}\right)^2\right) - 2Gm_{Mars}\left(\frac{1}{r_{Mars}} - \frac{1}{r}\right)$$	I		MarsToMoon.c:36
SL-OP-Alg-04	Where $v^2_{Rad,Moon}$ is less than zero, the ejecta does not have sufficient energy to reach the moon. The Differential Mass shall be set to zero	I	When the moon cannot be reached, no mass is transferred to the moon.	MarsToMoon.c:43
SL-OP-Alg-05	The velocity at the moon, shall be translated into the moons frame of reference via: $$v_{EW,Moon} \leftarrow v_{EW,Moon} - r\dot{\theta}$$	I	With this sign convention, and the formula used in SL-MR-Alg-09, this has the Moon and Mars rotating in the same sense. So the velocity that is added by the rotation of Mars, is subtracted n translation into the moons	MarsToMoon.c:47

			frame of reference.	
SL-OP-Alg-06	The ejecta velocities in the moons frame of reference shall be used to calculate the limiting impact point on edge of the moon via: $v_{Tan,Moon} = \sqrt{v_{EW,Moon}^2 + v_{NS,Moon}^2}$ $\tau = \sqrt{r_{Tan,Moon}^2 v_{Rad,Moon}^2 + r_{Rad,Moon}^2 v_{Tan,Moon}^2}$ $dy = \frac{r_{Tan,Moon}^2 \times v_{Rad,Moon}}{\tau}$ $dx = -\frac{r_{Rad,Moon}^2 \times v_{Tan,Moon}}{\tau}$	I,V	Where dx is the radial distance of the impact point from the centre of the moon. And dy is the rotational distance of the impact point from the centre of the moon.	MarsToMoon.c:51 MarsToMoon.c:55 MarsToMoon.c:56 MarsToMoon.c:57
SL-OP-Alg-07	The effective rotational radius of the moons size is calculated as: $dd = dy - dx\frac{v_{Tan,Moon}}{v_{Rad,Moon}}$	I,V	This gives the angular size of the moon, as seen by orbits from Mars. Hence this gives the apparent size of the Moon as seen by the orbit from Mars.	MarsToMoon.c:59
SL-OP-Alg-08	The apparent impact area of the moon is calculated from: $A_{Moon} = \pi \times dd \times r_{Oth,Moon}$	I		MarsToMoon.c:62
SL-OP-Alg-09	The size of Phobos shall be taken as: $r_{Rad,Moon} = 27km/2$ $r_{Tan,Moon} = 20km/2$ $r_{Oth,Moon} = 20km/2$	I	Phobos size is modified from 27×22×18km^3 as the orientation of the shorter axis in the gravitational potential is not clear. The longer axis is stabilised in the radial direction.	sterlim.h:50

THALES ALENIA SPACE OPEN

SL-OP-Alg-10	The angular size of the moon, as seen in orbit propagation from the surface of Mars is given by: $$\theta_{Moon} = \frac{A_{Moon}}{r^2}$$	I	Solid angle	MarsToMoon.c:64
SL-OP-Alg-11	The probability of impact of the ejecta on the moon, integrated over the whole Martian surface is given by: $$\varphi_{Moon} = \frac{\theta_{Moon}}{4\pi}$$	I	Solid angle of moon, over the solid angle of the whole sphere	MarsToMoon.c:66
SL-OP-Alg-12	The Differential Mass shall be scaled by φ_{Moon}	I	This scales the mass transferred, rather than using the probability that the ejecta hits Mars, as this gives a vastly more efficient Monte Calo, but obtains the same functional answer.	MarsToMoon.c:69
SL-OP-Alg-13	Where the ejection velocity from mars is below the ejection velocity from Mars, the differential mass shall be multiplied by 2.	I	Such orbits cross the Moons orbit twice, which double the collisional probability.	MarsToMoon.c:76
SL-OP-Alg-14	The impact velocity on the moon is calculated from $$v_{Impact} = \sqrt{v_{Rad,Moon}^2 + v_{EW,Moon}^2 + v_{NS,Moon}^2}$$	I	The orbital velocity in the Moons frame of reference.	MarsToMoon.c:86
SL-OP-Alg-15	The Impact Angle shall be modelled as: $\cos(2\theta)$ is uniformly random in range [-1:1]	I,V	This generates a $\sin(2\theta)$ distribution to θ.	MarsToMoon.c:89

5.5 Moon Hypervelocity impact and ejector

5.5.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-HVI-In-01	The inputs shall be: • Impact Velocity • Impact Angle • Impact Mass • Mass Mode • Depth Coef	I	Mass mode is a logical value, when material is ejected: MM=1 mass=0 MM=2 mass=impactor_mass * %age material ejected Depth Coef gives the number of impactor radii that material is deposited at. The default value is Depth Coef equals 0, and so material is at the surface.	HVImpact.c:12
SL-HVI-Out-01	The Outputs shall be: • Delta Temperature • Ejected • Depth • Ejected Velocity • Ejected Mass	I	Ejected is a logic value, that say if the material is ejected, or deposited: Ejected=0 : Material Deposited Ejected=-1 : Material rejected When Ejected=0, the Depth variable is returned, the depth at which the material is deposited. When Ejected=1, Ejecta Velocity (and Mass) is returned, the velocity at which the material is ejected, and the mass of the object in which the material is ejected. In either case the Sterilization variable is evolved (multiplicative)	HVImpact.c:12

5.5.2 Initialisation

Number	Requirement	Verification	Comment	
SL-HVI-Init-01	The distribution of energy transferred	I	These depend on HV modelling,	HVImpactParam.c:

	from kinetic to thermal shall be configured at initialisation.		they are taken from the file HVImpactParam.c	3

5.5.3 *Algorithmic Form*

Number	Requirement	Verification	Comment	Code
SL-HVI-Alg-01	The fractional distance from the front of the impact, and the distance from the central line will be modelled	I		HVImpact.c:14 HVImpact.c:15
SL-HVI-Alg-02	The fractional distance from the front of the impact, x, shall be modelled as flat between front and back	I	This gives flat distribution in $dV = dx\,dy\,2\,\pi\,y\,d\theta$ $= dx\,d(y^2)\,\pi\,d\theta$	HVImpact.c:22
SL-HVI-Alg-03	The fractional distance from the central line, y, shall be modelled as flat in y^2.	I		HVImpact.c:23
SL-HVI-Alg-04	The model will predict the kinetic energy transferred into thermal energy, as a function of position	I		HVImpact.c:96
SL-HVI-Alg-05	The model will be based on a look up table	I		HVImpactParam.c:3
SL-HVI-Alg-06	The look up table will interpolate between points in table	I		HVImpact.c:56 HVImpact.c:60
SL-HVI-Alg-07	The look up table used will depend on if the impact velocity is above or below a threshold	I	This threshold is held in HVISter.vsplit	HVImpact.c:28 HVImpactParam.c:6 HVImpactParam.c:7 HVImpactParam.c:13 HVImpactParam.c:19 HVImpactParam.c:25

SL-HVI-Alg-08	The fractional position in the impactor (both front-back, and radial) will be split into an integer position (on the look up table) and a fraction position (between points in the lookup table).	I	This gives interpolation between points in the lookup table.	HVImpact.c:25 HVImpact.c:26 HVImpact.c:51 HVImpact.c:52
SL-HVI-Alg-09	Interpolation between points on the look up table will be performed by bilinear interpolation	I		HVImpact.c:56 HVImpact.c:60
SL-HVI-Alg-10	The loop up will produce both a mean and a standard deviation for the fractional energy transferred from kinetic to thermal	I		HVImpactParam.c:7 HVImpactParam.c:13 HVImpactParam.c:19 HVImpactParam.c:25
SL-HVI-Alg-11	The kinetic energy transferred to thermal energy shall be modelled as a normal distribution with mean and standard deviation given from the lookup table	I		HVImpact.c:63
SL-HVI-Alg-12	Where the transferred energy is modelled as negative this will be moved to zero	I	Negative transferred energy corresponds to cooling, which is unphysical.	HVImpact.c:66
SL-HVI-Alg-13	Thermal Energy will be translated into temperature change via a fixed Heat Capacity	I		HVImpact.c:98 sterlim.h:83
SL-HVI-Alg-14	The probability of ejection will depend on the angle of impact from vertical: • Zero for impact angles <45° • Increase linearly from 0 to 100% for angles from 45° to 90°	I, V		HVImpact.c:69
SL-HVI-Alg-15	Material that is ejected the impact	I		HVImpact.c:73-74

	velocity shall be converted into radial and tangential			
SL-HVI-Alg-16	The radial velocity shall be: • Impact Velocity * cos(impact angle)	I	Impact angle is the angle between the velocity vector and a line joining the centre of the moon to the point of impact.	HVImpact.c:73
SL-HVI-Alg-17	The tangential velocity shall be: • Impact Velocity * sin(impact angle)	I		HVImpact.c:74
SL-HVI-Alg-18	The Ejected radial velocity shall be: • -40% impact radial velocity	I		HVImpact.c:75
SL-HVI-Alg-19	The Ejected tangential velocity shall be: • 76% at 45° impact angle • 100% at 90° impact angle • Increase linearly between 45° and 90	I,V		HVImpact.c:76
SL-HVI-Alg-20	The ejected velocity shall equal • Sqrt(radial vel^2 + tangential ^2)	I		HVImpact.c:77
SL-HVI-Alg-21	The ejected temperature change shall be zero	I	No heating of ejected material	HVImpact.c:78
SL-HVI-Alg-22	The ejected mass shall depend on MassMode • MM=1 ejected mass = 0 • MM=2 ejected mass is %age of the impactor mass, the %age being %age ejected	I	This gives 2 extremes: • MM=1, material powdered • MM=2, ejected material leaves as one object	HVImpact.c:79 HVImpact.c:82 HVImpact.c:85
SL-HVI-Alg-23	For deposited material the Kinetic Energy per mass shall be calculated as:	I		HVImpact.c:95

	$$\frac{E}{m} = \frac{v^2}{2}$$			
SL-HVI-Alg-24	For deposited material the Thermal Energy per mass shall be calculated as: • Kinetic Energy * fraction	I	Where fraction is given by SL-HVI-Alg-11	HVImpact.c:96
SL-HVI-Alg-25	The depth material is deposited at shall be calculated as a fixed multiple of the impactor radii	I	The multiple shall be configured as DepthCoef. Its default value will be which means material is at the surface.	HVImpact.c:101
SL-HVI-Alg-26	The impactor radii will be calculated from: $$r = \sqrt[3]{\frac{3m}{4\pi\rho}}$$	I		HVImpact.c:101

5.6 Heat Inactivation

5.6.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-TI-In-01	The inputs shall be: • Temperature	I		ThermSter.c:89
SL-TI-Out-01	The output shall be: • Logarithmic sterilization.	I	Natural logarithm, ln.	ThermSter.c:89
SL-TI-In-02	The initialisation inputs	I	T_0=50°C	ThermSter.c:6

	shall be: • T_0, temperature at which no sterilization occurs • Organism			

5.6.2 *Initialisation*

Number	Requirement	Verification	Comment	Code
SL-TI-Init-01	The initialisation function will construct a lookup table for logarithmic sterilization as a function of initial temperature.	I	This does not include the logarithmic term calculated analytically.	ThermSter.c:39 ThermSter.c:48 ThermSter.c:51 ThermSter.c:60 ThermSter.c:63 ThermSter.c:72
SL-TI-Init-02	The lookup table will include both: • Logarithmic Temperature • Logarithmic sterilization And have separate tables for lower 99, best, upper 99% confidence levels.	I		ThermSter.c:39 ThermSter.c:48 ThermSter.c:51 ThermSter.c:60 ThermSter.c:63 ThermSter.c:72
SL-TI-Init-03	Logarithmic Temperature will be defined as: $\ln(T - T_A)$	I	Where T_A is the ambient temperature on the Martian Moon.	ThermSter.c:15 ThermSter.c:23 ThermSter.c:31 ThermSter.c:39 ThermSter.c:51

				ThermSter.c:63
SL-TI-Init-04	The temperatures in the lookup table will start from T_0, increase by uniformly in logarithmic temperature	I	T_0 is an input parameters. Δln(T-T_A) is calculated from lookup table size and a maximal value for the lookup table.	ThermSter.c:14 ThermSter.c:15 ThermSter.c:22 ThermSter.c:23 ThermSter.c:30 ThermSter.c:31 ThermSter.h:21 ThermSter.h:22
SL-TI-Init-05	The size of the lookup table will be ThermSterSz	I	ThermSterSz is a parameter set in sterlim.h	sterlim.h:86
SL-TI-Init-06	The maximal temperature in the lookup table will be set to: $$b \times \mathrm{TableMultipler}$$	I	Table multiplier will be initially set to 1000. Which should keep the error in going beyond the end of the lookup table to ~0.1% Note that the three confidence levels will typically have different b values, and hence different maximal temperature.	ThermSter.c:11 ThermSter.c:19 ThermSter.c:27 sterlim.h:92
SL-TI-Init-07	The lookup table at T_0 shall be set to 0.	I	This corresponds to no sterilization at T_0.	ThermSter.c:16 ThermSter.c:24 ThermSter.c:32
SL-TI-Init-08	The lookup table shall have step size: $$\Delta x = \frac{\left(\ln(T_{max} - T_A) - \ln(T_0 - T_A)\right)}{\text{ThermSterSz-1}}$$	I	This sets the lookup table as a function of the variable x=ln(T-T_A).	ThermSter.c:14 ThermSter.c:22 ThermSter.c:30

			Note that the step size varies between the three confidence levels.	
SL-TI-Init-09	The lookup table shall be populated via: $L_n = L_{n-1} + \Delta x\left(1 - \exp(-b/T)\right)$	I	This gives a numerical approximation to: $L = \int_{\ln(T_0-T_A)}^{\ln(T-T_A)} dx\left(1 - \exp(-b/T)\right)$ And $T=T_A+\exp(x)$ There are separate tables for the three confidence levels.	ThermSter.c:48 ThermSter.c:60 ThermSter.c:72
SL-TI-Init-10	T in SL-TI-Init-9 shall be calculated: $T = \exp(0.5 \times \ln(T_L - T_A) + 0.5 \times \ln(T_U - T_A)) + T_A$	I	Takes the temperature at the centre of the x variable bin.	ThermSter.c:42 ThermSter.c:54 ThermSter.c:66

5.6.3 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-TI-Alg-01	The function will return logarithmic sterilization as a function of temperature	V		ThermSter.c:89
SL-TI-Alg-02	If the temperature is below T_0 the function will return zero logarithmic sterilization	I	Below T_0 no sterilization occurs	ThermSter.c:113 ThermSter.c:143 ThermSter.c:173
SL-TI-Alg-03	The bin in the lookup table shall be calculated from T, T_0, and Δx, and confidence level.	I		ThermSter.c:98 ThermSter.c:103

				ThermSter.c:108
SL-TI-Alg-04	The fractional distance between bins will be calculated	I		ThermSter.c:100 ThermSter.c:105 ThermSter.c:110
SL-TI-Alg-05	A check will be made that the temperature lines between the values of the lookup table bins around the point	I	This checks correct functionality of the code.	ThermSter.c:127 ThermSter.c:157 ThermSter.c:187
SL-TI-Alg-06	The returned logarithmic sterilization shall be linearly interpolated between the surrounding bins in the lookup table	I		ThermSter.c:139 ThermSter.c:169 ThermSter.c:199
SL-TI-Alg-07	For temperatures within the lookup table the logarithmic sterilization returned shall be: $$\ln(S) = \frac{k_0}{\beta}\left(LLT - \ln\left(\frac{T - T_A}{T_0 - T_A}\right)\right)$$	I	Where LLT is the linearly interpolated vale from the lookup table. k_0 and β are the organism parameters.	ThermSter.c:139 ThermSter.c:169 ThermSter.c:199
SL-TI-Alg-08	For temperatures over the maximal temperature in the lookup table, the logarithmic sterilization returned shall be: $$\ln(S) = \frac{k_0}{\beta}\left(LT_{\max} - \ln\left(\frac{T - T_A}{T_0 - T_A}\right)\right)$$	I	Where LT_{max} ist he last value in the lookup table. Using this value should make an error of about 1/TableMultipler	ThermSter.c:122 ThermSter.c:152 ThermSter.c:182
SL-TI-Alg-09	The returned logarithmic sterilization shall be the largest (least negative) of the three confidence levels	I	This gives “conservative” inactivation.	ThermSter.c:202

5.7 Martian cloud

5.7.1 Input/Output Form

Number	Requirement	Verification	Comment	Code

SL-MC-In-01	The inputs shall be: • Total Mass • Ejecta Velocity • Ejecta Mass	I		MartianCloud.c:24
SL-MC-Out-01	The Outputs shall be: • Total Mass • Impactor Velocity • Impactor Mass • Impactor Angle	I		MartianCloud.c:24

5.7.2 Initialisation

Number	Requirement	Verification	Comment	Code
SL-MC-Init-01	The angular rotation period of the moon shall be initialised to: $$\dot{\theta} = \sqrt{G\frac{m_{Mars} + m_{Moon}}{d_{Moon}^3}}$$	I	This leads to the orbital velocity of the Moon, which in turn allows to calculate if eject escape.	MartianCloud.c:8
SL-MC-Init-02	The escape velocity to escape the Martian system, from the radius of the moon is given by: $$v_E^2 = 2G\frac{m_{Mars}}{d_{Moon}}$$	I	Used to calculate if eject escape the Martian system.	MartianCloud.c:13
SL-MC-Init-03	The minimum mass particle that will remain in the Martian cloud is calculated from: $$m_{min} = \frac{\pi\rho}{6} d_{min}^3$$	I	This changes the minimum size to a minimum mass, at an assumed density.	MartianCloud.c:14

SL-MC-Init-04	The minimum size of particle that is not perturbed from the Martian Cloud is given by: $$d_{min} = 300\mu m$$	I	Value taken from [AD5].	sterlim.h:77
SL-MC-Init-05	The density of Moon Ejecta will be taken as: $$\rho = 2000 kg/m^3$$	I		sterlim.h:13

5.7.3 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-MC-Alg-01	Where the eject mass is below the minimum mass, it will be ejected from the cloud, and play no further part in the simulation.	I	See SL-MC-Init-03, taken from [AD5]	MartianCloud.c:35
SL-MC-Alg-02	The ejection from the moon shall be assumed to be uniform over the moons surface	I	Influences following requirements	MartianCloud.c:38
SL-MC-Alg-03	The angle between the moons velocity vector and the velocity vector of ejected particles shall be modelled as uniform in: $$\cos\theta$$ With value in range [-1:1]	I	This follows from SL-MC-Alg-02	MartianCloud.c:38
SL-MC-Alg-04	The azimuthal angle angle of the velocity vector of ejection about the moons velocity shall be modelled as flat in: $$\varphi$$ In a range [0:2π]	I	This follows from SL-MC-Alg-02	MartianCloud.c:48
SL-MC-Alg-05	The moon tangential velocity shall be calculated from:	I	Where $\dot{\theta}$ is the angular rate of rotation of the	MartianCloud.c:42

	$$v_{Moon} = \dot{\theta} d_{Moon}$$		moon about mars (SL-MC-Init-01); and d_{Moon} is the radius of the moons orbit about Mars.	
SL-MC-Alg-06	The velocity of ejection in the martian frame shall have magnitude calculated from: $$v_{Mars}^2 = v_{Moon}^2 + v_{Ejecta}^2 + 2v_{Moon}v_{Ejecta}\cos\theta$$	I	Vector addition of ejection velocity to velocity of the Moon.	MartianCloud.c:43
SL-MC-Alg-07	The velocity of ejection will be projected into a radial component (away from Mars), and a tangential component. $$v_{radial} = v_{Ejecta}\sin\theta\sin\varphi$$ $$v_{tangental} = \sqrt{v_{Mars}^2 - v_{radial}^2}$$	I		MartianCloud.c:50
SL-MC-Alg-08	The ejection velocities shall be propagated to the surface of Mars via: $$v_{tangental}^{Mars} = \frac{d_{Moon}}{r_{Mars}} v_{tangental}$$ $$v_{radial}^{Mars,2} = v_{Mars}^2 - v_{tangental}^{Mars,2} + 2Gm_{Mars}\left(\frac{1}{r_{Mars}} - \frac{1}{d_{Moon}}\right)$$	I	Conservation of angular momentum, and energy, in Martian gravitational potential.	MartianCloud.c:54
SL-MC-Alg-09	If $$v_{Mars}^2 > v_E^2$$ Then the ejecta escapes mars orbit and is lost	I	Velocity at the moon, greater than the escape velocity at the moon.	MartianCloud.c:59
SL-MC-Alg-10	If $$v_{radial}^{Mars,2} > 0$$ Then the ejecta collides with Mars and is lost	I	When the opposite occurs, the ejecta has sufficient angular momentum that it cannot reach mars.	MartianCloud.c:69

SL-MC-Alg-11	When the ejecta re-impacts Phobos, the Impact Angle shall be modelled as: cos(2θ) is uniformly random in range [-1:1]	I	This is the same as SL-OP-Alg15	MartianCloud.c:83
SL-MC-Alg-12	When the ejecta re-impacts Phobos, the Impact Mass shall be the same as the Ejecta Mass	I	No mass is lost when in the cloud	MartianCloud.c:88
SL-MC-Alg-13	When the ejecta re-impacts Phobos, the Impact Velocity shall be the same as the Ejecta velocity in the Moons frame of reference	I	Conservation of energy	MartianCloud.c:91

5.8 Radiation environment

5.8.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-RE-In-01	The inputs shall be: • Radiation Rate vs Depth at discrete points • Required Depth	I	Radiation Rate will be dependent on Radiation Type The Rate is per unit time	Spline.c:38
SL-RE-Out-01	The Outputs shall be: • Radiation Rate at Required Depth	I	Ejected is a logic value, that say if the material is	Spline.c:38

5.8.2 Initialisation

Number	Requirement	Verification	Comment	Code
SL-RE-Init-01	The spline must be initialised	I	SplineInit	Spline.c:4

SL-RE-Init-02	The initialisation shall calculate the second derivative at each discrete depth	I	The second derivative defines the cubic spline	Spline.c:32
SL-RE-Init-03	The second derivative at the two endpoints shall be zero	I	This is the natural spline, where the two free degrees of freedom in a cubic spline are set through the second derivative of the end points.	Spline.c:9 Spline.c:27

5.8.3 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-RE-Alg-01	The radiation variance with depth shall be calculated using a cubic spline	V		Spline.c:66
SL-RE-Alg-02	The cubic spline shall be evaluated from the radiation and the second derivative of the radiation at discrete depths	I		Spline.c:66
SL-RE-Alg-03	The radiation rate will be differential.	I	E.g. must be multiplied by the duration of exposure to obtain the total exposure in that period.	Radiation.c:21 RadSter.c:13-15
SL-RE-Alg-04	Galactic Cosmic Radiation shall have a spline fitted to linear depth.	I	Fits the distribution type	PhobosRadiation.c:19-23
SL-RE-Alg-05	Solar Energetic Photons shall have a split fitted to logarithm of depth	I	Due to steep increase at decreasing depth	PhobosRadiation.c:45-49
SL-RE-Alg-06	Primordial Radioactive Decay shall be uniform	I		PhobosRadiation.c:10

	with Depth			7-109
SL-RE-Alg-07	Solar Energetic Photons distribution will flatten below TBD depth	I	Physically cannot increase without limit	PhobosRadiation.c:76
SL-RE-Alg-08	The Primordial Radioactive Decay shall use the central values for concentration in soil	I		Radiation.c:149-151
SL-RE-Alg-09	Below a fixed depth DMin, the SEP will flatten to a constant value (equal to the dose at that depth).	I	DMin will be set to 0.4mm	PhobosRadiation.c:76: sterlim.h:80

5.9 Radiation Inactivation

5.9.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-RI-In-01	The inputs shall be: • Rate of Radiation Dose @LET>0 • Rate of Radiation Dose @LET>69MeVcm2/g • Rate of Radiation Dose @LET>534MeVcm2/g • Organism • Mass • Time mode	I		RadSter.c:9
SL-RI-Out-01	The outputs shall be: • Logarithmic sterilization • Time Duration • Mass	I		RadSter.c:9

5.9.2 Algorithmic Form

Number	Requirement	Verific	Comment	Code

		ation		
SL-RI-Alg-01	The time duration that the organism spends on the surface will be modelled uniform in time duration of the simulation.	I	Nominally the time duration is set to 10MY.	RadSter.c:11
SL-RI-Alg-02	The dose for a particular LET shall be calculated from the dose rate times the time duration	I		RadSter.c:13-15
SL-RI-Alg-03	The sterilization model used will be independent of LET.	I		RadSter.c:19
SL-RI-Alg-04	The logarithmic sterilization (natural log) shall be modelled as: $$\ln(S) = \lambda D$$	I	Where λ depends on organism, this is for LET≥0 only.	RadSter.c:35
SL-RI-Alg-05	When calculating mass transfer, the time spent in the radiation environment shall be uniform over the simulation duration.	I	Nominally 10MY	RadSter.c:17
SL-RI-Alg-06	When looking at unsterilized mass, the time spent in the radiation environment shall be calculated from: $$y = \frac{R}{\lambda\, dD/dt}$$ $$T = \frac{ln(y\lambda\, dD/dt)}{\lambda\, dD/dt}$$ $$dm \to -\frac{dm}{y(\lambda\, dD/dt)^2 T_{Total}}$$	I,V	Where R is uniformly random on [0:1]. This transform flattens the radiation sterilisation integral.	RadSter.c:20 RadSter.c:21 RadSter.c: 23

6. TESTING, VERIFICATION AND VALDIATION

6.1 Approach

Testing methodology in the Requirements in section 5, are of two types:

- I: Inspection
- V: Validation

Inspection is of the code, against the specified required. This is verified by explicit examination of the code, and it is recorded by commenting the code with the requirement it meets. This ensures that all Inspection requirements are met.

Validation is more subtle, it is does the code produce physically reasonable results against what can reasonably be expected. So specifically validation is used for the physical processes where there is an expectation as to what to expect.

This section documents those areas that needed validation.

6.2 Mars Ejecta

6.2.1 SL-ME-Alg-5

For SL-ME-Alg-05 the normalisation of the ejected mass needed to be matched to [AD2]. Specifically from [AD2] the normalisation is taken from Figure 6-1.

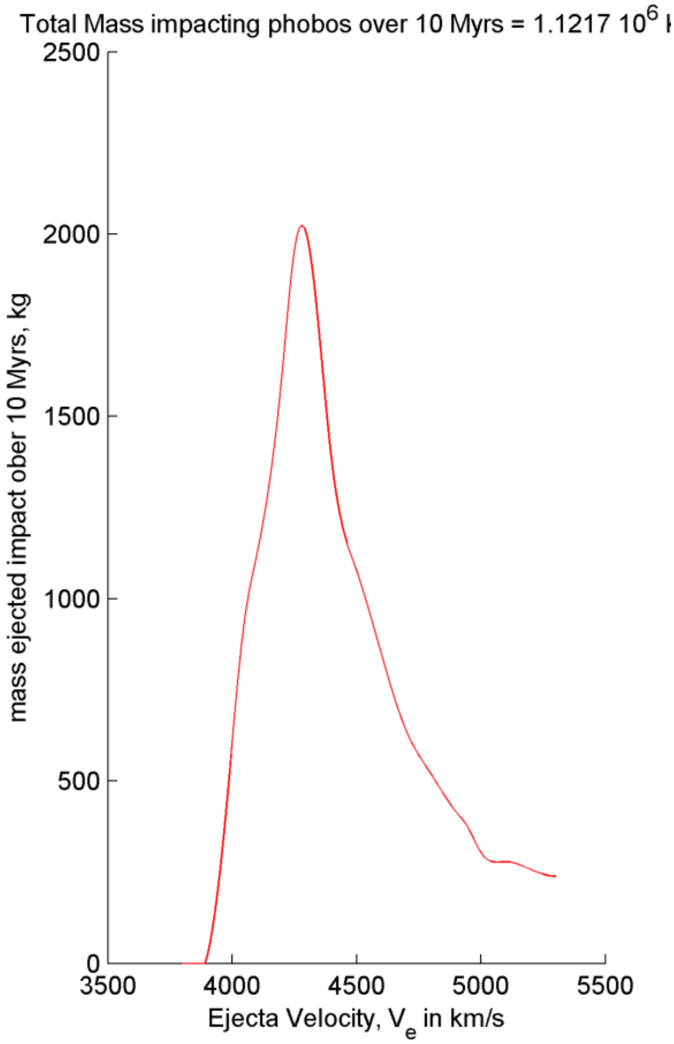


Figure 6-1. Figure V1.5.1 from [Ad2] The mass ejected over 10MY with an ejection cone angle of 45 degrees

Producing the same graph from the Monte Carlo simulation gives the graph shown in Figure 6-2.

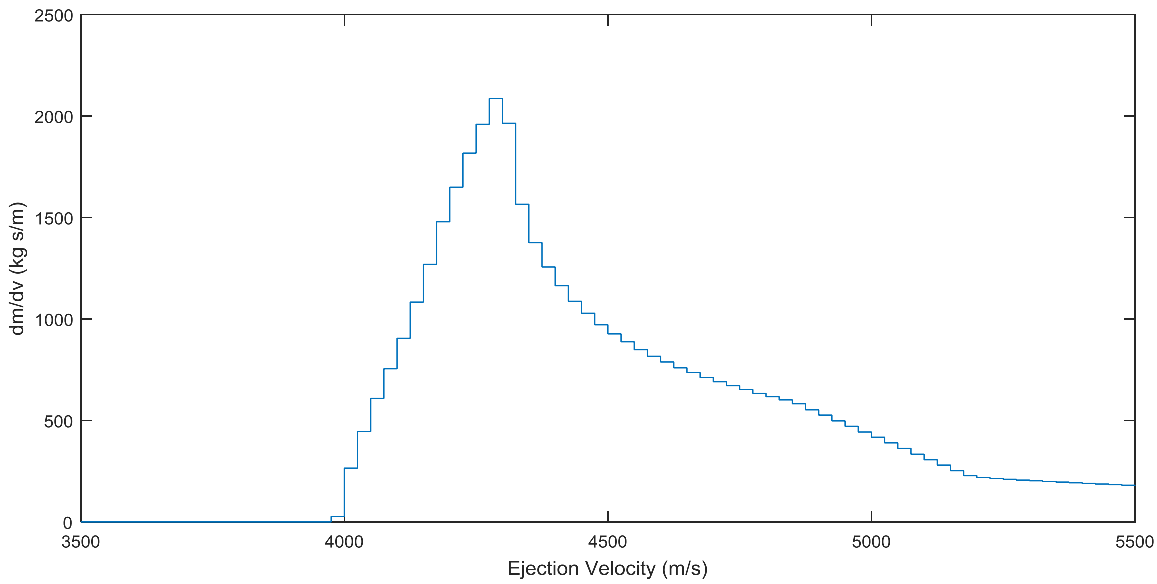


Figure 6-2. The mass distribution produced by the Monte Carlo for comparison with Figure 6-1. This is after normalisation.

This is after the graph has been normalised for agreement. Comparing the two graphs:

- The height of the peak is comparable ~ 2000 kg s /m
- The height of the tail at large velocities is comparable at 250 kg s/m
- The lower cut off differs, Melosh has ~3800m/s the Monte Carlo has 4000m/s. This is given by the orbital dynamics as the minimal ejection velocity to reach Phobos. Hence that is not tuned by the overall mass
- Melosh has total mass transferred as 1.1217e6 kg, the Monte Carlo produces 1.6302e6 kg. This difference has been traced to the Monte Carlo producing ejecta beyond 5.5km/s, whereas Melosh is normalised up to 5.5 km/s. It has been confirmed with Melosh that the size of the differential distribution is where there should be agreement. Hence the higher total mass estimate from the Monte Carlo is understood.

This agreement has been set by setting the reference mass ejected as 2.9e12kg at speeds above a cut off of 3.8km/s; this speed was chosen as being just below the minimal speed to reach Phobos.

The agreement here although not exact, is felt to be very good – and this brings confidence in both the ejecta modelling, and the orbit propagation of material between Mars and Phobos.

6.2.2 SL-ME-Alg-8

The mass distribution of the Mars Ejecta is tested in two ways, firstly the mass distribution is modelled as a γ=2 distribution, so a plot is taken of the differential mass distribution against Ejecta Mass. This is performed with a lower mass cut off of set to 1mg, and is shown in Figure 6-3.

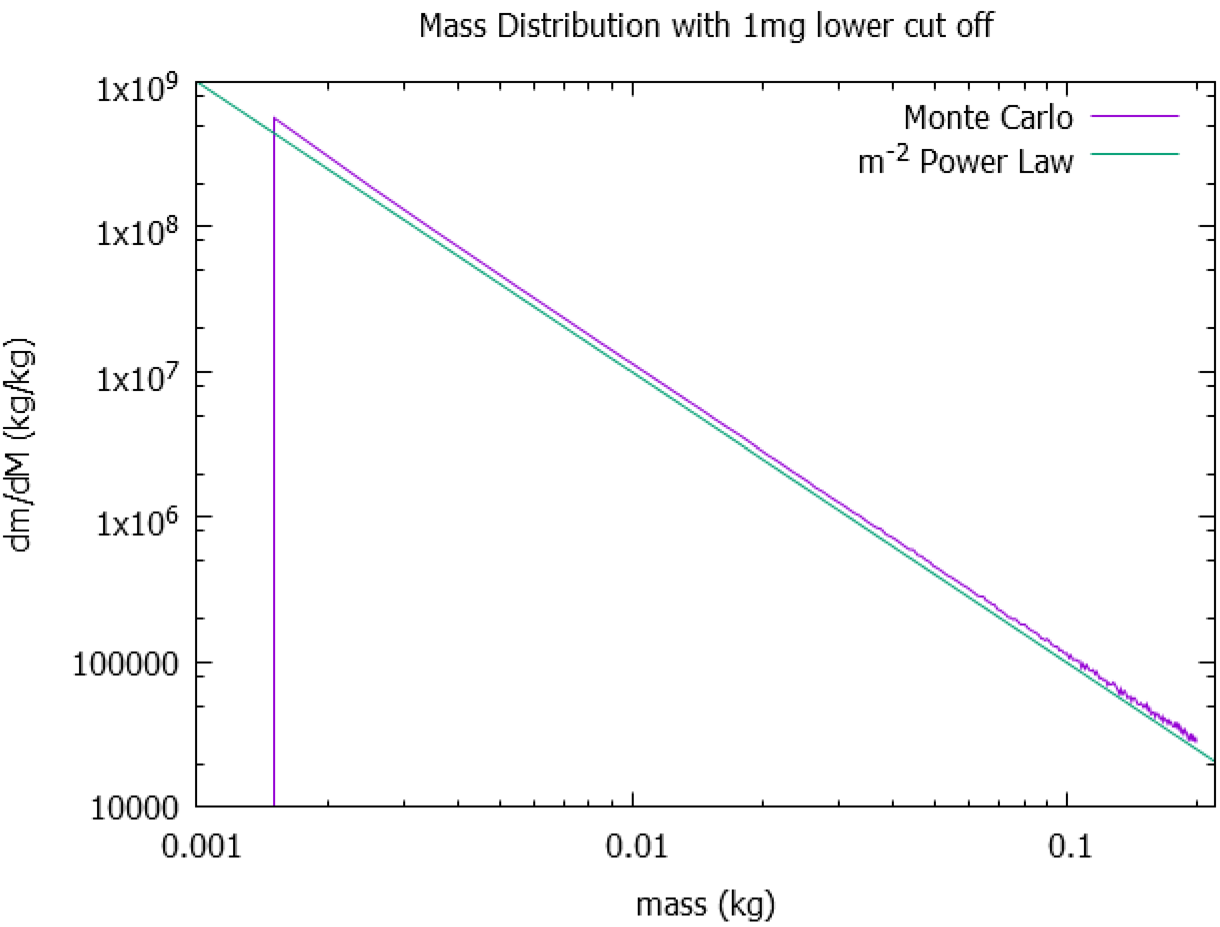


Figure 6-3. The differential distribution of the mass ejecta masses, this is compared against a γ=2 power law, with arbitrary normalisation. The agreement in slope is excellent.

Now as [AD2] described the power law in terms of ejecta diameter. The differential diameter is also plotted. To calculate the diameter from the mass the equation:

$$m = \frac{\pi\rho}{6} D^3$$

Is inverted:

$$\left(\frac{6m}{\pi\rho}\right)^{1/3} = D$$

Where the density used is 2000kg/m^3.

This is compared against a γ=4 power lab, and shown in Figure 6-4. Clearly the agreement is excellent. This confirms the implementation in the Monte Carlo.

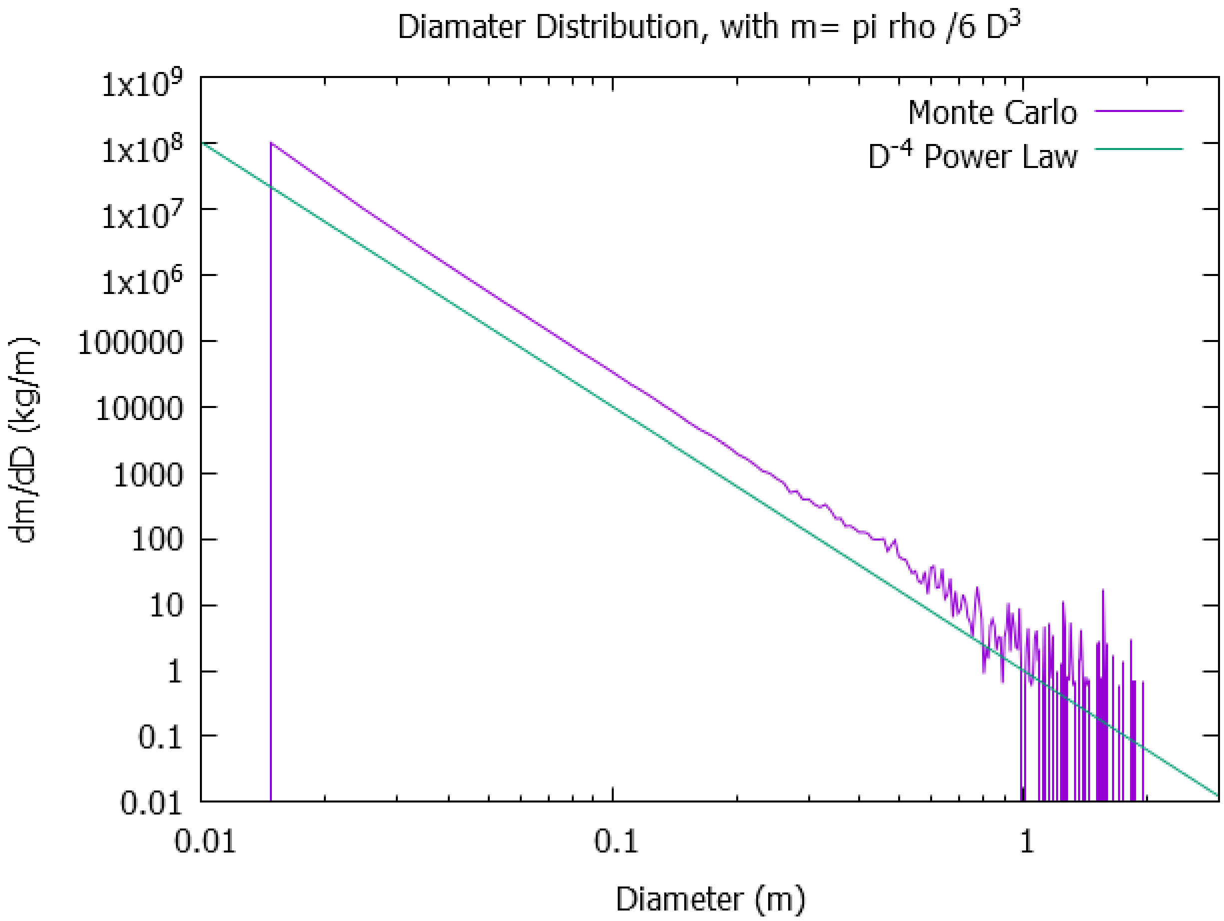


Figure 6-4. The diameter distribution for Mars Ejecta. It follows the very strongly falling γ=4 power law. Monte Carlo performed with 10^8 events, yet D=1m the rate falls down to 1 event per bin, and so the numerical errors increase.

6.3 Orbit Propagation Mars to Moon

6.3.1 SL-OP-Alg-06 & SL-OP-Alg-07

SL-OP-Alg-06 and SL-OP-Alg-07 consider the analytic orbit propagation from Mars to Phobos. Specifically the velocity of ejecta when they reach the moon, how this affects the impact points on the moon, and how this affects the effective diameter of the moon.
In order to test this, orbits have been numerically computed over a range of angles. The calculation has been performed in a rotating frame of reference, in which Mars and Phobs are stationary. The numerical integration is performed via 4th order Runge Kutta integration. Where these orbits cross Phobos orbit, it has been plotted the analytical values for:

- Velocity at the moon
- The calculated limits of the point on the surface of Phobos where the orbits are tangential
- The effective diameter of the moon, this is taken by projecting the points of the limits of impact in the direction of the velocity to a line perpendicular to the Mars direction in line with the centre of Phobos

This has been performed for two velocities:

- 4197m/s with 45° ejection angle – with only just enough velocity to reach Phobos
- 5573m/s with 45° ejection angle – greater than the Mars escape velocity

Plots of the orbits are shown in Figure 6-5 and Figure 6-6.

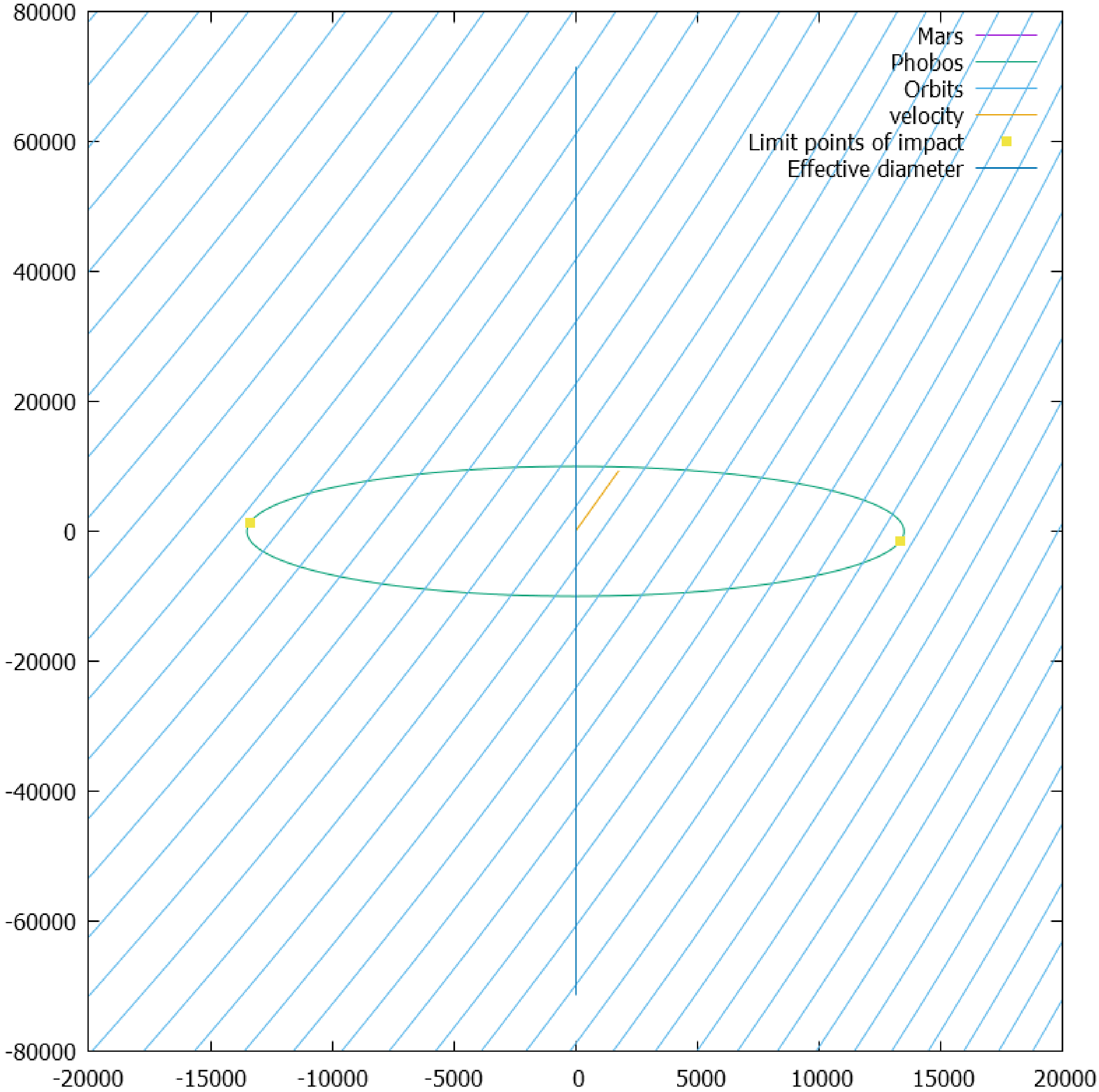


Figure 6-5. Orbit propagated numerically to Phobos with ejection speed from Mars of 4197m/s. The orbits on mars are separated by 10^{-3} rad. The critical points for impact are the limit points where impact on Phobos occurs. Shown in orange is an analytic calculation of the velocity at Phobos, it aligns well with the direction of the numerical trajectories. Analytic calculation of the limit points of Phobos impact, are shown by the yellow crosses, which also align well with the numerical trajectories. The vertical line is

an analytic calculation of the limit points projected back onto a vertical line through Phobos, this is used in calculating the effective size of Phobos.
The plot is centred on Phobos, with Mars some 9378km to the left (on the x axis). The ellipse is the approximate limit of Phobos, as used in the modelling. The x and y units are meters.

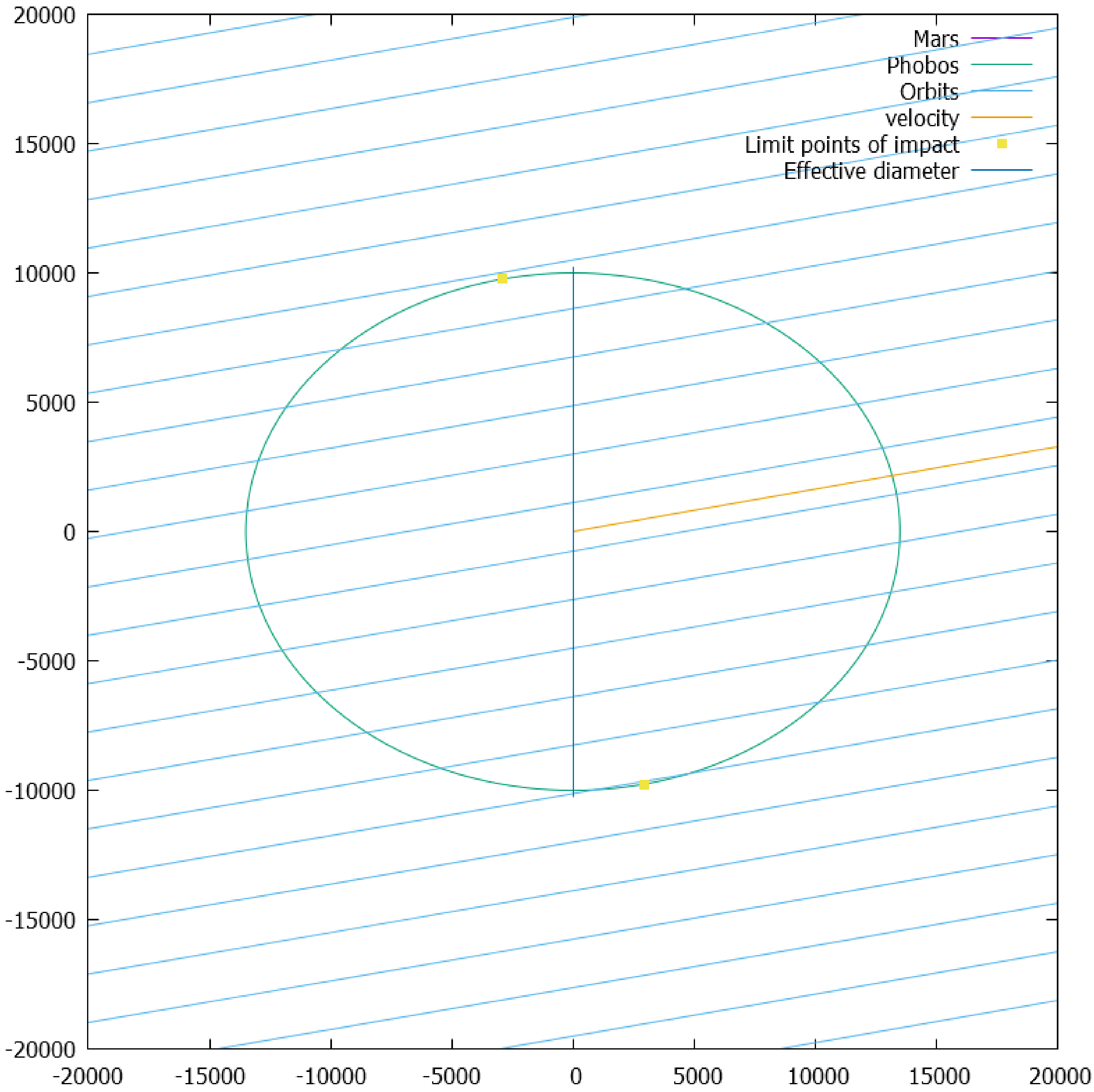


Figure 6-6. Orbit propagated to Phobos with ejection speed from Mars of 5573m/s. The orbits on mars are separated by $2{\cdot}10^{-4}$ rad. Hence $2.2{\cdot}10^{-3}$ rad wide range of angles that impact. The plot is centred on Phobos, with Mars some 9378km to the left (on the x axis).

The ellipse is the approximate limit of Phobos, as used in the modelling. The various elements are as described in caption to Figure 6-5.

Comments:

- For both launch velocities, the direction of the analytic velocity clearly aligns well with the numerically calculated orbits.
- The analytic calculations of the limits of impact also clearly align well with the modelled ellipse of Phobos
- The effective diameter should correspond to the limits of impact projected along the ejecta velocity direction to a vertical line through Phobos. This quite accurate for the higher speeds,
- With the 4197m/s ejection the diameter overshoots on one side of Phobos and undershoots on the other. This can be seen as due to the obvious curvature of the orbits at these low speeds. In particular note how the orbits bunch in the right (far from Mars) vs the left (closer to Mars). Overall though the effective diameter is still a good approximation to the numerical orbits.
- The lower speed clearly shows the expanded cross section. What is happening physically, is that the lower speeds just have enough velocity to reach Phobos altitude, but once there have little remaining velocity to pass through Phobos altitude, then Phobos velocity round Mars (~2km/s) sweeps up these slow moving ejecta. This can be clearly seen that at the lower velocity the impacts happen on the front face of Phobos (with respect to Phobos velocity); whilst the higher speed ejecta mainly impact on the Mars facing side. This is confirmed both in the numerical and the analytic calculation.

6.4 Phobos Hypervelocity impact and ejector

6.4.1 Angle of Impact with Phobos - SL-OP-Alg15

The angle of impact with the moon is modelled to follow a sin(2θ) distribution. Now as this is independent of all other variables, this distribution should be held to for all events, and also if the mass distribution is followed. Here is plotted both the distribution with respect to both events (Figure 6-7) and mass (Figure 6-8). Clearly the agreement is excellent.

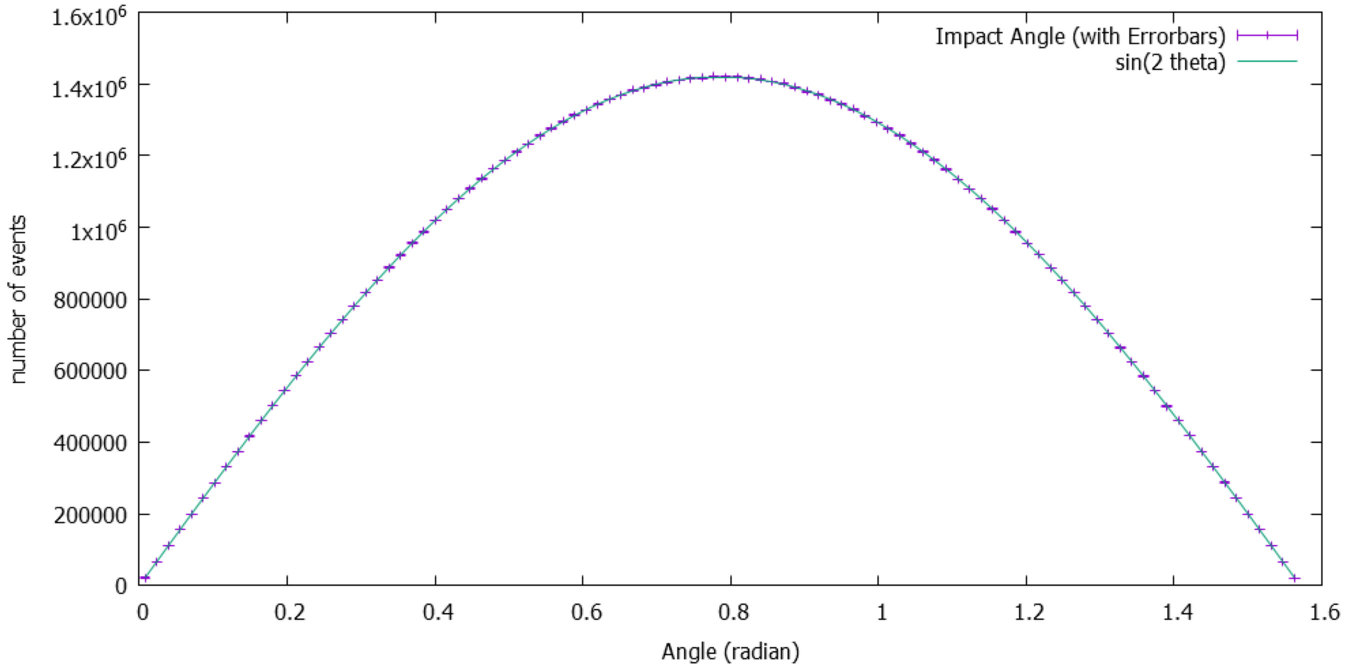


Figure 6-7. The angle of impact distribution plotted with respect to events. Also plotted is a sin(2θ) curve with approximately the same normalisation.

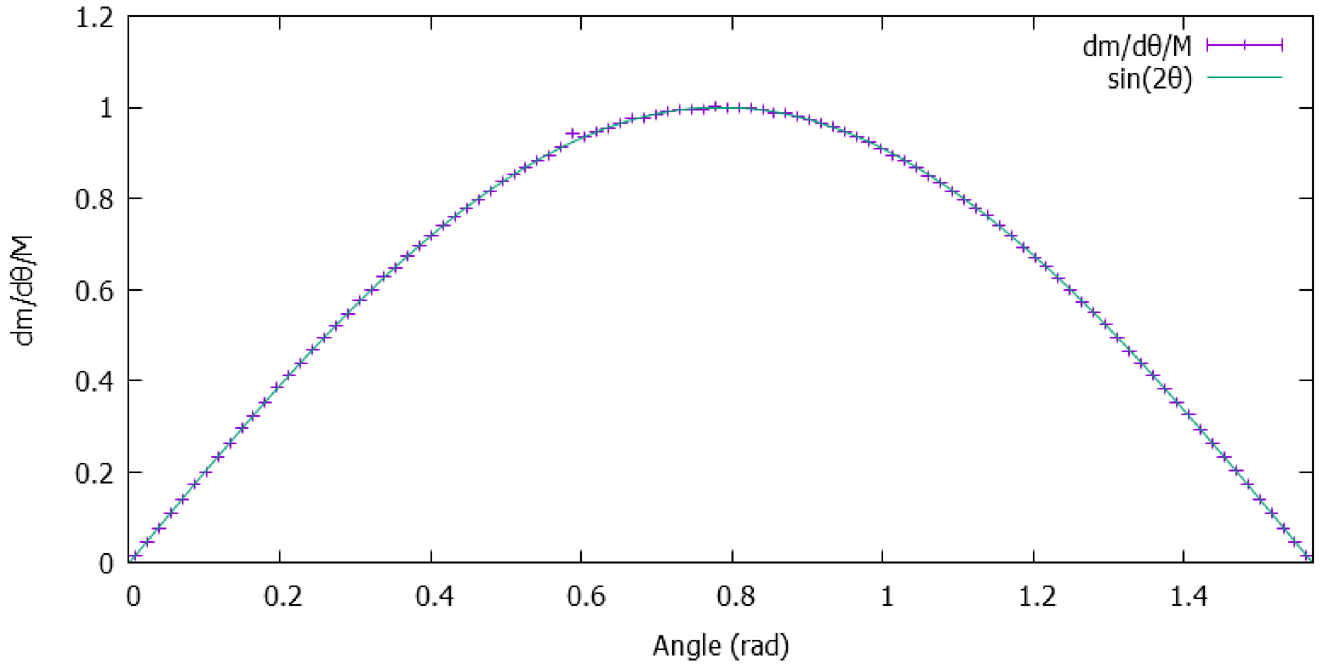


Figure 6-8. The angle of impact plotted by following mass through the simulation. This has been normalised to the total mass flow. Also plotted is sin(2θ), with the normalisation expected (e.g. area under curve is 1).

6.5 Heat inactivation

To verify the heat inactivation code, the comparison is to be made against the thermal sterilization model [RD6]. The thermal model with cooling included, is the hyper velocity modelling, where as an intermediate step the temperature was simulated, and the sterilization calculated against that temperature with cooling afterwards. The Hypervelocity modelling included a delay loss term, for the sterilization caused by contact time with the regolith simulant. As this contributed to the sterilization seen, it is included in both the HV modelling, and in this Heat Inactivation code. This can be compared with the heat inactivation code – and this is performed here.

N.B. This test was performed before the three confidence level models were introduced, and the comparison against the HV was also before the confidence level models were introduced.

Hence this test cannot easily be repeated with the three confidence level models.

However the code that this test verified, is mostly unchanged. Hence the confidence with this test brought is carried through to when confidence levels are added. So this section is left as before.

6.5.1 B atrophaeus

T(K)	Delay Loss	Ln(Ne/N0) HV Model	Ln(Ne/N0) Heat Inactivation
301.49	0.00	0.00000	0.000000
305.95	0.00	0.00000	0.000000
313.39	0.00	0.00000	0.000000
323.81	0.00	-0.11654	-0.113070
343.15	0.00	-0.65870	-0.653637
370.71	0.00	-1.52243	-1.515045
407.92	0.00	-3.12903	-3.107873
460.00	0.00	-6.70159	-6.651562
502.50	0.00	-11.21387	-11.124436
550.00	0.00	-18.45000	-18.296068

6.5.2 D. radiodurans

T (K)	Delay Loss	Ln(Ne/N0) HV Model	Ln(Ne/N0) Heat Inactivation
301.49	-3.25	-3.25000	-3.250000
305.95	-3.25	-3.25000	-3.250000
313.39	-3.25	-3.25000	-3.250000
323.81	-3.25	-3.26294	-3.262552
343.15	-3.25	-3.33493	-3.334192
370.71	-3.25	-3.49447	-3.492720

407.92	-3.25	-3.93034	-3.923949
460.00	-3.25	-5.43329	-5.408717
502.50	-3.25	-8.14449	-8.084226
550.00	-3.25	-13.94433	-13.804630

6.5.3 *B. diminuta*

T (K)	Delay Loss	Ln(Ne/N0) HV Model	Ln(Ne/N0) Heat Inactivation
301.49	-0.70	-0.70000	-0.700000
305.95	-0.70	-0.70000	-0.700000
313.39	-0.70	-0.70000	-0.700000
323.81	-0.70	-0.71002	-0.709720
343.15	-0.70	-0.77487	-0.774152
370.71	-0.70	-0.96351	-0.961099
407.92	-0.70	-1.67128	-1.659681
460.00	-0.70	-5.17025	-5.104815
502.50	-0.70	-13.56237	-13.356575
550.00	-0.70	-36.26520	-35.664054

6.5.4 *MS2*

T (K)	Delay Loss	Ln(Ne/N0) HV Model	Ln(Ne/N0) Heat Inactivation
301.49	-2.20	-2.20000	-2.200000
305.95	-2.20	-2.20000	-2.200000
313.39	-2.20	-2.20000	-2.200000
323.81	-2.20	-2.28703	-2.284442
343.15	-2.20	-2.69776	-2.693892
370.71	-2.20	-3.36976	-3.363887
407.92	-2.20	-4.65914	-4.642052
460.00	-2.20	-7.62995	-7.587888
502.50	-2.20	-11.49362	-11.416399
550.00	-2.20	-17.83649	-17.700115

6.5.5 *Comparison*

Comparing the HV Modelling [RD6] to the Heat Inactivation code – there are clearly small changes in sterilization, up to ~0.4 of a ln. This is larger than ideal, hence as both sterilizations are calculated using numerical integrals the number of points have been increased. This was performed for B Dim, where the larges errors were seen:

T (K)	Delay Loss	Ln(Ne/N0) HV Model 1000 points	Ln(Ne/N0) HV Model 5000 points	Ln(Ne/N0) Heat Inactivation	Ln(Ne/N0) Heat Inactivation

				1e6 points	5e6 points
301.49	-0.70	-0.70000	-0.70000	-0.700000	-0.700000
305.95	-0.70	-0.70000	-0.70000	-0.700000	-0.700000
313.39	-0.70	-0.70000	-0.70000	-0.700000	-0.700000
323.81	-0.70	-0.71002	-0.70978	-0.709720	-0.709720
343.15	-0.70	-0.77487	-0.77429	-0.774152	-0.774152
370.71	-0.70	-0.96351	-0.96166	-0.961099	-0.961099
407.92	-0.70	-1.67128	-1.66E+00	-1.659681	-1.659681
460.00	-0.70	-5.17025	-5.11784	-5.104815	-5.104815
502.50	-0.70	-13.56237	-13.39751	-13.356575	-13.356575
550.00	-0.70	-36.26520	-35.78375	-35.664054	-35.664053

The Hyper Velocity modelling shows significant change in answer with increased points. The heat inactivation code (which starts with 1000 times more points) – and the answers show no significant change.

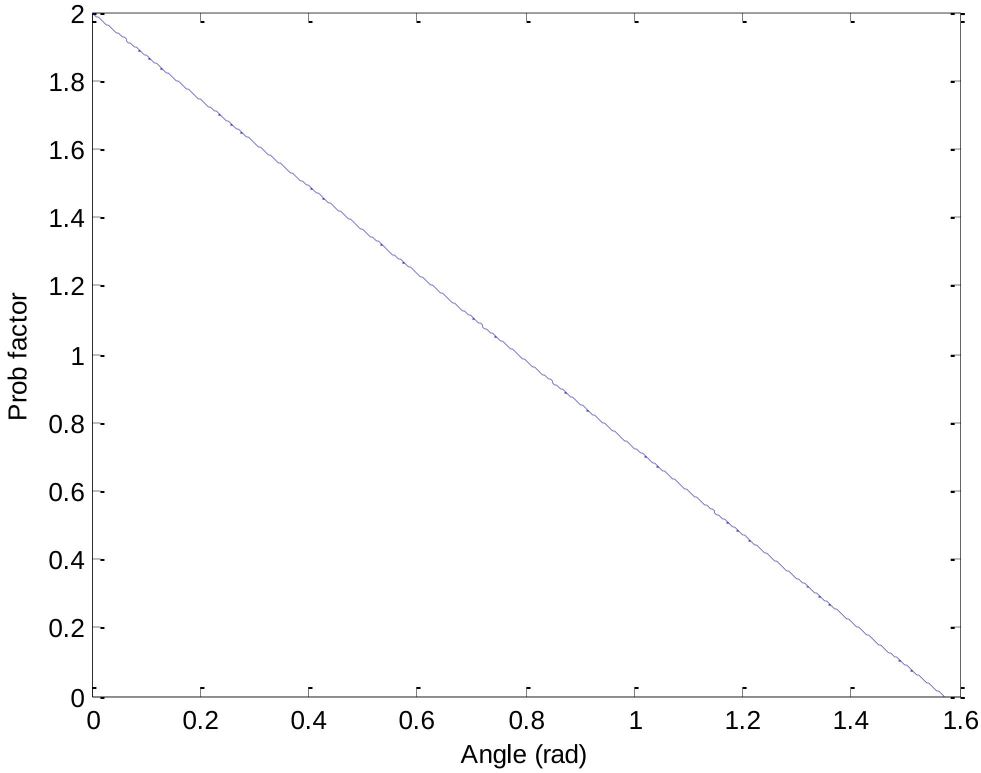
Hence the difference is attributed to numerical round in the Hyper Velocity model.

6.5.6 Ejection chance SL-HVI-Alg-14

SL-HVI-Alg-14 gives the probability of ejection:

- Zero for impact angles <45°
- Increase linearly from 0 to 100% for angles from 45° to 90°

This is performed by constructing a variable and comparing to a uniform random number in range [0:1]. The function used is shown below.



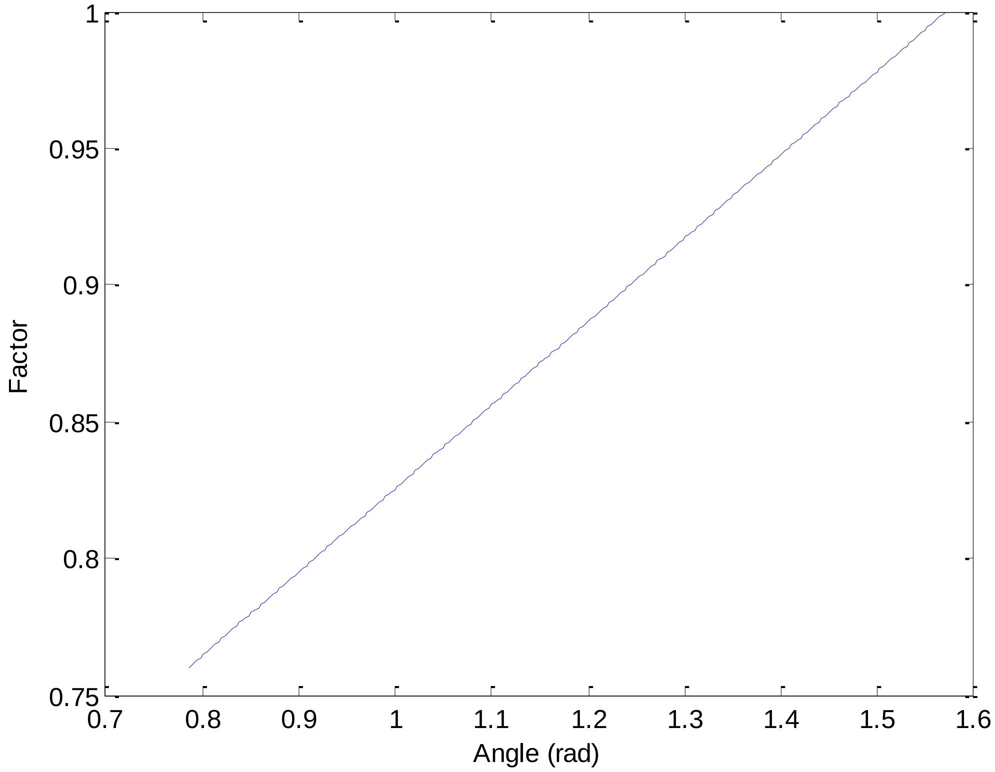
For angles less than pi/4, the probability factor is greater than 1.
For angles greater than pi/4, the probability factor tends from 1 to zero as the angle approaches pi/2.
Hence comparing this value to a random number in range [0:1] will give the desired probabilities for angles over 45°.

6.5.7 SL-HVI-Alg-19

SL-HVI-Alg-19 gives the Ejected tangential velocity:

- 76% at 45° impact angle
- 100% at 90° impact angle
- Increase linearly between 45° and 90°

A variable is constructed depending on impact angle and is shown below



At an angle of pi/4 the factor is 0.76, and tends linearly to 1 at pi/2. Hence this factor is as expected.

6.6 Radiation environment

6.6.1 Introduction

The primary direct verification that is required is that is a cubic spline sufficient to fit to the model data. A few test cases have been considered

6.6.2 Test Case 1

The test data used here is:

```
struct rad radGamma = {
  .size=20,
  .raddata={
    {0.0, 100.0, 0},
    {0.1, 98.0, 0},
    {0.2, 97.0, 0},
    {0.3, 98.0, 0},
    {0.4, 101.0, 0},
    {0.5, 95.0, 0},
    {0.6, 90.0, 0},
    {0.7, 85.0, 0},
    {0.8, 80.0, 0},
    {0.9, 75.0, 0},
    {1.0, 70.0, 0},
    {2.0, 50.0, 0},
    {3.0, 30.0, 0},
    {4.0, 20.0, 0},
    {5.0, 15.0, 0},
    {6.0, 10.0, 0},
    {7.0, 7.0, 0},
    {8.0, 5.0, 0},
    {9.0, 4.0, 0},
    {10.0, 3.0, 0}}
};
```

Specifically the radiation is specified at depth with separation of 0.1 down to 1; then separation of 1 down to 10. This probes that the spline passes through the points, and can cope with a change in resolution.

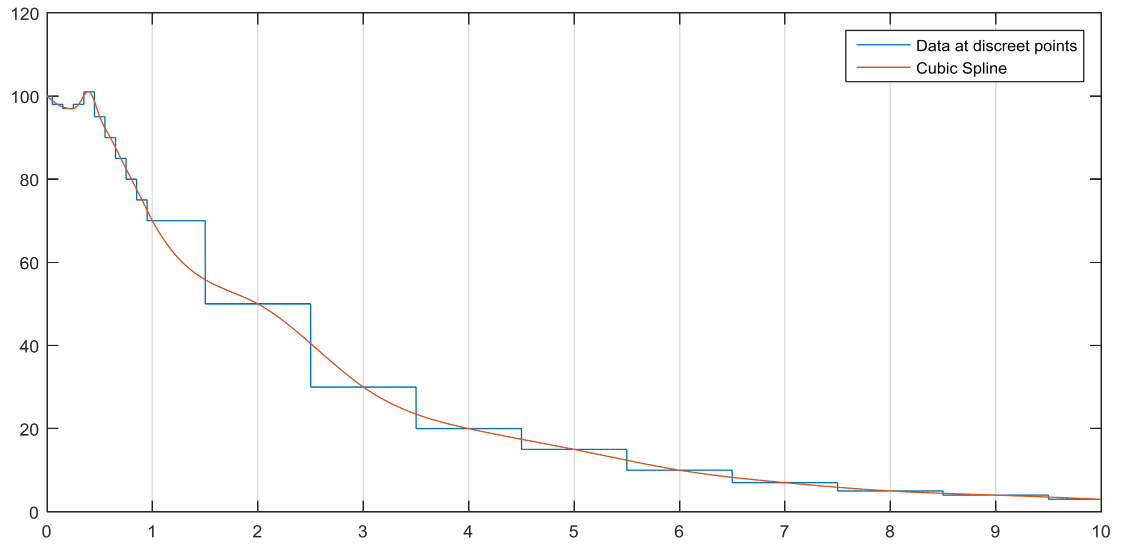


Figure 6-9. The discrete data plotted, and the spline through the data. The two should match at the data points (integers in this graph, and tenths in the next). The spline should be smooth everywhere.

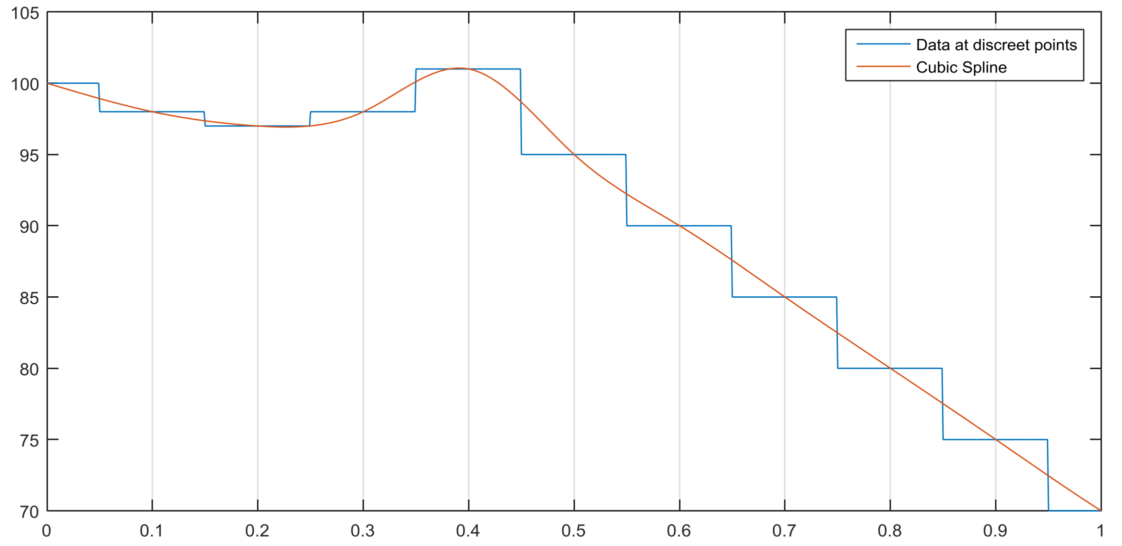


Figure 6-10. As Figure 6-9 but zoomed in on the range [0-1] to show the match at tenths below 1.

6.6.3 Test Case 2

The test data used here is:

```
struct rad radGamma = {
  .size=11,
  .raddata={
    {0.0, 1.0, 0},
    {1.0, 10.0, 0},
    {2.0, 100.0, 0},
    {3.0, 1000.0, 0},
    {4.0, 10000.0, 0},
    {5.0, 100000.0, 0},
    {6.0, 1000000.0, 0},
    {7.0, 10000000.0, 0},
    {8.0, 100000000.0, 0},
    {9.0, 1000000000.0, 0},
    {10.0, 10000000000.0, 0}}
};
```

That is a fairly strong power law.

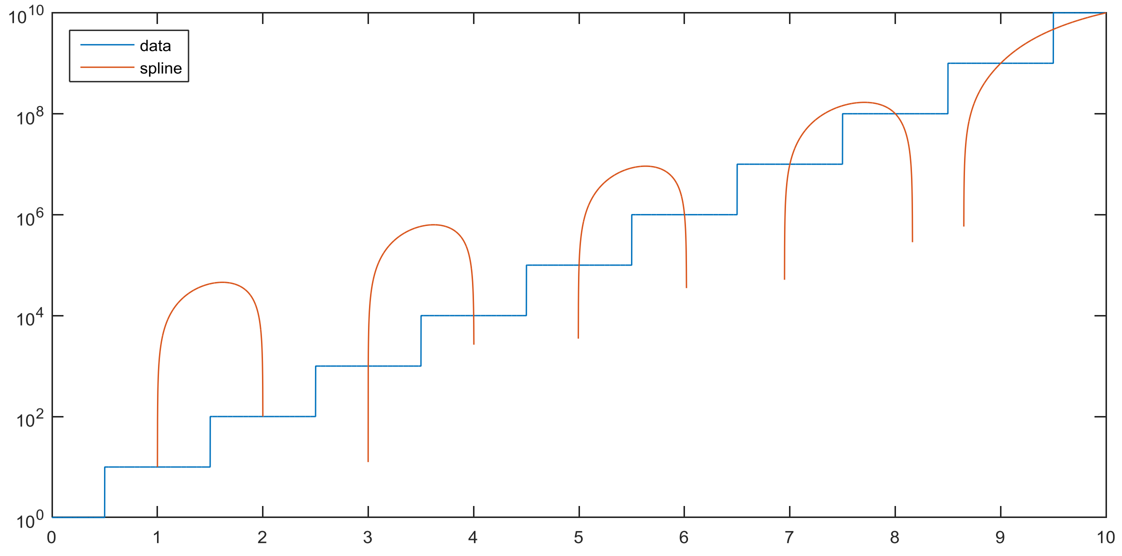


Figure 6-11. The fitted spline, although it does pass through all the data points, deviates wildly between the points, often going negative (so not shown on log graph), and between passing way above the value (factor of 10^3+).

Clearly such a strong power law, is not well represented by a cubic spline. The spline knows nothing of the data, in particular that it is a power law and always positive.

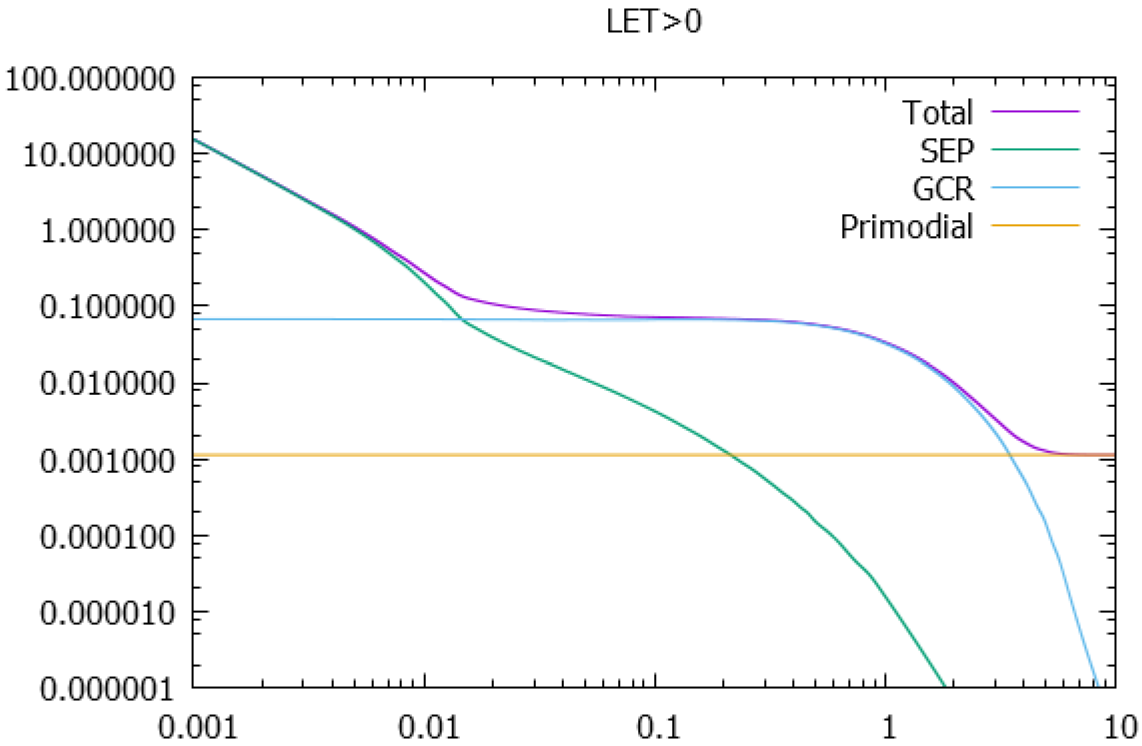
6.6.4 Conclusions

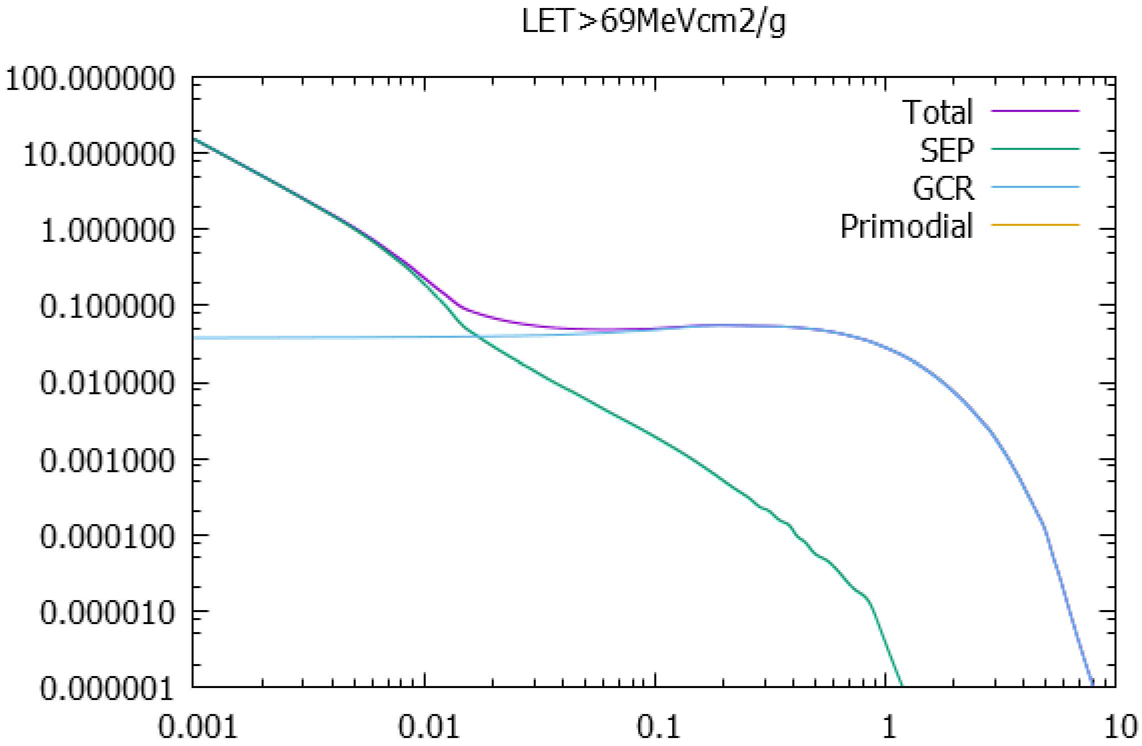
The fit of the spline to discrete data points clearly depends on the nature of the data. In particular a steep power law is problematic. Now as radiation from a solar system origin will be shielded by increasing depth of regolith, this will naturally give a power law – but not the steepness. Hence before fitting to the actual data, it cannot be said if the fit is sufficient.

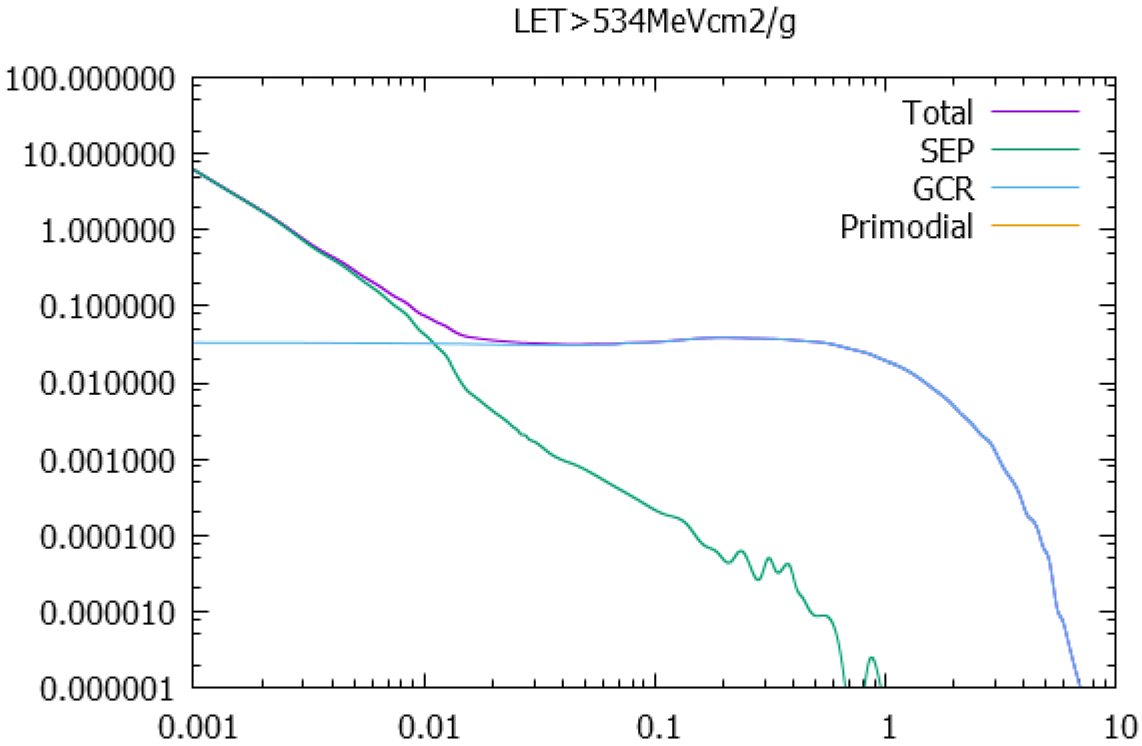
Practically though if issues are found, the fit can be modified to fit most forms. E.g. for the power law above, taking logarithms before the fit, would solve the problem.

Hence the suitability of the fit needs to be assessed after the radiation model for Phobos has been developed.

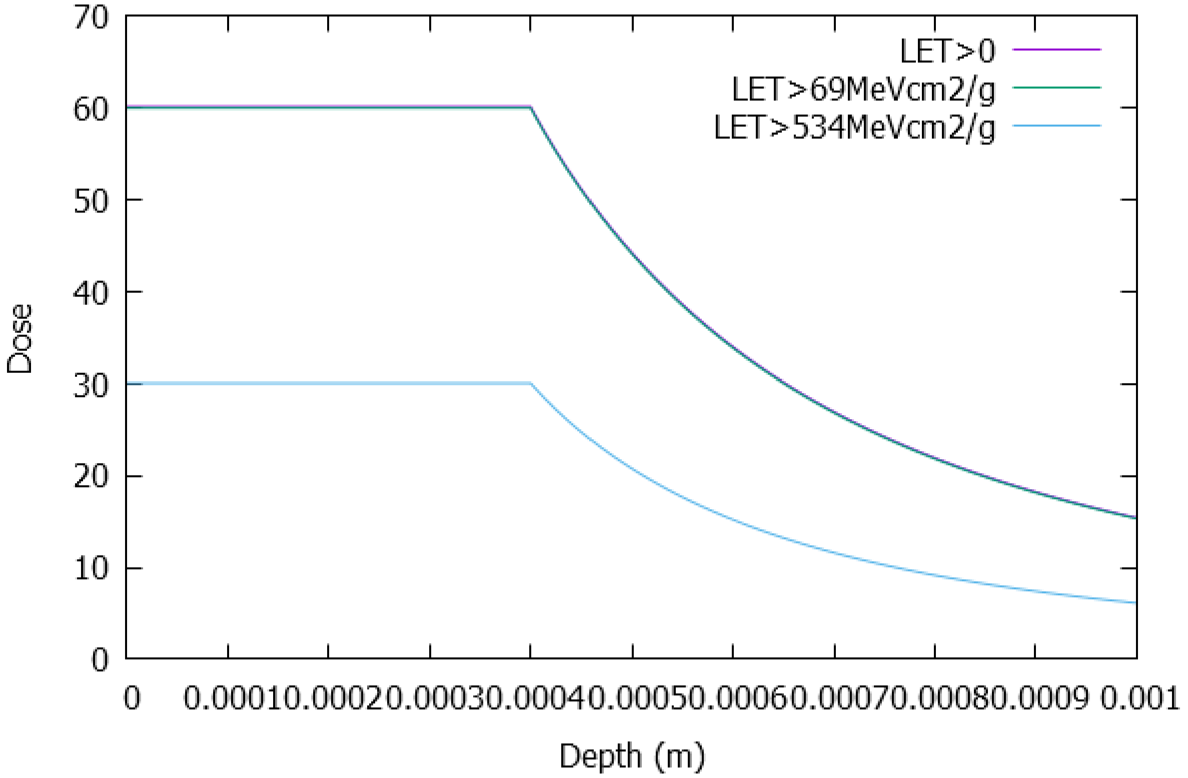
6.6.5 Fit to Data







On a linear graph, plotting the total radiation bellow 1mm (which is dominated by SEP) gives:



Which demonstrates the radiation flattening below a depth of 0.4mm, as the code is designed.

6.7 Radiation Inactivation

6.7.1 SL-RI-Alg-06

SL-RI-Alg-06 introduced a change in variable when looking at unsterilized mass, specifically the sterilisation caused by radiation is a gradual process which over time gives an exponential decay in unsterilized material. The variable change has time follow this decay, so more events are generated in the recent pass where there is more unsterilized material. The variable change though is done in a way that should not change distributions, whilst it changes the probability of choosing events, it affects the event weight to exactly compensate the change of variable. This then becomes a good test of the validity of the code, physical distributions produced by linear time (SL-RI-Alg-05) should be the same as for exponential time (SL-RI-Alg-06).

So for this test unsterilized mass has been plotted against time (deposited on a moon surface) for both linear and logarithmic time. The radiation should give an exponential decay, that is the same for both modes of generating time. This is shown in Figure 6-12.

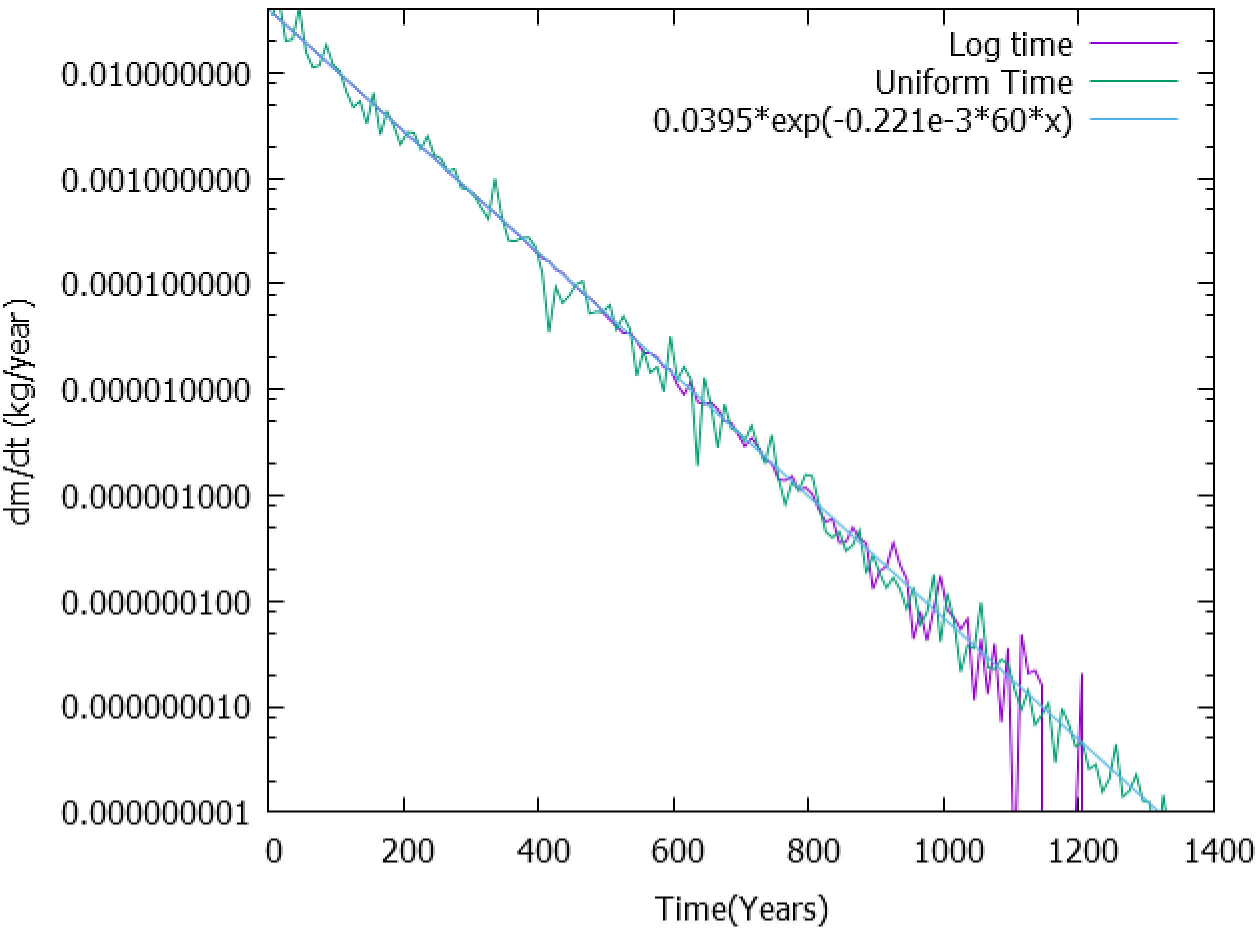


Figure 6-12. Unsterilized mass transfer against time, plotted where time is generated linearly or exponentially. The two mass lines align well, as expected; and also align with exponential decay with the parameters taken from the organism and radiation environment.

In this plot the mass distribution against time is clearly the same when generated using the two methods of generating time of exposure, they also line up well with an exponential decay that is expected from the organism (Super Bug in this plot) and the radiation environment.

What can also be seen is how the numerical errors differ between the two methods. Uniform time has large numerical errors everywhere, but the stay proportionally the same over time – this is because there are equal number of events per unit time. The logarithmic time puts far more events at short times, where most of the unsterilized mass is, this makes the errors there very small; however at longer times, where little mass is transferred the proportionate errors grow. This illustrates how careful choice of time should be made, depending on the plot being made.

7. DISCRETE MARS EJECTION EVENTS ALGORITHMS

7.1 Analysis

7.1.1 Introduction

This section considers the Mars Ejection Process happening as a series of discrete events (previously this was considered as a continuous process).

Each discrete ejection will have both a probability for material to reach the moon, and in the case where the moon is reached the fraction of the ejection mass which reaches the moon. Previously these were integrated over, however for discrete ejection, these need discrete values.

Now Section 4.3 and 4.4 considered the transfer of material from Mars to a Moon, it is strongly dependent ejection velocity, and weekly dependent on the ejection angle (the point of emission on Mars was integrated over). With discrete processes, more attention is needed as both the proportion of mass and the probability is required.

7.1.2 Transfer from Mars to the Moon

Previously the approach used was to follow the progress of mass through a sphere about Mars at a radius of the Moon. This approach is suited to discrete ejections as well. Consider Figure 7-1:

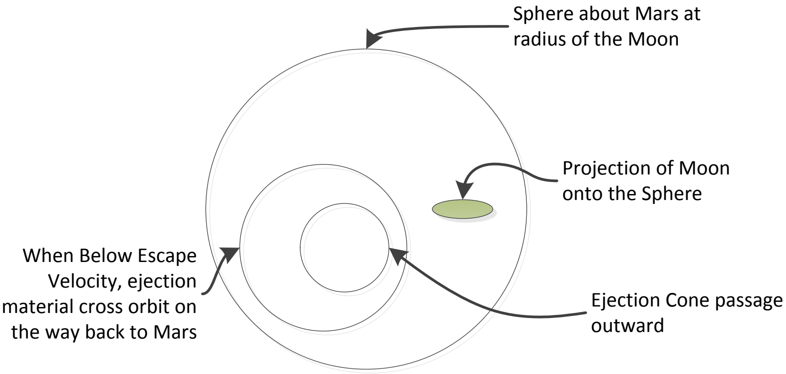


Figure 7-1. The Sphere about Mars at the orbit of the moon. The moon has been projected onto the sphere with rotational speed of the moon. The passage of the Mars ejection cone, both outward, and on the return to the surface is also shown.

- The ejection cone on cutting the sphere around Mars forms a circle
- Where the ejection velocity is below the Mars escape velocity, material will return to Mars. As it passes through the sphere a second time, it forms a second circle

- As the returning material passes through the sphere at a later time than the outgoing material, the returning circle is translated relative to the outgoing
- The projection of the moon onto the sphere depends on the relative velocity between the moon and the Mars ejecta, hence as the velocity varies around the ejection cone circle, the shape of the moon will vary as well, however the 2km/s orbital velocity of the moon is expected to dominate
- The probability for the ejection cone to deposit mass on the moon, is the probability that the circle crosses the ellipse of the moon.
- The mass transferred is the proportion of the circle which crosses the ellipse of the moon

So as before, by integrating over the launch position a probability for any part of the ejecta to collide with the moon can be established (previously just an isolated particle was considered). The size of the ejection cone circle for all ejection angles considered will be far larger than projection of the moon, so consider (in angular co-ordinates) the range of launch points (centre of the cone circle) which impact the moon, Figure 7-2.

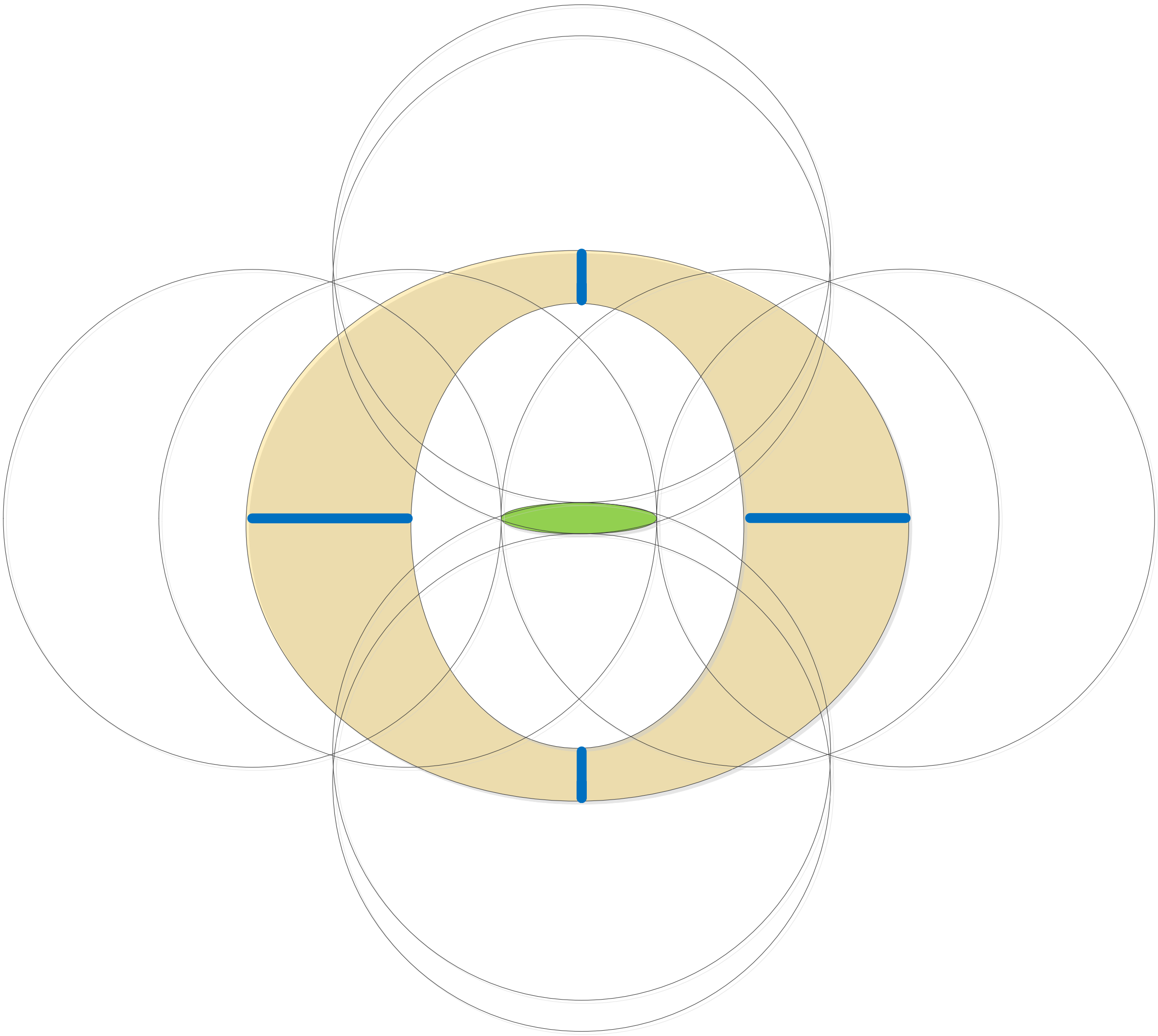


Figure 7-2. The extent of the circle of Mars Ejecta that will impact the moon. Projection of the moon shown in green, ejecta circles as outlines. Centre point of the circle, for 4 sides shown in blue lines, and the area that the centre of the circle must lie in brown area.

So if the projection of the moon is centred on (0,0) with semi-major axis, a, and semi-minor axis b; and the radius of the ejection circle, r; then the parameters of the inner and outer ellipse of the centre point of the ejecta circle are given by:

	LR "a/b"	UD "a/b"
Inner Ellipse	r-a	r-b
Outer Ellipse	r+a	r+b

And the area of the region where emission will collide with Phobos is given by:

$$A = \pi\big((r+a)(r+b) - (r-a)(r-b)\big)$$

$$= \pi\big((r^2 + r(a+b) + ab) - (r^2 - r(a+b) + ab)\big) = 2\pi r(a+b)$$

Note that if looked closely in Figure 7-2 the inner ellipse NS is slightly out, as the circle has greater curvature than the ellipse at that point. This will typically not happen in the simulation, where the ejection circle if far larger than projection of the moon.

For ejection below the Mars escape velocity, the fall back to Mars surface will produce a second circle on the sphere

7.1.3 Ejection Cone Evolution

Material ejected in a mass ejection event is ejected at an angle, this evolves in the gravity of Mars to the circle on sphere at the moons orbit. This evolution is ideally performed analytically, which is performed in this section. Consider Figure 7-3:

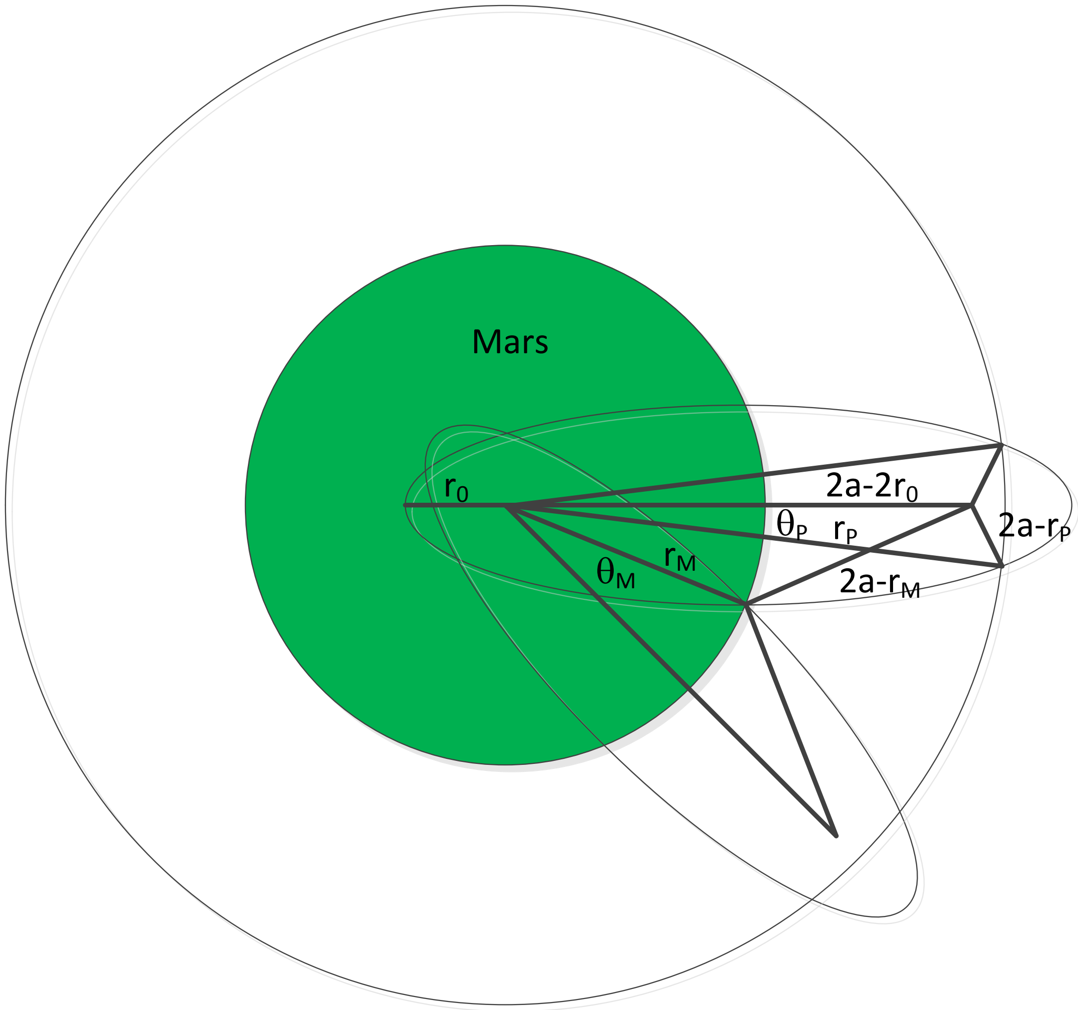


Figure 7-3. Ejection from Mars and orbit parameters.

For ejection velocity below the escape velocity, the ejecta travel on elliptical paths. The properties of the ellipse can be used to calculate the size of the circle at the distance of the moon. One focus of the ellipse is centred on Mars.

Specifically consider the properties of the ellipse. The radial velocity goes to zero at either end of the ellipse:

$$v_r^2 = v^2 - v^2\sin^2(\theta)\left(\frac{r_M}{r}\right)^2 - 2GM\left(\frac{1}{r_M} - \frac{1}{r}\right) = 0$$

Solving this for r gives:

$$r_{0\pm} = \frac{GM \pm \sqrt{G^2M^2 - \left(2GM/r_M - v^2\right)v^2 r_M^2 \sin^2\theta}}{2GM/r_M - v^2}$$

Now the semi-major axis is given by:

$$2a = r_{0+} + r_{0-}$$

$$a = \frac{GMr_M}{2GM - r_M v^2}$$

Now using the cosine rule θ_M and θ_P can be calculated:

$$\cos\theta_M = \frac{r_M^2 + (2a - 2r_0)^2 - (2a - r_M)^2}{2r_M(2a - 2r_0)} = \frac{r_0^2 - 2ar_0 + ar_M}{r_M(a - r_0)}$$

and

$$\cos\theta_P = \frac{r_0^2 - 2ar_0 + ar_P}{r_P(a - r_0)}$$

And the circle of the outward and return trajectories are given by:

$$\theta = \theta_M \pm \theta_P$$

7.1.4 *Ejection Cone velocity distribution*

In the following sections it will be shown that:

$$\frac{dm}{dv} = A\exp(-\lambda v)$$

Where A and λ depend on various parameters (impact speed, impactor size, crater size, etc), but the general form for the distribution of mass against velocity is common for a single mass ejection. Note that independent of other parameters most mass is transferred at the lowest velocity, e.g. the velocity needed to reach the Martian Moon.

Now as the velocity progresses the chance of collision with the moon varies, as does the amount of material transferred. Both these will be needed. The probability of impact will as before be evaluated with the point of ejection integrated over the surface of Mars.

In designing the simulation, the question is where the integral over velocity is performed. The properties of mass ejection, in particular probability and fraction of mass transferred, can only be established after the integral over velocity. However the circles at the moon orbit caused by the ejection cone are only for a single velocity, when a range of velocities are produced, the rings become density profiles. Also the spatial coherence of the ring has potential importance, if an early fast ejection deposits mass on a moon, a slower ejector with the longer transit time has arrives later when the moon has moved.

Also the velocity has other effects, the projection of the moon onto the sphere around Mars depends on the velocity of the ejector at Mars. Hence these effects cannot easily be separated.

Now the impact velocity distribution on Mars is independent of the other parameters. The scaling law of the hypervelocity impact means that to a good approximation the mass scales as the crater size cubed, velocities though are unaltered (as time scales in the same way as position). This means that the Mars ejection process scales, only depending on the impact velocity. Also as the impacts are approximately uniform over Mars surface, the mass ejections are also uniform. This suggests calculating only once the properties of the mass ejection as a function of impact velocity.

Firstly the nature of the distributions needs to be established. Consider the properties of the ejection cones as a function of velocity, this is plotted for an ejection cone of 45°.

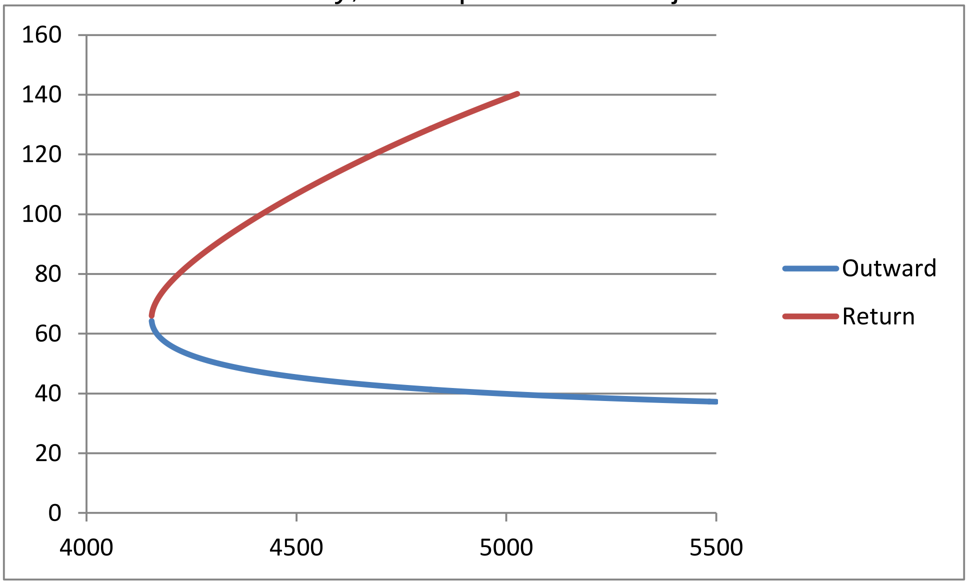


Figure 7-4. The angle (in degrees) of the circle at Phobos radius, as a function of the ejection velocity (in m/s).

There is a minimum speed, just under 4155m/s need to reach the attitude of Phobos, below the escape velocity the orbit return to Mars gives a second crossing. What this shows is over a range of velocities, much of the sky is covered by the ejected material. There is a cone (immediately overhead where the material cannot hit Phobos), also on the opposite side of Mars there is a small cone that cannot be reached. So much of the sky can be reached, in one dimension due to the rotation symmetry of the mass ejection, in the other dimension due to the range of velocities.

Now the crux is that the Martian moon position in spherical co-ordinates $\cos(\theta)$ ϕ is a small (very small) ellipse. Hence it will practically be at a single $\cos(\theta)$ angle. This means it will be a single ejection velocity which reaches the moon, which in turn means a single impact velocity. This simplifies the calculation, as only a single impact velocity is important, all be it distributed across 50% of the surface of the Moon. This leaves the important question of the amount of mass transferred.

The limits on size of the cone at the Martian moon also need calculating:

- The inner limit, is given by very fast ejecta and is given by

$$\sin(\phi_{min}) = \frac{\sin(\theta)\left(\sqrt{r_P^2 - \sin^2(\theta)\, r_M^2} - r_M \cos(\theta)\right)}{r_P}$$

- The outer limit is given when the ejection velocity is exactly the escape velocity:

$$\phi_{max} = \operatorname{acos}(1 - 2\sin^2(\theta)) + \operatorname{acos}\left(1 - 2\sin^2(\theta)\frac{r_M}{r_P}\right)$$
$$= 2\theta + \operatorname{acos}\left(1 - 2\sin^2(\theta)\frac{r_M}{r_P}\right)$$

The other useful parameter is the minimum velocity which reaches the moon, this is given by,

$$v_{min}^2 = \frac{2GM\left(\frac{1}{r_M} - \frac{1}{r_P}\right)}{1 - \sin^2(\theta)\left(\frac{r_M}{r_P}\right)^2}$$

And the angle of the cone in this case can be calculated.

Consider also the time of flight, this to a first approximation is given by the altitude of the moon, over the speed of ejection. This is shown in Figure 7-5.

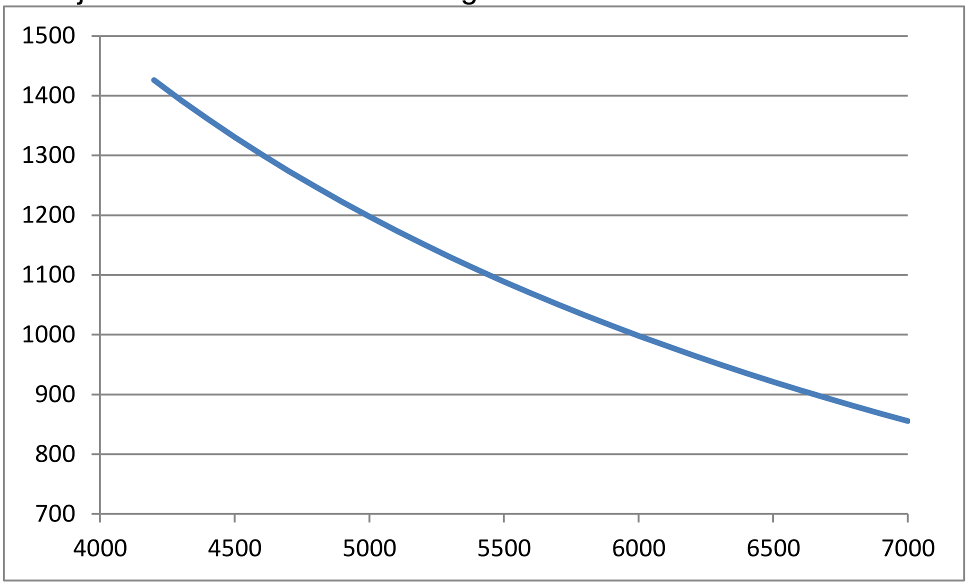


Figure 7-5. The (approximate) time of flight (s) for the outward flight to Phobos against the velocity of emission (m/s).

The time of flight for the slower emissions is just under 30 minutes, the faster the emission the quicker the transit. Now in 30minutes Phobos orbital speed means it will move ~3,000km. This will also smear the ejection cone over, however this is by most 18°, which is comparable to the variation in the size of the outward cone due to the variation in velocity. Practically this will apply a transform to the velocity distribution over the spherical co-ordinates, but only change its volume and magnitude to a second order. Hence this is not considered further here.

Key is the density of mass over the phase space, specifically integrated over the physical size of a martian moon projected onto the phase space. Specifically the mass transferred per steradian of spherical co-ordinates. This is formed in several steps:

- $\frac{d\cos(\theta)}{dv}$ can be calculated in an involved, but analytic form, from the equations above.
- $\frac{dm}{dv}$ can be calculated from the properties of the Mass ejection on Mars
- $\frac{dm}{d\cos(\theta)} = \frac{dm}{dv} / \frac{d\cos(\theta)}{dv}$ gives the desired mass distribution. This is allowable if the relationship between θ and v is exact, which it is.

So the critical component left is calculating $\frac{d\cos(\theta)}{dv}$ or $\frac{dv}{d\cos(\theta)}$. Consider the following steps:

$$d\cos(\theta) = d(\cos(\theta_m))\ \cos(\theta_p) + \cos(\theta_m)\ d(\cos(\theta_p)) \mp d(\sin(\theta_m))\ \sin(\theta_p) \mp \sin(\theta_m)\ d(\sin(\theta_p))$$

$$\cos\theta_M\, r_M(a - r_0) = r_0^2 - 2ar_0 + ar_M$$

$$d(\cos\theta_M) r_M(a - r_0) + \cos\theta_M\, r_M(da - dr_0) = 2r_0 dr_0 - 2d(a)r_0 - 2ad(r_0) + d(a)r_M$$

$$d(\cos\theta_M) = \frac{2r_0 dr_0 - 2d(a)r_0 - 2ad(r_0) + d(a)r_M - \cos\theta_M\, r_M(da - dr_0)}{r_M(a - r_0)}$$

$$d(\cos\theta_P) = \frac{2r_0 dr_0 - 2d(a)r_0 - 2ad(r_0) + d(a)r_P - \cos\theta_P\, r_P(da - dr_0)}{r_P(a - r_0)}$$

$$1 = \sin^2(\theta) + \cos^2(\theta)$$

$$0 = 2\sin(\theta) d(\sin(\theta)) + 2\cos(\theta)\, d(\cos(\theta))$$

$$d(\sin(\theta)) = -\frac{\cos(\theta)}{\sin(\theta)} d(\cos(\theta))$$

$$\frac{dr_0}{dv} = \frac{{r_0}^3 v(1 - \sin^2(\theta_E)\,(r_M/r_0)^2)}{r_0 GM - v^2 \sin^2(\theta_E)\, r_M^2}$$

$$\frac{da}{dv} = \frac{2ar_M v}{2GM - r_M v^2}$$

This can be combined to give the required $\frac{d\cos(\theta)}{dv}$.

Now whilst $\cos(\theta)$ can be calculated numerically from v , and for outgoing and incoming orbits there is a unique v for each $\cos(\theta)$, there is no clear way of inverting this equation and calculating v from $\cos(\theta)$. Hence this needs to be solved numerically.

7.1.5 Crater Density

[AD2] takes the crater density against time from Ivanov and Hartman, 2007. This has been updated in [RD12] so the update here is used. This is shown in Figure 7-6.

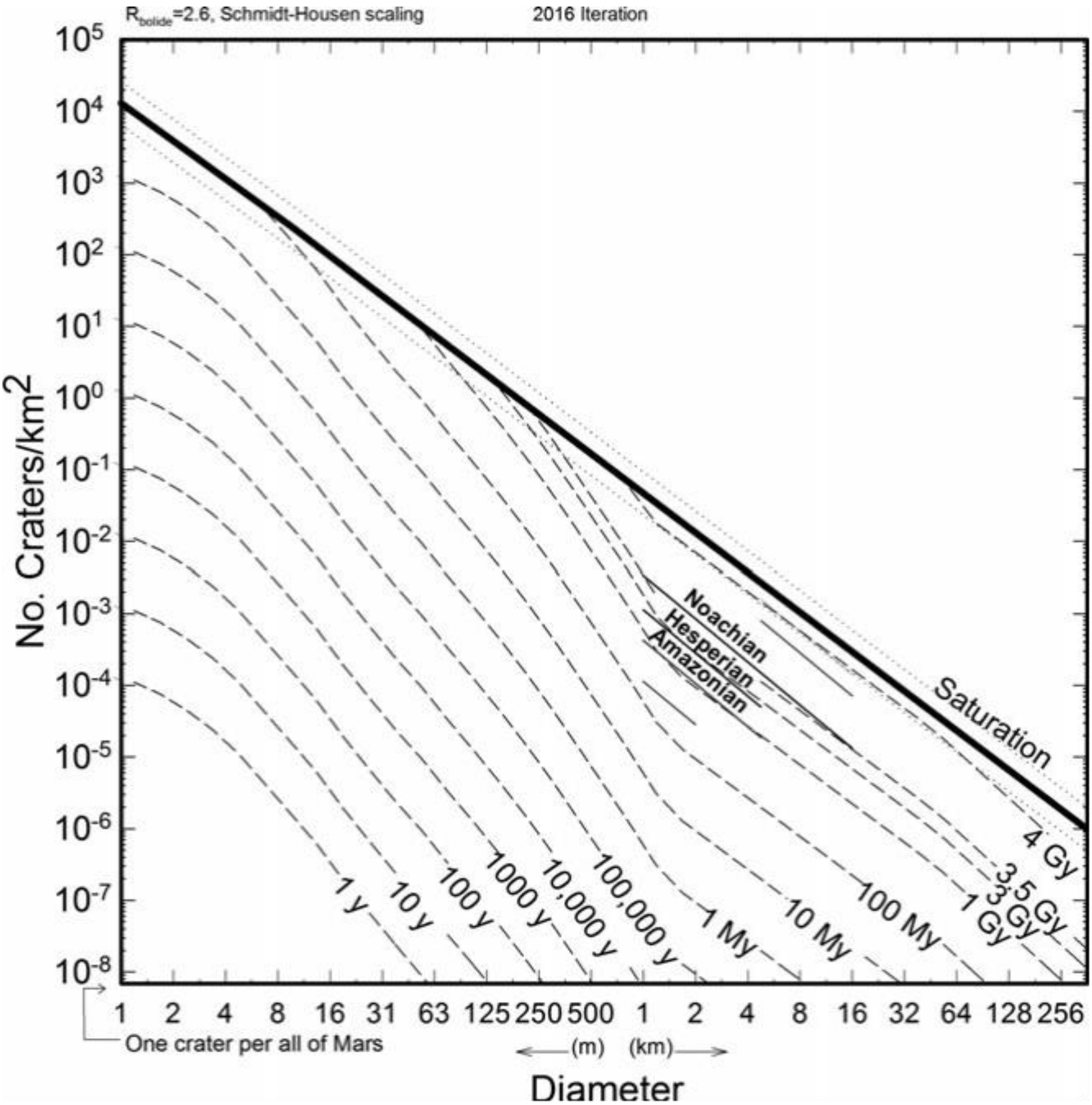


Figure 7-6. Figure 2 from [RD12]. Crater size distribution against time period.

This shows power law behaviour, with a knee at about 1km crater diameter. Fitting to this graph suggests the density of craters as:

$$\frac{d}{dt}\frac{d}{ds}\frac{dn}{dA} = \left(\frac{10^{-3}}{(s/[m])^4} + \frac{10^{-6}}{(s/[m])^3}\right)\frac{1}{\sqrt{\sqrt{2}} - \sqrt{\sqrt{0.5}}}\ [\text{craters / km}^2\text{ / m / y}]$$

[Note there is a little confusion over if the equation should be:

$$\frac{d}{dt}\frac{d}{ds}\frac{dn}{dA}$$

Or

$$\frac{d}{dt}\frac{d}{d\log_{\sqrt{2}} s}\frac{dn}{dA}$$

This has been checked original author [RD12] which confirms the data is histogramed in bins of width √2 – this gives rise to the $s\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)$ factor in the above equation.].

This is plotted for several time periods in Figure 7-7.

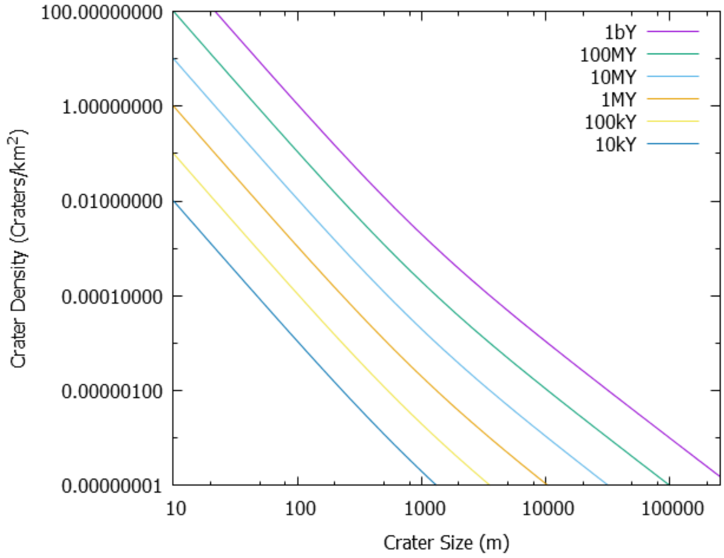


Figure 7-7. The fitted distribution. Whilst comparable to Hartman and Daubar, the knee at 1km is not so apparent.

Now the density distribution is written as the sum of two terms, the density can be multiplied by the area of Mars to get the rate of cratering at a size. Now as this is shown differential with respect to crater size, how this is derived is described in [AD2] that the craters are binned in logarithmic bins of width $\sqrt{2}$(e.g. each bin is form a diameter D to $\sqrt{2}$D).

Splitting into two terms:

$$\frac{d}{dt}\frac{d}{ds}\frac{dn_1}{dA}=\frac{10^{-3}}{s^4\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}[\text{craters / km}^2\text{ /m / y}]$$

$$\frac{d}{dt}\frac{d}{ds}\frac{dn_2}{dA}=\frac{10^{-6}}{s^3\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}[\text{craters / km}^2\text{/ m / y}]$$

The first term dominates for crater size below 1km, and the second dominates over 1km. Now Figure 7-6 suggests that over 1km impacts only become important for time periods of 10,000 years – on this periods it is expected that life will be sterilised by radiation. ~~Hence for the purposes of this study only the first term n_1 needs to be considered.~~

If mass ejections are to be followed, then the rate of cratering is followed.

$$\frac{d}{dt}\frac{dn_1}{dA}=\int d\left(\frac{-1}{3s^3}\right)\frac{10^{-3}}{\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}[\text{craters / km}^2\text{ / y}]$$

This equation is divergent toward zero, this corresponds to ever increasing numbers of craters as the size tends to zero. Each though carries less and less mass, and hence potentially less and less life.

To consider this transform the equation to mass transferred:

$$m \sim s^3$$

Which can be used:

$$\frac{d}{dt}\frac{d}{ds}\frac{dm_1}{dA} \sim \frac{d}{dt}\frac{d}{ds}\frac{dn_1}{dA}s^3 = \frac{s^3}{s^4} = \frac{1}{s}$$

This still has a soft logarithmic divergence at zero – this means eventually other physics will take over. A hint of this can be seen in Figure 7-6 where the graph flattens at 4m, possibly due to depletion in the Martian atmosphere. This can be modelled by:

$$\frac{d}{dt}\frac{d}{ds}\frac{dn_1}{dA} = \frac{10^{-3}}{(s+s_L)s^3\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}\,[\text{craters / km}^2\text{ /m / y}]$$

With s_L set to 4m.

There is a similar logarithmic divergence to large size, which eventually other physics will take over (e.g. size of objects from the asteroid belt). Here is used:

$$\frac{d}{dt}\frac{d}{ds}\frac{dn_1}{dA} = \frac{10^{-3}\,s_H}{(s+s_L)s^3\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)(s+s_H)}\,[\text{craters / km}^2\text{ /m / y}]$$

With s_L being the lower cut off, placed at 4m. s_H is an upper cut off, here we set it to 256km as the point where it takes ~1Gy for there to be on average 1 Martian crater, but it could just as easily be set to 2300km, as the size of Hellas Planitia the largest known crater on Mars.

The integral form is given by:

$$\frac{d}{dt}\frac{dn_1}{dA} = \int ds\,\frac{10^{-3}\,s_H}{(s+s_L)s^3\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)(s+s_H)}$$

$$= \frac{10^{-3}s_H}{\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}\int ds\left(\frac{1}{s_Ls_Hs^3} - \frac{(s_L+s_H)}{s_L^2s_H^2s^2} + \frac{(s_L^2+s_Ls_H+s_H^2)}{s_L^3s_H^3s} - \frac{1}{s_L^3(s_H-s_L)(s+s_L)} + \frac{1}{s_H^3(s_H-s_L)(s+s_H)}\right)$$

$$= \frac{10^{-3}s_H}{\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}\int\left(-\frac{1}{s_Ls_H}d\left(\frac{1}{2s^2}\right) + \frac{(s_L+s_H)}{s_L^2s_H^2}d\left(\frac{1}{s}\right) + \frac{(s_L^2+s_Ls_H+s_H^2)}{s_L^3s_H^3}d(\ln(s)) - \frac{1}{s_L^3(s_H-s_L)}d(\ln(s+s_L)) + \frac{1}{s_H^3(s_H-s_L)}d(\ln(s+s_H))\right)$$

$$= \frac{10^{-3} s_H}{\left(\sqrt{\sqrt{2}} - \sqrt{\sqrt{0.5}}\right)} \int d\left(\left(-\frac{1}{s_L s_H}\frac{1}{2s^2}\right) + \left(\frac{(s_L+s_H)}{s_L^2 s_H^2}\frac{1}{s}\right) + \left(\frac{(s_L^2+s_L s_H+s_H^2)}{s_L^3 s_H^3}\ln(s)\right) + \left(-\frac{1}{s_L^3(s_H-s_L)}\ln(s+s_L)\right) + \left(\frac{1}{s_H^3(s_H-s_L)}\ln(s+s_H)\right)\right)$$

This can be simplified by noting that $\ln(s+s_L) = \ln\left(s\left(1+\frac{s_L}{s}\right)\right) = \ln(s) + \ln\left(1+\frac{s_L}{s}\right)$ and collecting terms in $\ln(s)$ and they cancel so giving:

$$= \frac{10^{-3} s_H}{\left(\sqrt{\sqrt{2}} - \sqrt{\sqrt{0.5}}\right)} \int d\left(\left(-\frac{1}{s_L s_H}\frac{1}{2s^2}\right) + \left(\frac{(s_L+s_H)}{s_L^2 s_H^2}\frac{1}{s}\right) - \left(\frac{1}{s_L^3(s_H-s_L)}\ln\left(1+\frac{s_L}{s}\right)\right) + \left(\frac{1}{s_H^3(s_H-s_L)}\ln\left(1+\frac{s_H}{s}\right)\right)\right)$$

$$= \frac{10^{-3} s_H}{\left(\sqrt{\sqrt{2}} - \sqrt{\sqrt{0.5}}\right)} \left[-\frac{1}{s_L s_H}\frac{1}{2s^2} + \frac{(s_L+s_H)}{s_L^2 s_H^2}\frac{1}{s} - \frac{1}{s_L^3(s_H-s_L)}\ln\left(1+\frac{s_L}{s}\right) + \frac{1}{s_H^3(s_H-s_L)}\ln\left(1+\frac{s_H}{s}\right)\right]_{s_{min}}^{\infty}$$

$$= \frac{10^{-3} s_H}{\left(\sqrt{\sqrt{2}} - \sqrt{\sqrt{0.5}}\right)} \left(\frac{1}{s_L s_H}\frac{1}{2s_{min}^2} - \frac{(s_L+s_H)}{s_L^2 s_H^2}\frac{1}{s_{min}} + \frac{1}{s_L^3(s_H-s_L)}\ln\left(1+\frac{s_L}{s_{min}}\right) - \frac{1}{s_H^3(s_H-s_L)}\ln\left(1+\frac{s_H}{s_{min}}\right)\right)$$

To construct a Monte Carlo, the last line gives the volume of phase space. A random variable uniform over that phase space can be constructed, and means solving:

$$X = \left(-\frac{1}{s_L s_H}\frac{1}{2s^2}\right) + \left(\frac{(s_L+s_H)}{s_L^2 s_H^2}\frac{1}{s}\right) + \left(-\frac{1}{s_L^3(s_H-s_L)}\ln\left(1+\frac{s_L}{s}\right)\right) + \left(\frac{1}{s_H^3(s_H-s_L)}\ln\left(1+\frac{s_H}{s}\right)\right)$$
$$= \left(-\frac{1}{s_L s_H}\frac{1}{2s^2}\right) + \left(\frac{(s_L+s_H)}{s_L^2 s_H^2}\frac{1}{s}\right) - \frac{1}{s_L^3(s_H-s_L)}\left(\ln\left(1+\frac{s_L}{s}\right)\right) + \frac{1}{s_H^3(s_H-s_L)}\left(\ln\left(1+\frac{s_H}{s}\right)\right)$$

Where X is the uniform variable.

This equation has numerical instabilities, as the first two terms cancel the first terms of the Laurent expansion of the logarithm. Hence as a result this equation is better written:

$$X = \frac{1}{s_H - s_L}\left(\frac{1}{s_L^3} f\left(\frac{s_L}{s}\right) - \frac{1}{s_H^3} f\left(\frac{s_H}{s}\right)\right)$$

Where,

$$f(x) = x - x^2/2 + x^3/3 - \ln(1+x)$$

For values of x>1e-2 this equation is calculated directly, below that value,

$$f(x) = x^4/4 - x^5/5 + x^6/6 + \cdots$$

Note that breaking up the equation between a $\frac{s_L}{s}$ term and $\frac{s_H}{s}$ term is important, the simple Laurent expansion of $f(x)$ only converges for $|x| < 1$ – however due to the fine cancellations in of $f(x)$ for x not much smaller the Laurent expansion is accurate. Now as s_L and s_H are so hugely different, the two terms $f\left(\frac{s_L}{s}\right)$ and $f\left(\frac{s_H}{s}\right)$ often need calculating using different methods. This maintains accuracy for values of s between 1m and 1e10m and so covers the size of craters measured on Mars.

The equation strongly follows a power law (as is expected for the integral of a power law). So it is quickly inverted though use of Newton-Raphson on the logarithm of the integral against the logarithm of crater size:

$$\frac{d\ln X}{d\ln s} = \frac{1}{X}\frac{dX}{ds}\Big/\frac{d\ln s}{ds} = \frac{1}{X}\frac{s}{(s+s_L)s^3(s+s_H)}$$

And considering mass

$$\frac{d}{dt}\frac{d}{ds}\frac{dm_1}{dA} \sim \frac{d}{dt}\frac{d}{ds}\frac{dn_1}{dA}s^3 = \frac{s^3 s_H}{(s+s_0)s^3(s+s_H)} = \frac{s_H}{(s+s_0)(s+s_H)}$$

Which converges when integrated from zero to infinity (hence finite mass ejected!).

This does give a problem, in looking at discrete ejections – the divergence can't be handled, but it can be seen that the small craters carry little mass. This will be solved by using a cut off for size of crater considered, by varying this cut off the mass transferred will eventually become constant, despite the increasing rate of mass ejections. Once this stage is reached the cut off would not need decreasing. This divergence is physical – it says that the smaller the crater the more regular the impact, and when very small craters are considered (~1m) they will happen very regularly on Mars (~3 times a day).

Note that the cut at the large scale is very unlikely to affect the sterilization measurement, large craters despite ejecting significant mass, are very rare. The rarity means such events are typically in the distant past, so such material will have a long duration during which it is sterilized.

Turning to the s^3 term:

$$\frac{d}{dt}\frac{d}{ds}\frac{dn_2}{dA} = \frac{10^{-6}}{s^3\left(\sqrt{\sqrt{2}} - \sqrt{\sqrt{0.5}}\right)}\,[\text{craters / km}^2\text{/ m / y}]$$

As before the large s form is modified the give finite mass transfer:

$$\frac{1}{s^3} \to \frac{1}{s^3}\left(\frac{s_H}{s+s_H}\right)^2$$

And as before this can be partial fractioned:

$$\frac{1}{s^3}\left(\frac{s_H}{s+s_H}\right)^2 = \frac{3}{{s_H}^2 s} - \frac{2}{s_H s^2} + \frac{1}{s^3} - \frac{3}{{s_H}^2(s+s_H)} - \frac{1}{s_H(s+s_H)^2}$$

And this integrated:

$$\frac{1}{s^3}\left(\frac{s_H}{s+s_H}\right)^2 ds = d\left(\frac{3}{{s_H}^2}\ln s + \frac{2}{s_H s} - \frac{1}{2s^2} - \frac{3}{{s_H}^2}\ln(s+s_H) + \frac{1}{s_H(s+s_H)}\right)$$
$$= d\left(-\frac{3}{{s_H}^2}\ln\left(1+\frac{s_H}{s}\right) + \frac{2}{s_H s} - \frac{1}{2s^2} + \frac{1}{s_H(s+s_H)}\right)$$

And from this:

$$\frac{d}{dt}\frac{dn_2}{dA} = \frac{10^{-6}}{\left(\sqrt{\sqrt{2}} - \sqrt{\sqrt{0.5}}\right)} d\left(-\frac{3}{{s_H}^2}\ln\left(1+\frac{s_H}{s}\right) + \frac{2}{s_H s} - \frac{1}{2s^2} + \frac{1}{s_H(s+s_H)}\right) [\text{craters / km}^2/\text{ y}]$$

Now in combining the n_1 and n_2 terms, they each have different normalisations – so to get the right balance between the two the normalisation needs to be included

7.1.6 Mars Impact Velocity

The impact velocity of objects on Mars is taken from [RD7] which models the impact of objects from the asteroid belt impacting on planets. This gives the best understanding of the impact velocity of objects on Mars (which in turn drives the ejection velocity and mass from Mars).

For Mars [RD7] models the impactors with the velocity distribution shown in Figure 7-8.

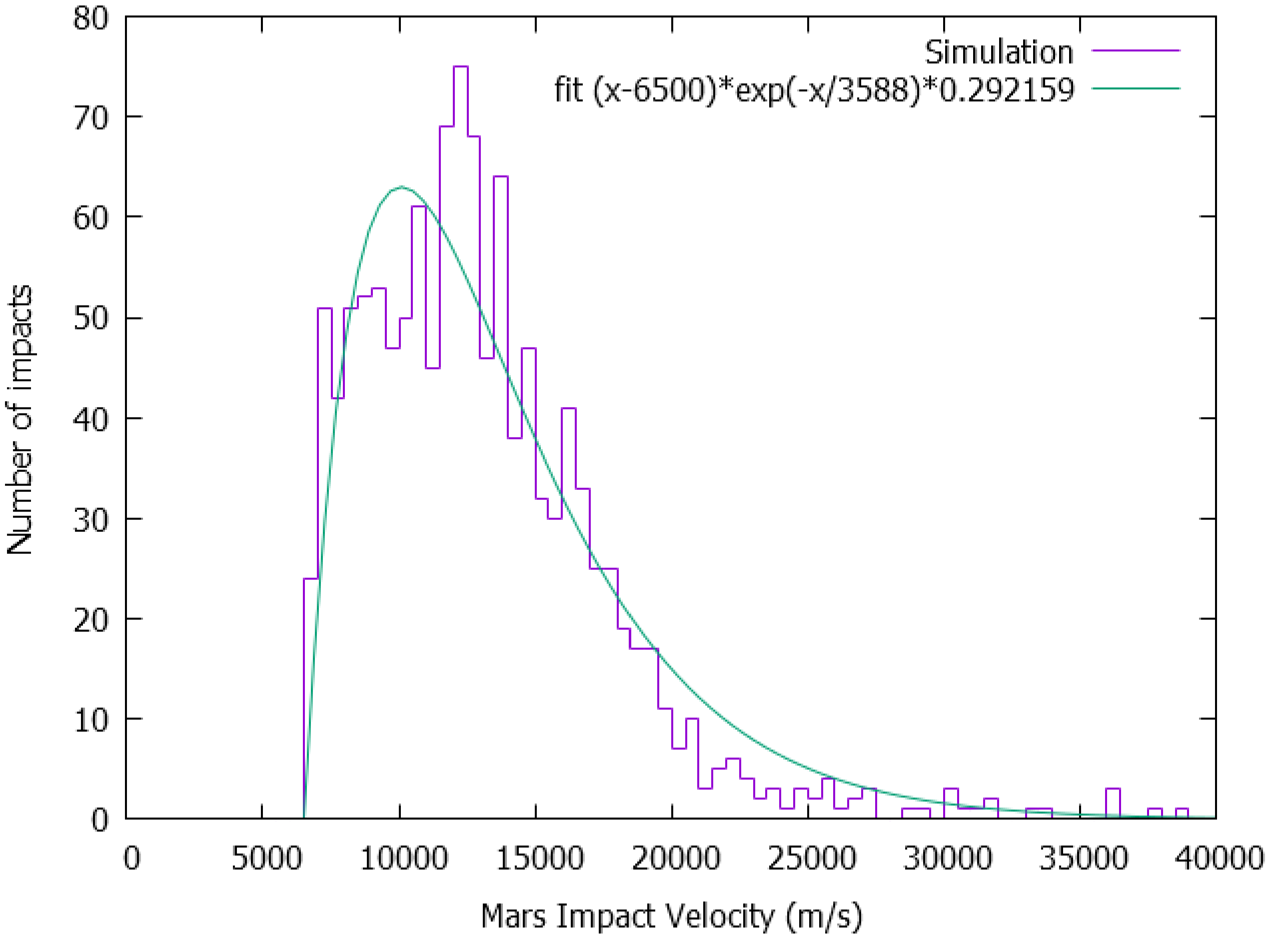


Figure 7-8. Modelled impact velocity on Mars. The simulated data is taken from [RD7]. The solid curve is a fitted distribution with similar features.

The shape of this distributions suggests a fit of the form:

$$\frac{dP}{dv} \sim (v-a)\exp\left(-\frac{v}{b}\right)$$

Where the zero gives the minimal velocity, and the exponential term gives the tail to large velocity. The minimum velocity from inspection is given by a=6500m/s. Fitting for the exponential gives b=3588m/s. The normalisation is also fitted – but is not important.

This form is useful for Monte Carlo as:

$$d\left(-\frac{(v-a+b)\exp\left(-\frac{v}{b}\right)}{b\exp\left(-\frac{a}{b}\right)}\right)=\frac{1}{b^2\exp\left(-\frac{a}{b}\right)}(v-a)\exp\left(-\frac{v}{b}\right)dv$$

The right hand side is proportional to the desired differential form, the left hand side when integrated from v=a to v=∞ has unit area, and so can be implemented as a Monte Carlo:

$$X=\frac{(v-a+b)}{b}\exp\left(\frac{(a-v)}{b}\right)$$

Where X is generated uniformly on [0:1], and the equation solved for v. This is performed iteratively:

$$y_0=\sqrt{-2\ln X}$$
$$y_{i+1}=\ln\left(\frac{1+y_i}{X}\right)$$
$$v=by+a$$

Which in 20 or so steps gives an accurate solution, v_0 is designed to give an accurate result when X~1 (based on a 2nd order Laurent expansion).

[Note that y<1, or x>2/e, a quicker convergence is given by

$$y_{i+1}=\sqrt{2\left(\ln(1+y_i)-y_i+\frac{{y_i}^2}{2}-\ln(X)\right)}$$

].

7.1.7 Crater size

Ejector properties are scaled on impactor size and velocity, size though is measured through Martian crater diameters. Hence there is a need to relate impactor size to crater size to connect the two. [RD9] gives a good description of the relevant concepts.

In the gravity-dominated cratering (which dominates at large diameter), the volume of the crater is given by:

$$V_g=K_1\left(\frac{m_i}{\rho_t}\right)\left(\frac{ga_i}{v_i^2}\right)^{-\frac{3\mu}{2+\mu}}\left(\frac{\rho_t}{\rho_i}\right)^{\frac{\mu}{2+\mu}}$$

The subscript, *i*, referring to the impactor; and *t* to the target (Martian surface).

Taking both the impactor (from the asteroid belt) and the target (Martian surface) to be hard rock, we have the parameters [RD9]:

Variable	Value
K_1	0.20
μ	0.55

ρ	2500kgm^{-3}

The volume of a crater can be related to its diameter via:

$$V = \frac{\pi}{24} D^3$$

Where the crater depth of about 1/3 of its diameter.

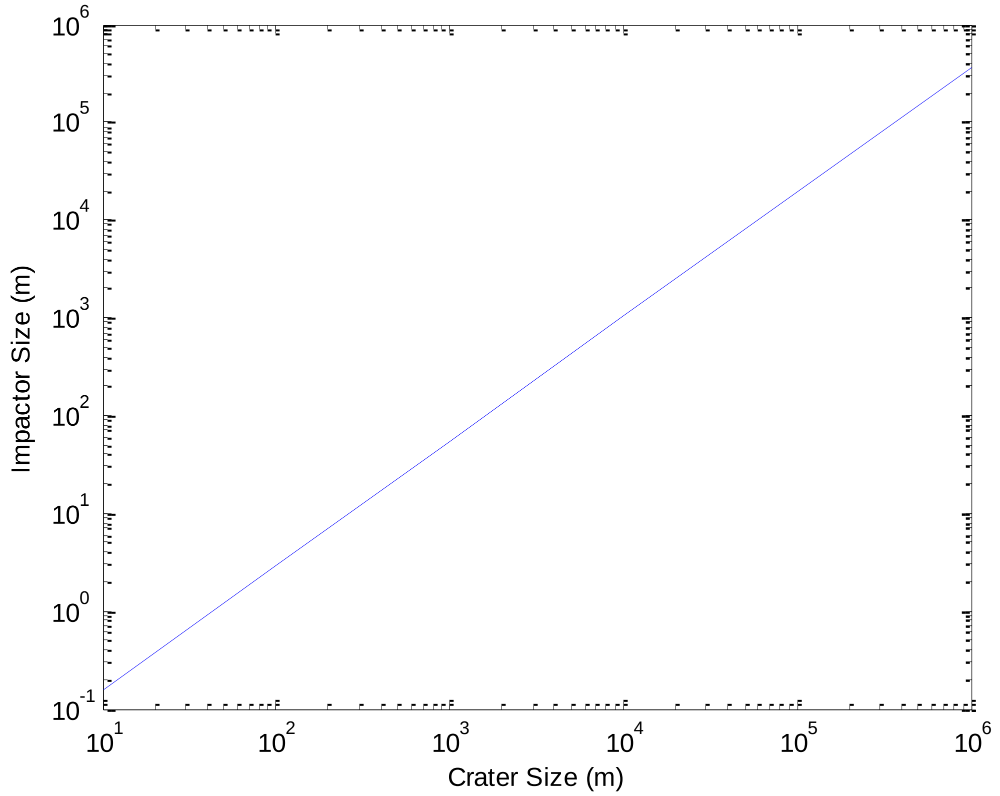
The mass of the impactor, is given by its size and density for a sphere

$$m_i = \frac{4}{3} \pi a_i^3 \rho_i$$

So assembling the various terms:

$$V_g = K_1 \left(\frac{4}{3}\pi a_i^3\right) \left(\frac{g a_i}{v_i^2}\right)^{-\frac{3\mu}{2+\mu}} \left(\frac{\rho_t}{\rho_i}\right)^{\frac{\mu}{2+\mu}-1} = \frac{\pi}{24} D^3$$

$$a_i = \left(\frac{1}{32K_1}\right)^{\frac{2+\mu}{6}} D \left(\frac{Dg}{v_i^2}\right)^{\frac{\mu}{2}} \left(\frac{\rho_t}{\rho_i}\right)^{\frac{1}{3}}$$



7.1.8 *Velocity and mass of ejection*

The speed with which material is ejected from Mars, depends primarily on the impactor velocity. Now most material is ejected at low speeds, but for transfer to Martian moons it is only the high end tail that is important. Modelling of this tail is quite limited, at the suggestion of Jay Melosh [RD10] has been used, this using hydrodynamic modelling has modelled a 10km sized basalt impact on a Martian basalt target. Scaling to other sizes can be performed using hydrodynamic scaling.

Now [RD10] models at three impact velocities 7.5,13.1,20.0 km/s. This maps well to impacts on Mars from the asteroid belt (§7.1.6). The results are plotted at a 3 dimensional plot, of ejection velocity vs radius and depth. This is shown in Figure 7-9.

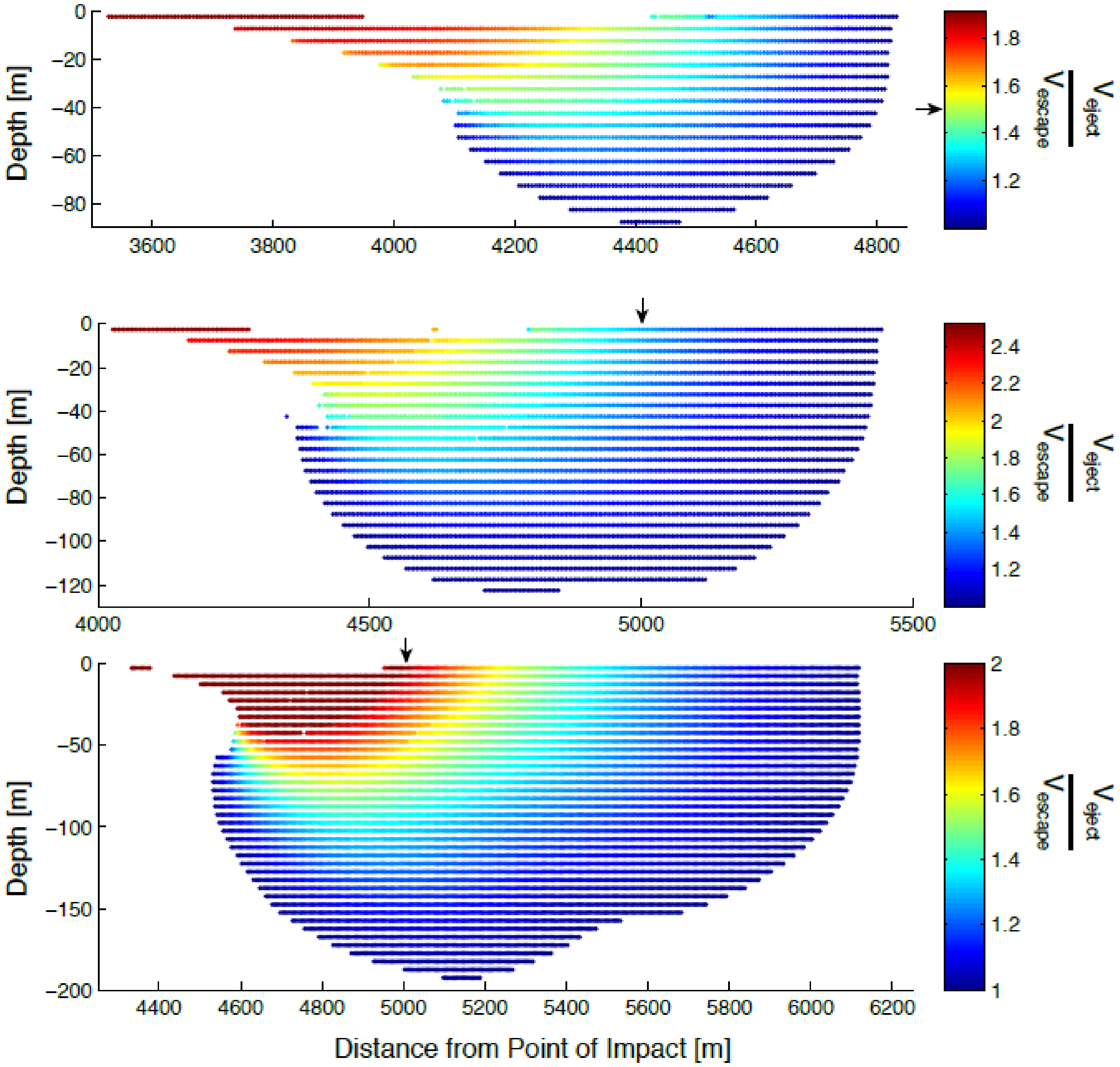


Figure 7-9. Figure 5.2 from [RD10] the speed of ejection of material following impact of a 10km object. This is shown top – 7.5km/s; middle 13.1 km/s; bottom 20km/s.

As the velocity increases so does the mass ejected:

Speed	Mass ejected over Mars Escape velocity
7.5km/s	4.7e12 kg
13.1km/s	1.0e13kg
20.0km/s	2.33e13kg

Now the three dimensional graphs, and total mass ejected have been processed to give a two dimensional plot of mass ejected vs velocity. This is shown in Figure 7-10.

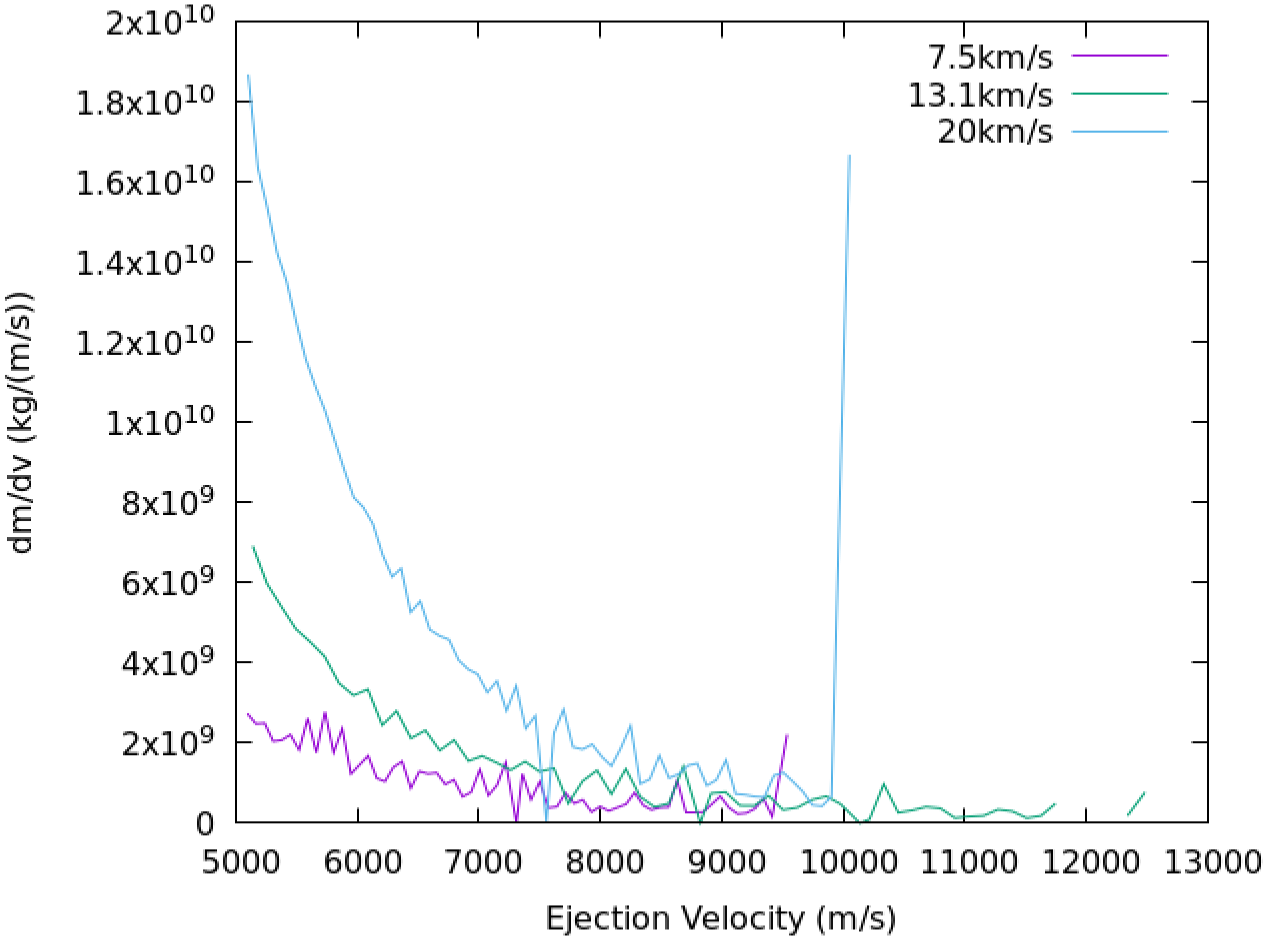


Figure 7-10. Processing results of [RD10] to produce mass ejected vs ejection velocity, for three values of impact velocity.

The general falling distribution with increased ejection velocity is clear, as the increase in ejected material with impact velocity. The one oddity is the highest velocity bin for 7.5km/s, and in particular 20.0km/s is unexpectedly high – this is taken as being due to the last bin collecting all ejection over the maximum speed – and so not well represented in a differential distribution. In addition this makes it questionable as if to fit to the distribution should be performed for this last point.

The steeply falling distribution suggests and exponential fit:

$$y = A\exp(\lambda x)$$

Where y is the mass distribution against the ejection velocity x.

Forming the χ^2 distribution for this:

$$\chi^2 = \sum_i \left(y_i - A\exp(\lambda x_i)\right)^2$$

This can be minimized by solving the following equations:

$$\sum_i y_i \exp(\lambda x_i) = A \sum_i \exp(2\lambda x_i)$$

$$\sum_i y_i x_i \exp(\lambda x_i) = A \sum_i x_i \exp(2\lambda x_i)$$

Which can be solved via the solution to:

$$\frac{\sum_i y_i \exp(\lambda x_i)}{\sum_i y_i x_i \exp(\lambda x_i)} = \frac{\sum_i \exp(2\lambda x_i)}{\sum_i x_i \exp(2\lambda x_i)}$$

Followed by:

$$A = \frac{\sum_i y_i \exp(\lambda x_i)}{\sum_i \exp(2\lambda x_i)}$$

This gives fits:

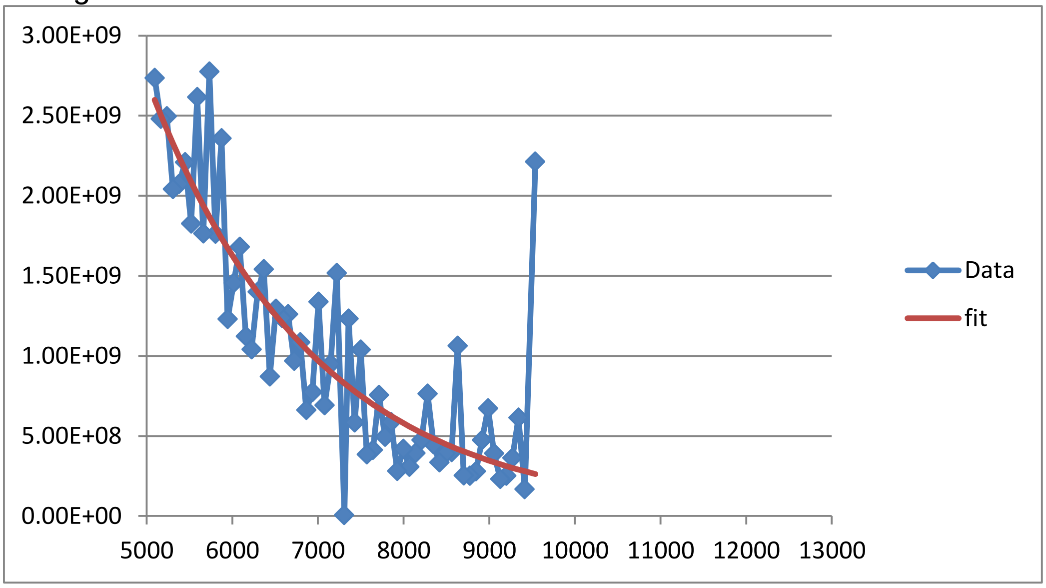


Figure 7-11. The fit for 7.5km/s

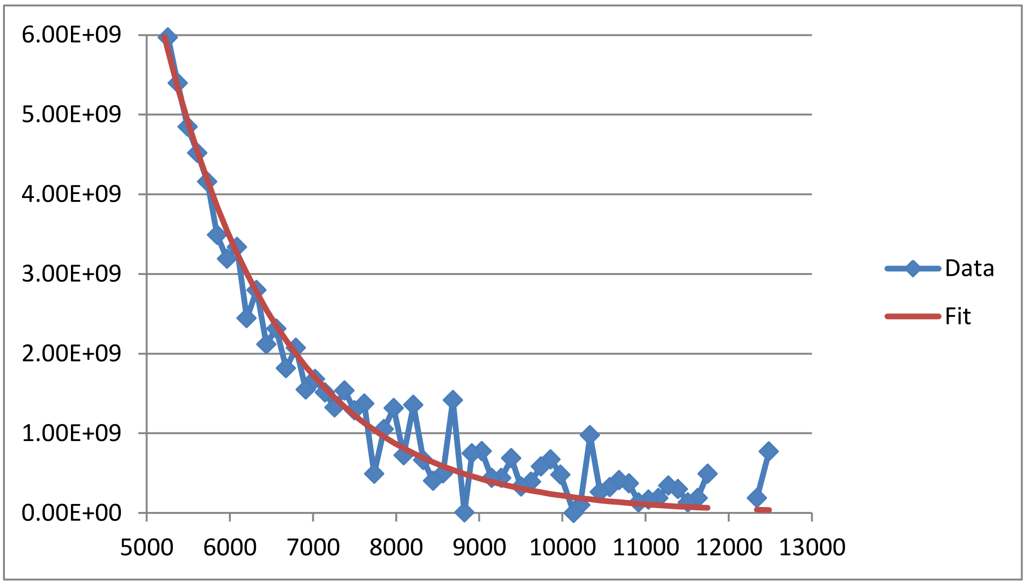


Figure 7-12. The fit for 13.1km/s

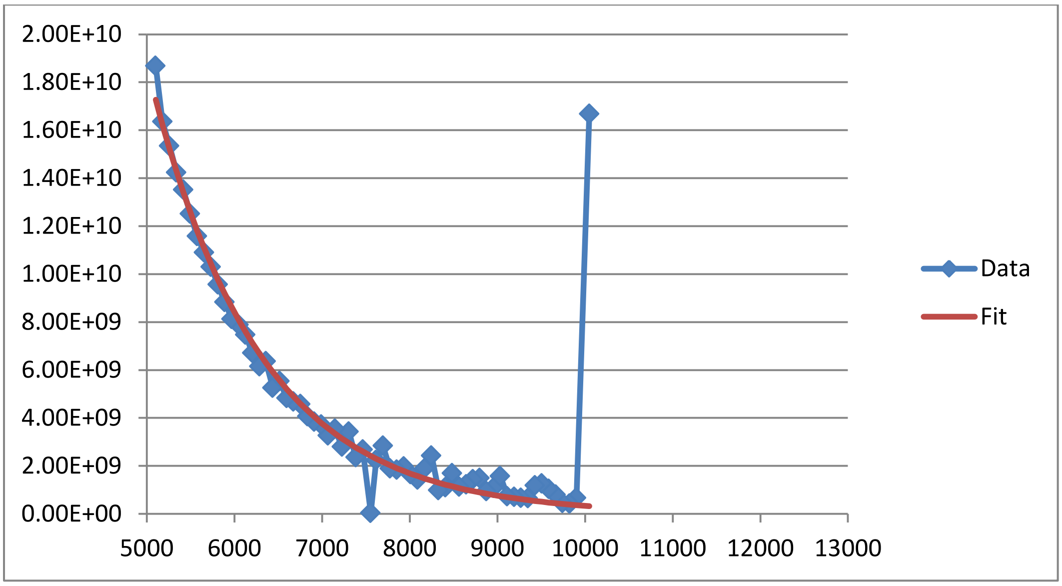


Figure 7-13. The fit for 20.0km/s

Which clearly gives a good fit. Next comparing the values of λ for the three impact velocities:

Speed (km/s)	Fitted λ (m^{-1})
7.5	-5.16E-04
13.1	-6.93E-04
20.0	-8.01E-04

Now this variation over velocity can be well fitted by a straight line:

$$\lambda = -2.25 \times 10^{-8} \left(\frac{s}{m}\right)^2 (v_I) - 3.64 \times 10^{-4} \left(\frac{s}{m}\right)$$

And the logarithm of normalisation A with variation in impact velocity:

Speed (km/s)	A $\left(\frac{kg}{m/s}\right)$	ln A
7.5	4.00E+10	24.41321
13.1	1.83E+11	25.93298
20.0	1.12E+12	27.74315

Which also fits a straight line.

$$\mathrm{A} = \exp\left(2.66 \times 10^{-4}\left(\frac{s}{m}\right)(v_I) + 2.24 \times 10^{1}\right)\left(\frac{kg}{m/s}\right)$$

With this fit over velocity it gives the full fit as:

$$\frac{dm}{dv_E} = \exp\left(\left(2.66 \times 10^{-4}\left(\frac{s}{m}\right)v_I + 22.4\right) - \left(2.25 \times 10^{-8}\left(\frac{s}{m}\right)^2 v_I + 3.64 \times 10^{-4}\left(\frac{s}{m}\right)\right)v_E\right)\left(\frac{kg}{m/s}\right)$$

This fit is shown below – it clearly has no significant differences from the fit without taking into account impact velocity.

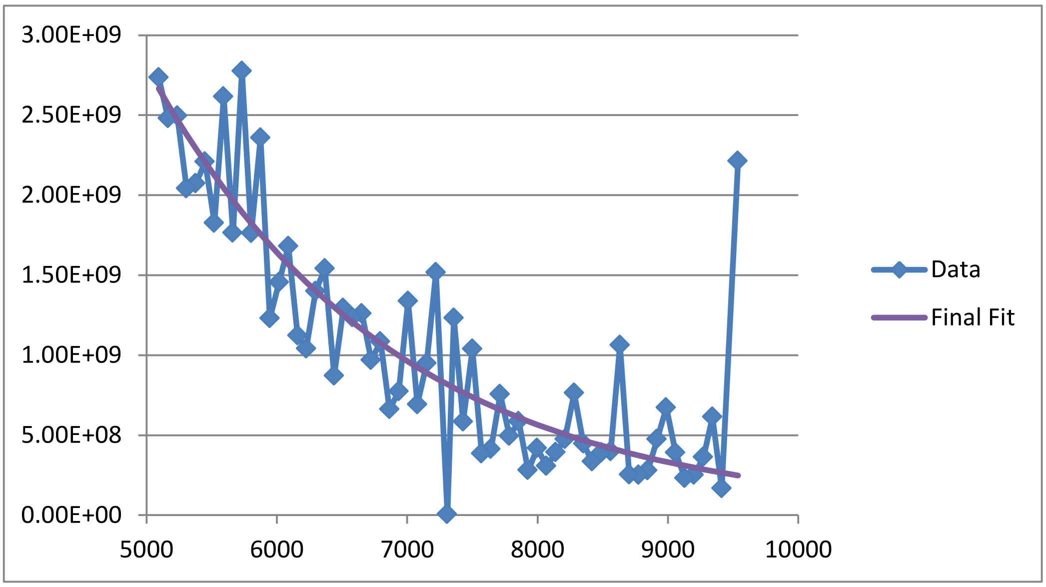


Figure 7-14. The final fit of the mass distribution at 7.5km/s impact

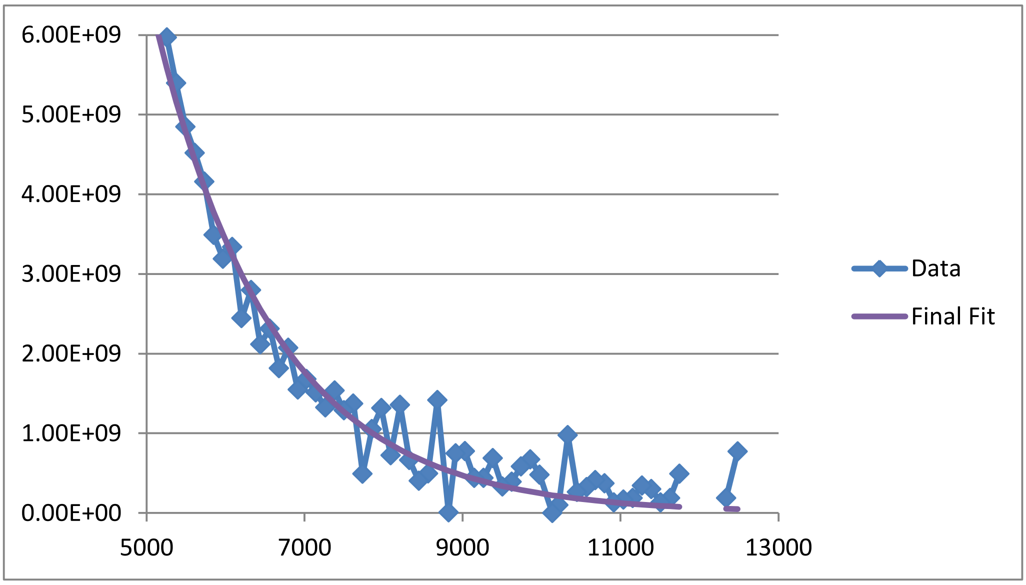


Figure 7-15. The final fit of the mass distribution at 13.1km/s impact

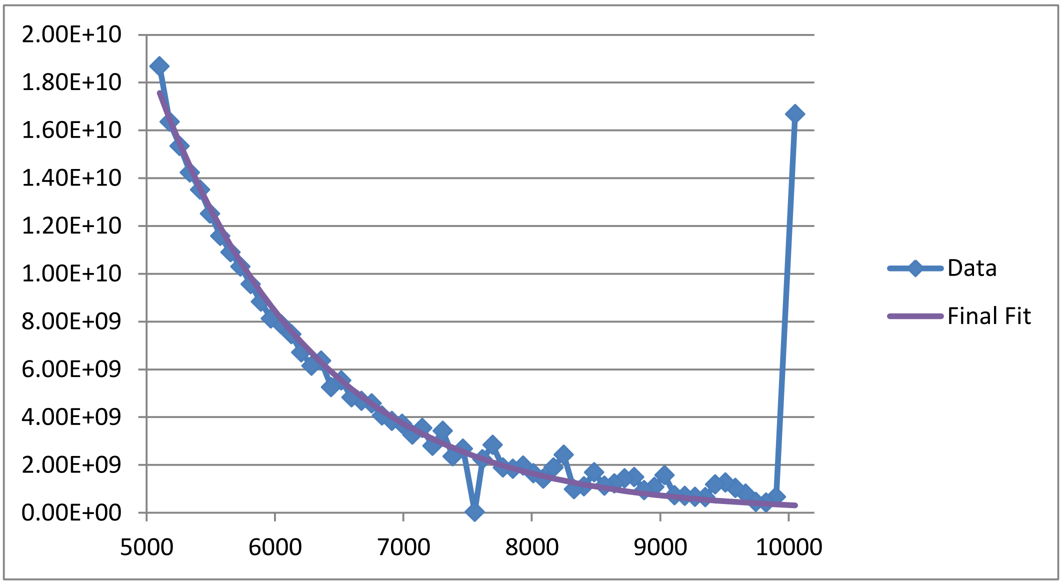


Figure 7-16. The final fit of the mass distribution at 20.0km/s impact

Now the resulting distribution:

$$\frac{dm}{dv}dv = A\exp(\lambda v)dv = \frac{A}{\lambda}d\big(\exp(\lambda v)\big)$$

Can be flatted by choosing $\exp(\lambda v)$ uniformly and:

$$\frac{\lambda}{A\exp(\lambda v_{min})}\frac{dm}{dv}dv = d\left(\frac{\exp(\lambda v)}{\exp(\lambda v_{min})}\right)$$

Integrates to unity – making choice of the random number simple. With X chosen uniformly:

$$v = \left[\frac{\ln(X)}{\lambda} + v_{min}\right]$$

And the mass transferred is given by:

$$\frac{A\exp(\lambda v_{min})}{|\lambda|}$$

This if for a 10km sphere of basalt. So finally how does this scale with impactor size, the velocity distribution, this is best described through hydrodynamic invariance [RD10] where scaling the impactor by α, scales variables:

$$x \rightarrow \alpha x$$
$$t \rightarrow \alpha t$$

This means that the velocity distribution is unchanged, the ejected mass scales as the cube of the impactor diameter. So this modifies the ejected mass to:

$$\left(\frac{D}{10km}\right)^3 \frac{A\exp(\lambda v_{min})}{|\lambda|}$$

Now consider the possible errors:

- The fit to the simulated hypervelocity collision, is typically better than the errors from the simulation.
- The fit between the measured extremes has no reason not to be reasonable
- Hence for impact velocity between 7.5km/s and 20km/s the fit should be good
- There are few impacts outside that range from objects from the asteroid belt
- The ejection speed is fitted between approximately 5-10km/s
- There is relatively little material ejected at over 10km/s
- The velocity to reach Phobos is ~3.8km/s – hence the extrapolation here is questionable.

7.1.9 Phobos Hypervelocity impact

With discrete ejection, the ejecta is expected to hit Phobos as a discrete object. This changes the calculation of the Hypervelocity Impact. Previous the variables of the impact where integrated over using a Monte Carlo, this was possible as the process was broken down into infinitesimal masses. Each mass followed its own trajectory through the process of transfer to the moon, and so everything that effected its route was chosen by random variables which had the correct distribution. This was chosen because of the fast convergence of Monte Carlo Integrals. With discrete ejection, during the collision with the Moon and object as a whole will impact, the question becomes what fraction of the object remains unsterilized.

This is calculated by integrating over the parameters which describe the impact:

$$\Delta E = \left(\frac{v^2}{2}\right) F(x, y, \theta, p)$$

Where:

- x: distance along a cylinder
- y: radius from the centre of the cylinder
- θ: the angle about the cylinder
- p: the distribution of the random component of the impact
- F: fraction of kentic energy turned into heat
- ΔE: The fraction of kinetic energy at that point that is converted into heat

This will give the energy which can be converted to temperature at each point inside the impactor. The temperature can be converted into the fraction of mass that is unsterilized.

This then needs integrate over the internal volume:

$$M_{Unsterilized} = \rho \int dV\,dp = \iiint dx\;ydy\;d\theta\;dp\;S$$

Where this integrates over a cycling on which the fraction of sterilization is defined.

This is compared to the total mass:

$$M_{Unsterilized} = \rho \int dV = \iiint dx\;ydy\;d\theta\; = \rho h \pi y^2$$

As F depends just on the fractional position in the cylinder, this can be simplified, but as energy depends on velocity, it must be calculated separately for each velocity, e.g. for each impact with the moon.

As kinetic energy conversion to heat data is a grid of x and y values, the integral is most easily performed as a sum:

$$\widetilde{M}_{unsterilized} = \sum_{x,y} y\;dp\;S(x,y,p)$$

$$\widetilde{M} = \sum_{x,y} y\;dp$$

$$M_{unsterilised} = M\frac{\widetilde{M}_{unsterilized}}{\widetilde{M}}$$

This utilises that the dθ integral is flat (e.g. no dependence on the azimuthal angle). It leaves only the integral over “p” the normal distribution of the energy transfer to heat. The distribution about p is given by:

$$dP = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2}{2\sigma^2}\right)dx$$

Integrating over this distribution was previously shown as to how to generate this distribution randomly, by generating it twice:

$$dP = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2}{2\sigma^2}\right)dx\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{y^2}{2\sigma^2}\right)dy$$

And changing to radial coordinates:

$$dP = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)rdr\;d\theta$$

$$= \frac{d\theta}{2\pi}d\left(\exp\left(-\frac{r^2}{2\sigma^2}\right)\right)$$

Which can be performed by integrating θ and $\exp\left(-\frac{r^2}{2\sigma^2}\right)$. Now unfortunately this means performing two integrals, for just the one variable.

This makes the full integral over hyper velocity collision with the moon is at least 4 dimensional, which is about the point where a Monte Carlo becomes most efficient. Hence the previous simulation as implemented as a Monte Carlo is maintained, just now applied to an individual mass ejections.

7.2 Evolution to an architecture

With the various distributions modelled, this section turns to how they should be integrated; what architecture supports the physics of the process.

The process to follow is:

- Select a minimum Martian crater size to study
- At initiation evaluate the average time period between impacts creating craters larger than the minimum
- Using Poisson distribution generate Martian impacts with the correct history distribution
- Continue this back to some history cut off.
- For each impact generate the crater size
- From the crater size evaluate the impactor size
- Evaluate the impact velocity for material from the asteroid belt
- For the ejection cone angle, calculate the probability of mass ejection hitting a Martian moon
- Generate the position of the moon with respect to the ejection (note as Martian impact is expected to be uniform on Mars, the position of the moon with respect to the ejection is uniform)
- Where the ejection will hit the moon, evaluate the position in spherical co-ordinates of the moon with respect to mass ejection
- Evaluate the ejection speed for that point in phase space
- For that ejection speed, transform to the altitude of the moon, and include the moons velocity
- Calculate the mass transferred for the point in spherical co-ordinates
- Deposit a *finite* mass on the moon (note previous Monte Carlo deposited an infinitesimal mass)
- Pass the moon impact onto the previous code
- Iterate the history several times, to evaluate the variability

The crux of this change, is to directly follow the discrete nature of transfer from Mars to the moon, a side effect is it will give an independent prediction of the rate of mass transfer to the moon.

8. DISCRETE MARS EJECTION EVENTS REQUIREMENTS

8.1 Mars Impact Velocity

8.1.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-MI-In-01	The Mars Impact Velocity shall take no inputs	I		MarsImpactVelocity.c:3
SL-MI-Out-01	The outputs shall be: • Mars Impact velocity	I		MarsImpactVelocity.c:3

8.1.2 Parameters

Number	Requirement	Verification	Comment	Code
SL-MI-Parm-01	The minimum impact velocity shall be set to 65000m/s.	I	Variable MarsImpactVelocityA	sterlim.h:107
SL-MI-Parm-02	The exponential fall of factor shall be set to 3588 m/s.	I	Variable MarsImpactVelocityB	sterlim.h:108
SL-MI-Parm-03	The integrations to solve the inverse problem shall be set to 20.	I	Variable MarsImpactVelocityItt	sterlim.h:109

8.1.3 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-MI-Alg-01	The initial point on the inverse	I	X a random variable [0:1]	MarsImpactVelocity.c:6

	solution shall be: $y_0 = \sqrt{-2 \ln X}$			
SL-MI-Alg-02	The iterative step forward shall be: $y_{i+1} = \ln\left(\frac{1 + y_i}{X}\right)$	I		MarsImpactVelocity.c:8
SL-MI-Alg-03	The iterative step forward shall be evaluated a fixed number of times	I		MarsImpactVelocity.c:7
SL-MI-Alg-04	The returned velocity shall be $v = by + a$	I		MarsImpactVelocity.c:10

8.2 Crater Properties

8.2.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-CR-In-01	For the total rate of cratering over a minimum velocity, the inputs shall be: • Minimum Crater Size	I		crater.c:6
SL-CR-Out-01	The outputs shall be: • Crater Rate per year	I		crater.c:6
SL-CR-In-02	For the distribution of crater sizes, the inputs shall be: • Minimum Crater Size	i		crater.c:21
SL-CR-Out-02	The outputs shall be: • Crater size	i		crater.c:21

8.2.2 Parameters

Number	Requirement	Verification	Comment	Code
SL-CR-Parm-01	The minimum crater size will be set to s_{min}.	I	Initial value s=1m. Variable MarsMinCraterSize	sterlim.h:116
SL-CR-Parm-02	The roll off of the crater at small size shall be set to s_L.	I	Initial value s_L=4m Variable MarsCraterSLower	sterlim.h:112
SL-CR-Parm-03	The roll off of the crater at large size shall be set to s_H.	I	Initial value s_H=256km Variable MarsCraterSHiger	sterlim.h:113
SL-CR-Parm-04	The surface area of Mars, A_{Mars} shall be 144798500km^2…	I	Initial value A=144798500 Variable MarsSurfaceArea	sterlim.h:119
SL-CR-Parm-05	The normalisation of the crater rate shall be: $A_{Mars}\frac{10^{-3}s_H}{\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}$	I	Variable MarsCraterNormalisation	sterlim.h:114

8.2.3 Initiation

Number	Requirement	Verification	Comment	Code
SL-CR-Init-01	The average rate of crater production shall be calculated at initialisation of the simulation	I		sterlimDisc.c:34
SL-CR-Init-02	The rate shall be initialised to:			

	$\frac{dn}{dt} = A_{Mars}\left(\frac{10^{-3}s_H}{\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}\left(\frac{1}{s_L s_H}\frac{1}{2{s_{min}}^2} - \frac{(s_L+s_H)}{s_L^2 s_H^2}\frac{1}{s_{min}} + \frac{1}{s_L^3(s_H-s_L)}\ln\left(1+\frac{s_L}{s_{min}}\right) - \frac{1}{s_H^3(s_H-s_L)}\ln\left(1+\frac{s_H}{s_{min}}\right)\right) + \frac{10^{-6}}{\left(\sqrt{\sqrt{2}}-\sqrt{\sqrt{0.5}}\right)}\left(-\frac{3}{{s_H}^2}\ln\left(1+\frac{s_H}{s}\right)+\frac{2}{s_H s}-\frac{1}{2s^2} + \frac{1}{s_H(s+s_H)}\right)\right)$			

8.2.4 *Algorithmic Form*

Number	Requirement	Verification	Comment	Code
SL-CR-Alg-01	The time of the previous ejection shall be calculated as $\Delta t = \frac{-\ln X}{dn/dt}$	I	X a random variable [0:1] dn/dt taken from SL-CR-Init-01	sterlimDisc.c:151
SL-CR-Alg-02	Time will be initialised at zero	I	The present	sterlimDisc.c:35
SL-CR-Alg-03	Time will advance into the pass in steps of Δt	I	Taken from SL-CR-Alg-01, and calculated fresh for each step.	sterlimDisc.c:151
SL-CR-Alg-03	The crater rate calculation shall be evaluated as the sum of the 4th and 3th power law distribution	I	Both 3rd and 4th order distributions have fall off, which is described below	crater.c:20 crater.c:45
SL-CR-Alg-04	The 4th order power law distribution (with fall off) shall be calculated as: $-\mathrm{Norm}\times A_{Mars}$		Fall off from 4th order at the lower limit of s_L and increased roll of at s_H.	crater.c:12 crater.c:33 crater.c:35

	$$\frac{1}{s_H - s_L}\left(\frac{1}{s_L^3} f\left(\frac{s_L}{s}\right) - \frac{1}{s_H^3} f\left(\frac{s_H}{s}\right)\right)$$			
SL-CR-Alg-05	The function f is defined as: $$f(x) = x - x^2/2 + x^3/3 - \ln(1+x)$$	I		SL-CR-Alg-06 SL-CR-Alg-07
SL-CR-Alg-06	For values of x>0.01, f shall be evaluated directly.	I	This maintains accuracy	crater.c:91
SL-CR-Alg-07	For values of x<0.01 f shall be evaluated as: $$f(x) = x^4/4 - x^5/5 + x^6/6$$	I	First 3 non zero terms of Laurent expansion, maintains accuracy better than direct evaluation	crater.c:93
SL-CR-Alg-07	The normalisation of the forth order power law shall be set to $$\text{Norm} = \frac{10^{-3} s_H}{\sqrt[4]{2} - \sqrt[4]{0.5}}$$	I		sterlim.h:114
SL-CR-Alg-08	The 3rd power law (with fall off) shall be calculated as: $$-\text{Norm} \times A_{Mars}$$ $$-\frac{3}{{s_H}^2}\ln\left(1+\frac{s_H}{s}\right) + \frac{2}{s_H s} - \frac{1}{2s^2} + \frac{1}{s_H(s+s_H)}$$	I		crater.c:15 crater.c:20 crater.c:38 crater.c:40 crater.c:45
SL-CR-Alg-09	The Normalisation of the 3rd order power law shall be set to: $$\text{Norm} = \frac{10^{-6}}{\sqrt[4]{2} - \sqrt[4]{0.5}}$$			sterlim.h:115
SL-CR-Alg-10	For the distribution of crater sizes, the volume of phase space shall	I	This does not include the normalisation factor used for the rate measurement;	crater.c:54 crater.c:56

	be initialised to: $vol = \text{Norm3}\frac{1}{s_H - s_L}\left(\frac{1}{s_L^3} f\left(\frac{s_L}{s_{min}}\right) - \frac{1}{s_H^3} f\left(\frac{s_H}{s_{min}}\right)\right) + \text{Norm4}\left(-\frac{3}{{s_H}^2}\ln\left(1+\frac{s_H}{s}\right) + \frac{2}{s_H s} - \frac{1}{2s^2} + \frac{1}{s_H(s+s_H)}\right)$		the normalisation does not affect the distribution of crater sizes	
SL-CR-Alg-11	The crater distribution shall generated from: $Integ = X \times vol$	I	Integ is the integrated distribution, X is a random variable in [0:1], vol is taken from SL-CR-Alg-07	crater.c:58
SL-CR-Alg-12	The solution to the crater size is given by: $Integ = \text{Norm3}\frac{1}{s_H - s_L}\left(\frac{1}{s_L^3} f\left(\frac{s_L}{s}\right) - \frac{1}{s_H^3} f\left(\frac{s_H}{s}\right)\right) + \text{Norm4}\left(-\frac{3}{{s_H}^2}\ln\left(1+\frac{s_H}{s}\right) + \frac{2}{s_H s} - \frac{1}{2s^2} + \frac{1}{s_H(s+s_H)}\right)$	I		crater.c:29
SL-CR-Alg-13	The crater size distribution integral equation shall be solved by	I	For a power law, which the crater size is approximately, Newtons method solves	

	Newtons method in logarithm of integral against logarithm of crater size		the solution to a straight line with speed.	
SL-CR-Alg-14	The initial size crater used in Newtons search shall be: $s = 1m$	I	Typically this is the minimal size of the simulation, where the integral has a large value.	crater.c:47
SL-CR-Alg-15	The logarithmic derivate used is: $\frac{d\ln(I)}{d\ln(s)} = \frac{s}{I}\frac{dI}{ds}$	I		crater.c:117
SL-CR-Alg-16	$\frac{dI}{ds}$ shall be calculated as: $\frac{dI}{ds} = Norm3\frac{1}{(s+s_L)s^3(s+s_H)} + Norm4\frac{{s_H}^2}{s^3(s+s_H)^2}$	I		crater.c:113
SL-CR-Alg-17	The step forward in the crater size shall be calculated as: $\Delta\ln(s) = \frac{\ln(-I_{target}) - \ln(-I)}{d\ln(I)/d\ln(s)}$	I	Note the –ve of the integral, as the volume of phase space evaluates negative.	crater.c:117
SL-CR-Alg-18	Newtons method is iterated until $\left\|\ln(-I_{target}) - \ln(-I)\right\| < 0.000001$	I	This gives rapid convergence, and 0.0001 typically means at machine accuracy.	crater.c:121

8.3 Impactor Size

8.3.1 Input/Output Form

Number	Requirement	Verification	Comment	Code

SL-IS-In-01	For impactor size the inputs shall be: • Crater Size (diameter) • Impactor Velocity	I		ImpactorSize.c:3
SL-IS-Out-01	The outputs shall be: • Impactor Size (radius)	I		ImpactorSize.c:3

8.3.2 Parameters

Number	Requirement	Verification	Comment	Code
SL-IS-Parm-01	The Mars Crater Mu term shall be set to 0.55	I	Variable MarsCraterMu	sterlim.h:122
SL-IS-Parm-02	The Mars Crater K_1 term shall be set to 0.20	I	Variable MarsCraterK1	sterlim.h:123
SL-IS-Parm-03	The Mars Impactor Density shall be set to 2500 kg/m3	I	Variable MarsImpactorDensity	sterlim.h:124
SL-IS-Parm-04	The Mars Surface Density shall be set to 2500 kg/m3	I	Variable MarsSurfaceDensity	sterlim.h:125
SL-IS-Parm-04	The Mars surface Gravity shall be set to 3.711	I	Variable MarsGravity	sterlim.h:126

8.3.3 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-IS-Alg-01	The function shall calculate the	I, T		ImpactorSize.c:5

	impactor diameter via: $a_i = \left(\frac{1}{32K_1}\right)^{\frac{2+\mu}{6}} D \left(\frac{Dg}{v_i^2}\right)^{\frac{\mu}{2}} \left(\frac{\rho_t}{\rho_i}\right)^{\frac{1}{3}}$			

8.4 Mars Ejection Mass Distribution

8.4.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-MF-In-01	The Mars Ejection shall take inputs: • Impact velocity • Ejection velocity • Impactor diameter	I		MarsEjection.c:3
SL-MFOut-01	The outputs shall be: • Mars Ejection Mass distribution against ejection velocity	I		MarsEjection.c:3

8.4.2 Parameters

Number	Requirement	Verification	Comment	Code
SL-MF-Parm-01	The slope of the normalisation factor A shall be set to • 2.66e-4 $\left(\frac{s}{m}\right)$	I	Variable MarsEjectionAslope	sterlim.h:131
SL-MF-Parm-02	The offset of the normalisation factor A shall be set to • 22.4	I	Variable MarsEjectionAoffset	sterlim.h:132

SL-MF-Parm-03	The slope of the velocity factor λ shall be set to • 2.25e-8 $\left(\frac{s}{m}\right)^2$	I	Variable MarsEjectionLamslope	sterlim.h:133
SL-MF-Parm-04	The offset of the velocity factor λ shall be set to • 3.64e-4 $\left(\frac{s}{m}\right)$	I	Variable MarsEjectionLamoffset	sterlim.h:134
SL-MF-Parm-05	The reference diameter of a impactor on Mars shall be set to • 10km	I	Variable MarsEjectionRefS	sterlim.h:135

8.4.3 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-MF-Alg-01	The differential mass with respect to ejection velocity shall be calculated from	I	For a reference size of 10km	MarsIEjection.c:7

	$\frac{dm}{dv_E} = \exp\left(\left(2.66 \times 10^{-4}\left(\frac{s}{m}\right)v_I + 22.4\right) - \left(2.25 \times 10^{-8}\left(\frac{s}{m}\right)^2 v_I + 3.64 \times 10^{-4}\left(\frac{s}{m}\right)\right)v_E\right)\left(\frac{kg}{m/s}\right)$			
SL-MF-Alg-02	The constants in SL-MF-Alg-01 shall be set to the parameter values	I		MarsIEjection.c:7
SL-MF-Alg-03	The scaling of the ejection mass with impactor size shall be calculated; $\frac{dm}{dv_E}(s_I) = \frac{dm}{dv_E}(s_{\text{ref}})\left(\frac{s_I}{s_{\text{ref}}}\right)^3$	I	With $s_{\text{ref}} = 10\text{km}$ by default	MarsIEjection.c:11

8.5 Mars To the Moon

8.5.1 Input/Output Form

Number	Requirement	Verification	Comment	Code
SL-MM-In-01	The Mars to Moon shall take inputs: • Cos theta	I		MarsToMoon2.c:44

	• Ejection Cone Angle			
SL-MM-Out-01	The outputs shall be: • ejection velocity	I		MarsToMoon2.c:44
SL-MM-In-02	The initialisation routine will take inputs: • Ejection Cone Angle	I		MarsToMoon2.c:10
SL-MM-Out-02	The initialisation routine shall have no outputs	I		MarsToMoon2.c:10
SL-MM-In-03	The $d\cos\theta/dv$ routine will take inputs • Velocity • Angle between ejection and the moon • Ejection angle		Note that velocity and angle are related (and calculated in MarsToMoon2 and VelToAngle), this route takes both as input for computational efficiency.	MarsToMoon2.c:149
SL-MM-Out-03	The $d\cos\theta/dv$ routine will return • The rate of change of the cosine of theta with respect to changes in velocity			MarsToMoon2.c:149

8.5.2 Parameters

Number	Requirement	Verification	Comment	Code
SL-MM-Parm-01	The gravitation constant shall be a parameter	I	Variable G	sterlim.h:11
SL-MM-Parm-02	The mass of Mars shall be a parameter	I	Variable MassMars	sterlim.h:39
SL-MM-Parm-03	The radius of Mars shall be a parameter	I	Variable RadiusMars	sterlim.h:41
SL-MM-Parm-04	The distance between Mars	I	Variable DistanceMarsMon	sterlim.h:68

	and the Moon shall be a parameter			

8.5.3 Initialisation

Number	Requirement	Verification	Comment	Code
SL-MM-Init-01	The initialisation route shall initialise: • $\cos(\theta)_{min}$ • $\cos(\theta)_{max}$ • $\cos(\theta)_{minVel}$ • v_{min} • v_{escape}	I		MarsToMoon2c:2-6
SL-MM-Init-02	$\cos(\theta)_{min}$ shall be the smallest $\cos(\theta)$ where ejecta can reach the moon.	I		MarsToMoon2c:2
SL-MM-Init-03	$\cos(\theta)_{max}$ shall be the largest $\cos(\theta)$ where ejecta can reach the moon.	I		MarsToMoon2c:3
SL-MM-Init-04	$\cos(\theta)_{minVel}$ shall be the $\cos(\theta)$ where ejecta with least velocity can reach the moon.	I		MarsToMoon2c:4
SL-MM-Init-05	v_{min} shall be the least velocity to reach the moon.	I		MarsToMoon2c:5
SL-MM-Init-06	v_{escape} shall be the escape velocity from Mars.	I		MarsToMoon2c:6
SL-MM-Init-07	The minimum θ where ejecta can reach the moon shall be calculated from	I		MarsToMoon2c:14

	$\theta_{min} = \cos^{-1}(-\sin(\theta_{cone})) - \cos^{-1}\left(-\frac{r_{Mars}}{d_{moon}}\sin(\theta_{cone})\right)$			
SL-MM-Init-08	$\cos(\theta)_{max}$ shall be calculated from $\cos(\theta)_{max} = \cos(\theta_{min})$	I		MarsToMoon2c:16
SL-MM-Init-09	The maximum θ where ejecta can reach the moon shall be calculated from $\theta_{max} = 2\theta_{cone} + \cos^{-1}\left(1 - 2\sin^2(\theta_{cone})\frac{r_{Mars}}{d_{moon}}\right)$	I		MarsToMoon2c:17
SL-MM-Init-10	Where $\theta_{max} > \pi$: $\cos(\theta)_{min} = -1$	I		MarsToMoon2c:20
SL-MM-Init-11	Where $\theta_{max} < \pi$: $\cos(\theta)_{min} = \cos(\theta_{max})$	I		MarsToMoon2c:22
SL-MM-Init-12	The minimum velocity to reach the moon shall be calculated from: $v_{min} = \sqrt{\frac{2GM\left(\frac{1}{r_{Mars}} - \frac{1}{d_{moon}}\right)}{1 - \sin^2(\theta_{cone})\frac{r_{Mars}{}^2}{d_{moon}{}^2}}}$	I		MarsToMoon2c:24
SL-MM-Init-13	$\cos(\theta)_{minVel}$ shall be calculated from: $\cos(\theta)_{minVel} = \frac{r_0^2 - 2ar_0 + ar_M}{r_M(a - r_0)}$	I		MarsToMoon2c:34
SL-MM-Init-14	r_0 shall be calculated from:	I	In Req SL-MM-Init-13	MarsToMoon2c:28

	$r_0 = \dfrac{GM - \sqrt{G^2M^2 - \left({2GM}/{r_M} - {v_{min}}^2\right){v_{min}}^2 r_M^2 \mathrm{si}}}{{2GM}/{r_M} - {v_{min}}^2}$			
SL-MM-Init-15	a shall be calculated from: $$a = \frac{GMr_M}{2GM - r_M {v_{min}}^2}$$	I	In Req SL-MM-Init-13	MarsToMoon2c:27
SL-MM-Init-16	v_{escape} shall be calculated from: $$v_{escape} = \sqrt{\frac{2GM}{r_M}}$$	I		MarsToMoon2c:36

8.5.4 Algorithmic Form

Number	Requirement	Verification	Comment	Code
SL-MM-Alg-01	The MarsToMoon2 routine will calculate the velocity needed to reach a moon, at a selected angle between the ejection and the moon.	I		MarsToMoon2.c
SL-MM-Alg-02	The MarsToMoon2 routine will search for the velocity using the method of bracketing and bisection.	I		MarsToMoon2.c
SL-MM-Alg-03	For the forward calculation (velocity-to-angle) the routine VelToAngle shall be used	I		MarsToMoon2.c:120
SL-MM-Alg-04	VelToAngle will take the arguments: • Velocity	I		MarsToMoon2.c:120

	• Ejection Angle • Outward or Inward direction			
SL-MM-Alg-05	VelToAngle will return: • Angle	I		MarsToMoon2.c:120
SL-MM-Alg-06	VelToAngle will calculate the outward direction using: $\theta = \cos^{-1}(\cos\theta_M - \cos\theta_P)$	I		MarsToMoon2.c:140
SL-MM-Alg-07	VelToAngle will calculate the return direction using: $\theta = \cos^{-1}(\cos\theta_M + \cos\theta_P)$	I		MarsToMoon2.c:145
SL-MM-Alg-08	VelToAngle will calculate $\cos\theta_M$ using: $\cos\theta_M = \frac{r_0^2 - 2ar_0 + ar_M}{r_M(a - r_0)}$	I		MarsToMoon2.c:135
SL-MM-Alg-09	VelToAngle will calculate $\cos\theta_P$ using: $\cos\theta_P = \frac{r_0^2 - 2ar_0 + ar_P}{r_P(a - r_0)}$	I		MarsToMoon2.c:137
SL-MM-Alg-10	VelToAngle will calculate a using: $a = \frac{GMr_M}{2GM - r_M v^2}$	I		MarsToMoon2.c:129
SL-MM-Alg-11	VelToAngle will calculate r_0 using: $r_0 = \frac{GM - \sqrt{G^2M^2 - \left({}^{2GM}/_{r_M} - v^2\right)v^2 r_M^2 \sin^2\theta}}{{}^{2GM}/_{r_M} - v^2}$	I		MarsToMoon2.c:130
SL-MM-Alg-12	VelToAngle when calculating the return direction, if the velocity is over the Martian escape velocity, the code will flag an error.	I		MarsToMoon2.c:143
SL-MM-Alg-13	The MarsToMoon2 routine when $\cos\theta < \cos\theta_{min}$ shall return -1.	I		MarsToMoon2.c:51
SL-MM-Alg-14	The MarsToMoon2 routine when $\cos\theta > \cos\theta_{max}$ shall	I		MarsToMoon2.c:53

	return -1.			
SL-MM-Alg-15	The MarsToMoon2 routine when $\cos\theta_{min} < \cos\theta < \cos\theta_{minVel}$ shall return • The velocity of the return orbit at the moon	I		MarsToMoon2.c:89
SL-MM-Alg-16	The MarsToMoon2 routine when $\cos\theta_{minVel} < \cos\theta < \cos\theta_{max}$ shall return • The velocity of the outward orbit at the moon	I		MarsToMoon2.c:55
SL-MM-Alg-17	The outward orbit shall set velocity limits by search.	I		MarsToMoon2.c:73 MarsToMoon2.c:98
SL-MM-Alg-18	The initial point of the search for the outward orbit limits search shall be set to $v = v_{min}$	I		MarsToMoon2.c:60
SL-MM-Alg-19	The velocity for the search for the outward limits shall be doubled every step, until the last two span the desired $\cos\theta$.	I		MarsToMoon2.c:64 MarsToMoon2.c:67
SL-MM-Alg-20	Once the spanning velocity limits for the outward orbit have been determined the desired velocity shall be search by bisection.	I		MarsToMoon2.c:73
SL-MM-Alg-21	The bisection will be continued until the difference between $\cos\theta_{low}$ and $\cos\theta_{high}$ are below 1e-6.	I		MarsToMoon2.c:73
SL-MM-Alg-22	At the end of the bisection the code shall return $$\frac{v_{low} + v_{high}}{2}$$	I		MarsToMoon2.c:67
SL-MM-Alg-23	For the return orbit shall set the velocity limits to: $$[v_{min}: v_{escape}]$$	I		MarsToMoon2.c:92 MarsToMoon2.c:95
SL-MM-Alg-24	The return orbit shall be calculated by bisection.	I		MarsToMoon2.c:98
SL-MM-Alg-25	The dcosthetadv routine when $\cos\theta < \cos\theta_{min}$ shall return -1	I		MarsToMoon2.c:152
SL-MM-Alg-26	The dcosthetadv routine when $\cos\theta > \cos\theta_{max}$ shall return -1.	I		MarsToMoon2.c:154

SL-MM-Alg-27	The dcosthetadv routine when $\cos\theta_{min} < \cos\theta < \cos\theta_{minVel}$ shall return $$\frac{d\cos\theta}{dv} = \frac{d\cos\theta_M}{dv}\cos\theta_P + \cos\theta_M\frac{d\cos\theta_P}{dv} - \frac{d\sin\theta_M}{dv}\sin\theta_P - \sin\theta_M\frac{d\sin\theta_P}{dv}$$	I		MarsToMoon2.c:186
SL-MM-Alg-28	The dcosthetadv routine when $\cos\theta_{minVel} < \cos\theta < \cos\theta_{max}$ shall return: $$\frac{d\cos\theta}{dv} = \frac{d\cos\theta_M}{dv}\cos\theta_P + \cos\theta_M\frac{d\cos\theta_P}{dv} + \frac{d\sin\theta_M}{dv}\sin\theta_P + \sin\theta_M\frac{d\sin\theta_P}{dv}$$	I		MarsToMoon2.c:221
SL-MM-Alg-29	dcosthetadv will calculate $\frac{d\sin\theta}{dv}$ using: $$\frac{d\sin\theta}{dv} = -\frac{\cos\theta}{\sin\theta}\frac{d\cos\theta}{dv}$$	I		MarsToMoon2.c:185 MarsToMoon2.c:184
SL-MM-Alg-30	dcosthetadv will calculate $\sin\theta$ shall be calculated using: $$\sin\theta = \sqrt{1-\cos^2\theta}$$	I		MarsToMoon2.c:182 MarsToMoon2.c:183
SL-MM-Alg-31	dcosthetadv will calculate $\frac{d\cos\theta_M}{dv}$ shall be clouted using: $$\frac{d\cos\theta_M}{dv} = \frac{\left(2r_0\frac{dr_0}{dv} - \frac{da}{dv}r_0 - 2a\frac{dr_0}{dv} + r_M\frac{da}{dv} - \cos\theta_M r_M\left(\frac{da}{dv} - \frac{dr_0}{dv}\right)\right)}{r_M(a-r_0)}$$	I		MarsToMoon2.c:174
SL-MM-Alg-32	dcosthetadv will calculate $\frac{d\cos\theta_P}{dv}$ using:	I		MarsToMoon2.c:178

	$\frac{d\cos\theta_P}{dv} = \frac{\left(2r_0\frac{dr_0}{dv} - \frac{da}{dv}r_0 - 2a\frac{dr_0}{dv} + r_P\frac{da}{dv} - \cos\theta_M r_P\left(\frac{da}{dv} - \frac{dr_0}{dv}\right)\right)}{r_P(a-r_0)}$			
SL-MM-Alg-33	dcosthetadv will calculate $\cos\theta_M$ using: $\cos\theta_M = \frac{r_0^2 - 2ar_0 + ar_M}{r_M(a-r_0)}$	I		MarsToMoon2.c:170
SL-MM-Alg-34	dcosthetadv will calculate $\cos\theta_P$ using: $\cos\theta_P = \frac{r_0^2 - 2ar_0 + ar_P}{r_P(a-r_0)}$	I		MarsToMoon2.c:172
SL-MM-Alg-35	dcosthetadv will calculate $\frac{dr_0}{dv}$ using: $\frac{dr_0}{dv} = \frac{{r_0}^3 v(1-\sin^2(\theta_E)\,(r_M/r_0)^2)}{r_0GM - v^2\sin^2(\theta_E)\,r_M^2}$	I		MarsToMoon2.c:168
SL-MM-Alg-36	dcosthetadv will calculate $\frac{da}{dv}$ using: $\frac{da}{dv} = \frac{2ar_Mv}{2GM - r_Mv^2}$	I		MarsToMoon2.c:161
SL-MM-Alg-37	dcosthetadv will calculate r_0 using: $r_0 = \frac{GM - \sqrt{G^2M^2 - \left(2GM/r_M - v^2\right)v^2r_M^2\sin^2\theta}}{2GM/r_M - v^2}$	I		MarsToMoon2.c:162
SL-MM-Alg-38	dcosthetadv will calculate a using: $a = \frac{GMr_M}{2GM - r_Mv^2}$	I		MarsToMoon2.c:160

9. TEST

9.1 Mars Impact Velocity (from Asteroid belt)

The simulated impact velocity of Mars was used to produce 10^8 events, these were histogrammed and compared to the theoretical curve. This is shown in Figure 9-1.

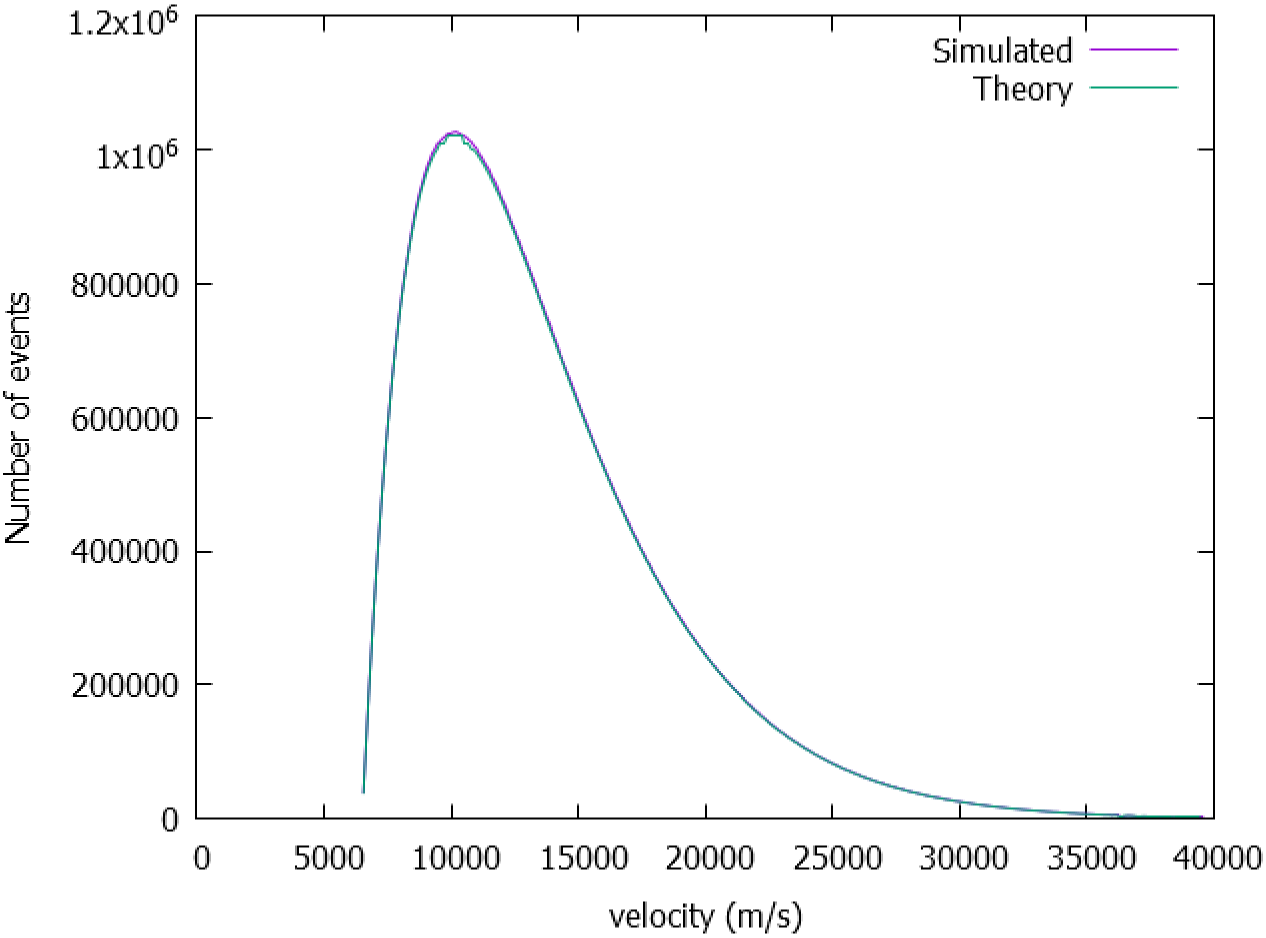


Figure 9-1. 10^8 simulated impacts from the asteroid belt, plotted against the theoretical model, with the normalisation set equal.

The fit is clearly excellent and so the test is passed.

9.2 Crater Isochrones

Two tests were done, first that the analytic integral of the crater rate reproduces [RD12] and Figure 7-6. Secondly that the Monte Carlo has the correct crater size distribution.

As the cone scales linearly with time, there is nothing to gain from plotting isochrones for many time periods, instead a single time period of 10My has been chosen, as this is a timescale of Figure 7-6 that spans most of the graph. Reproducing this graph with the same scaling (craters per km squared in sqrt 2 bin widths) gives the graph shown in Figure 9-2.

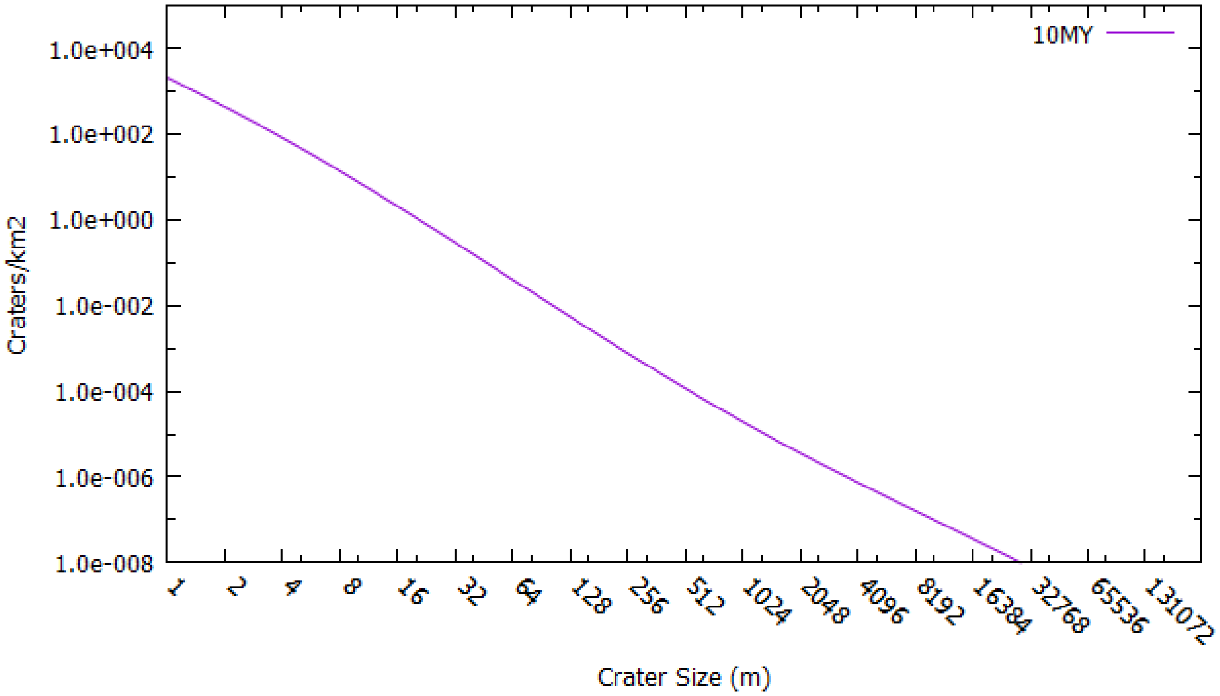


Figure 9-2. The isochrone for 10My for cratering per km on Mars vs size. This is to be compared with Figure 7-6.

The general shape between 4m and 1km agrees well between the two curves. The analytic code does not roll off at sharply at 4m, this is expected as a single pole takes an order of magnitude to roll off, so only has minimal effect between 4m and 1m. The curve also flattens at 1km (not this is not easily apparent, but can be seen with lines drawn on the graph, Figure 9-3 makes this clearer). Hence the fit is as expected, and the anomalies understood.

Also if straight lines are drawn over the curve (extended in both x and y), the flattening below 4m, and over 256km, is apparent, but as changing the only slightly it isn't clear without visual aids like this. Note that change in power laws as has been chosen typically take about one order of change in magnitude to change the slope, this is more gradual than shown in Figure 7-6 – however sharp changes as shown in Figure 7-6 are not easily modelled in maths.

To show this, the easiest plot to demonstrate this is $dn/ds \times s^3$ – as this was used to motivate the distribution. It should have the properties:

- Flat below s=4m
- At 4m steepens to a 1/s distribution
- Flattens at 1km
- At 256km steps to a $1/s^2$ distribution

This is plotted using the same isochrones data as above, and is shown in Figure 9-3 – and the behaviour desired is clear.

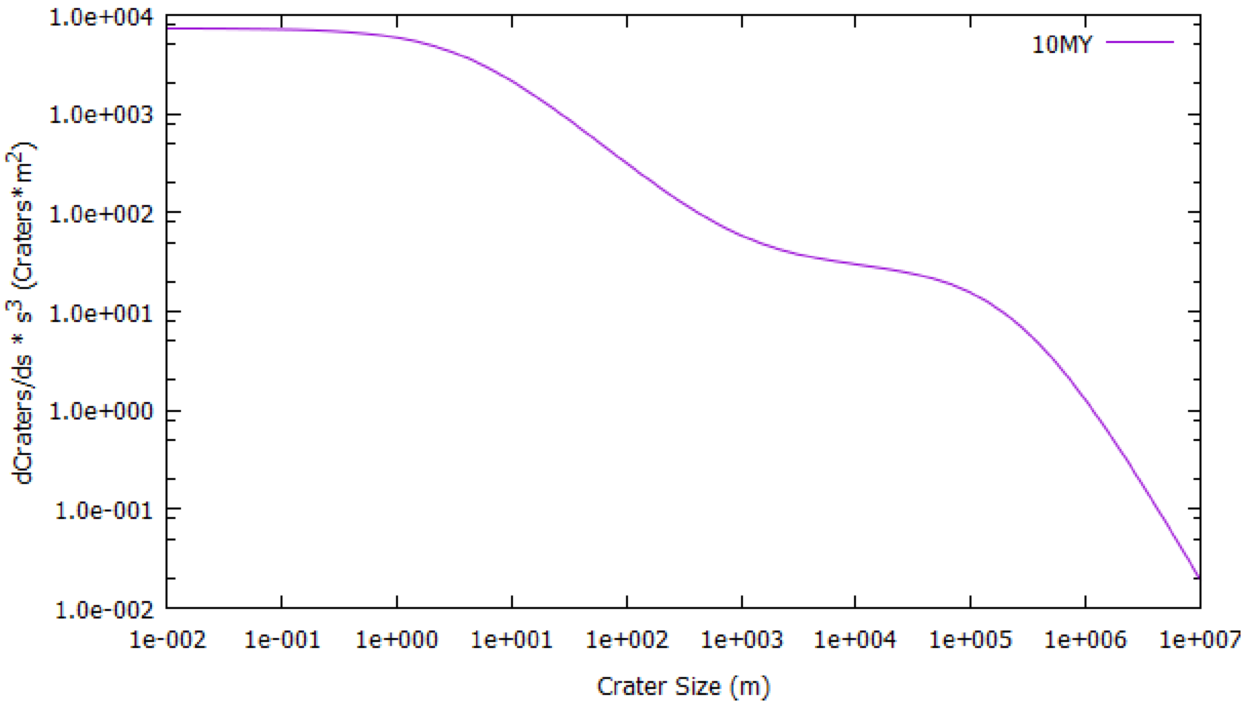


Figure 9-3. The approximate dm/ds distribution which for modelling is designed to be finite when integrated from zero to infinity, but with poles and zeros to fit the isochrone distribution. The graph changes slope at the expected points

For the crater size distribution 10^8 craters are created above a cut off, and histogramed again in windows of sqrt(2). The normalisation isn't set – as each call to the code produces a crater, so no time is defined. That action is performed by crater rate calculation.

The expected distribution is a 4th order power law between 4m and 1km, rolls off at below 4m and over 1km to 3rd order, and declines after 256km to 2nd order. As a 4th order power law even with 10^8 craters it will only just cover 2 orders of magnitude of crater size, hence the comparison is performed with two minimum crater sizes:

- s_{min}=1m
- s_{min}=100m

This is shown in Figure 9-4 and Figure 9-5

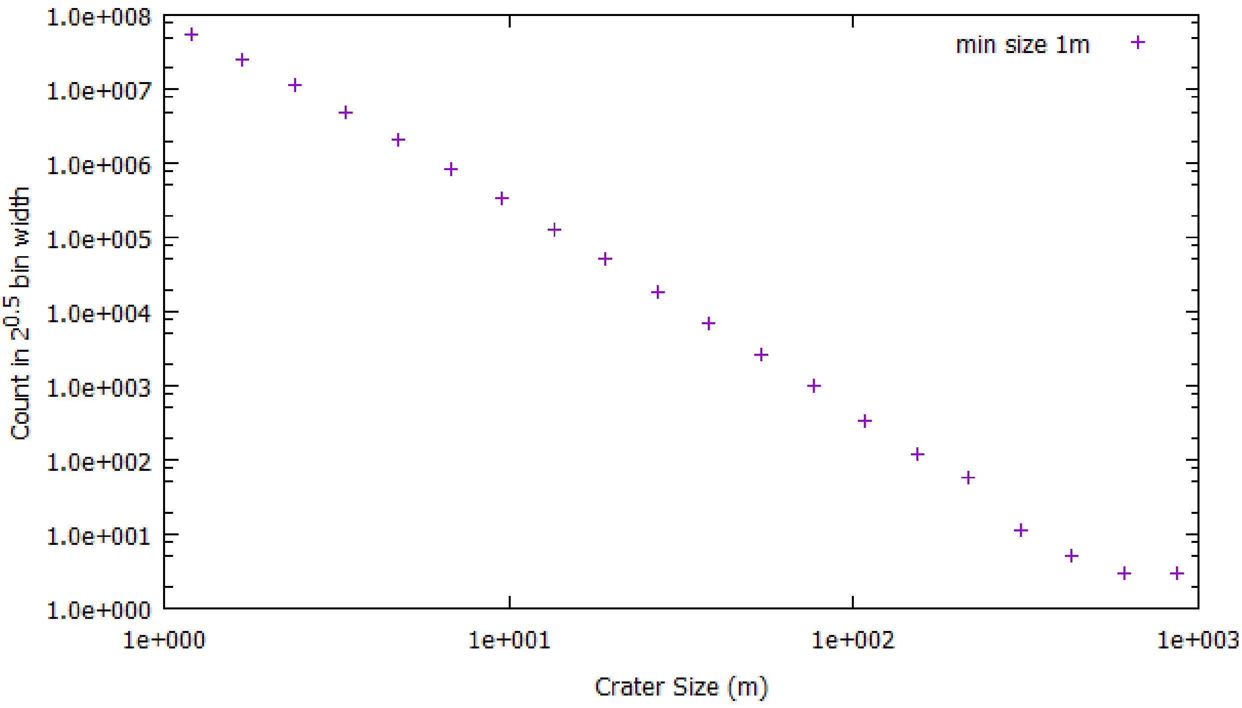


Figure 9-4. Crater distribution for 10^8 craters with size over 1m, the graph falls slightly slower than a 4th power law.

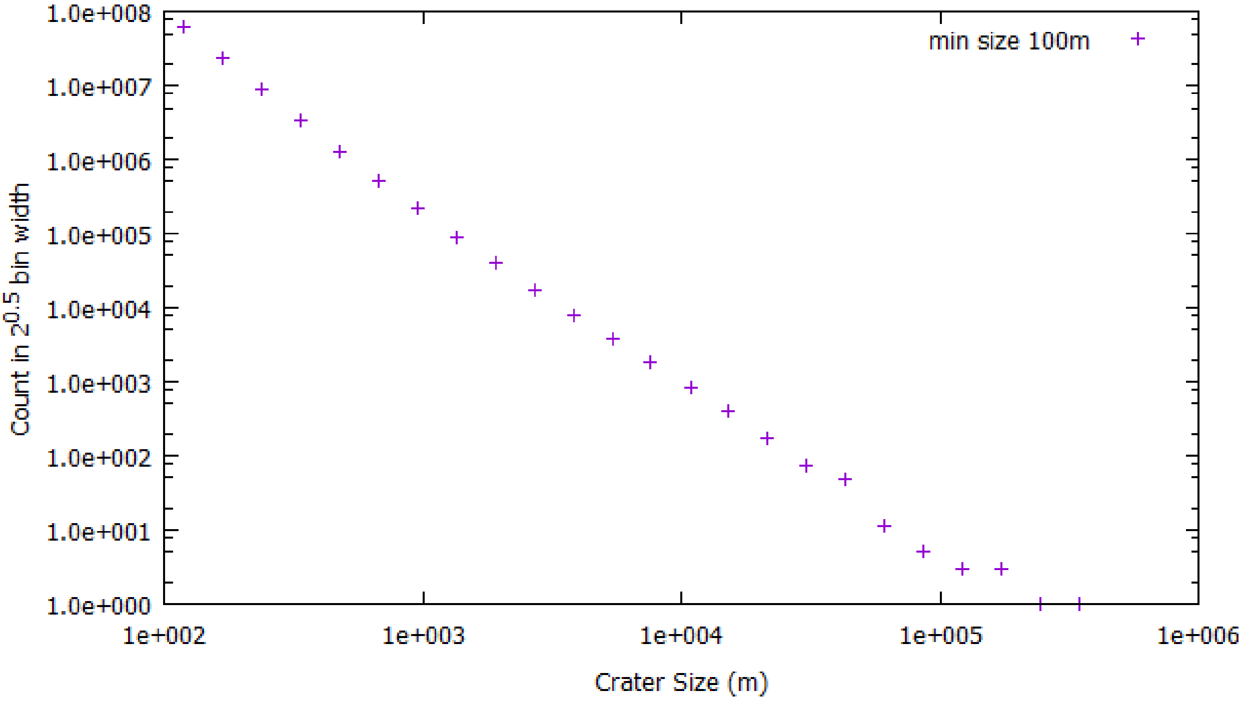


Figure 9-5. Crater distribution for 10^8 craters with size over 100m, the graph falls slightly slower than a 4th power law, but closer to 4 than Figure 9-4.

The power law is hard to see, but clearly is approximately correct – being slightly flatter over 1km. 256km and 4m are only just in the scale, and so not easily seen.

9.3 Impactor Size

The calculated impactor size for craters between 1m and 1km has been plotted, and is shown in Figure 9-6.

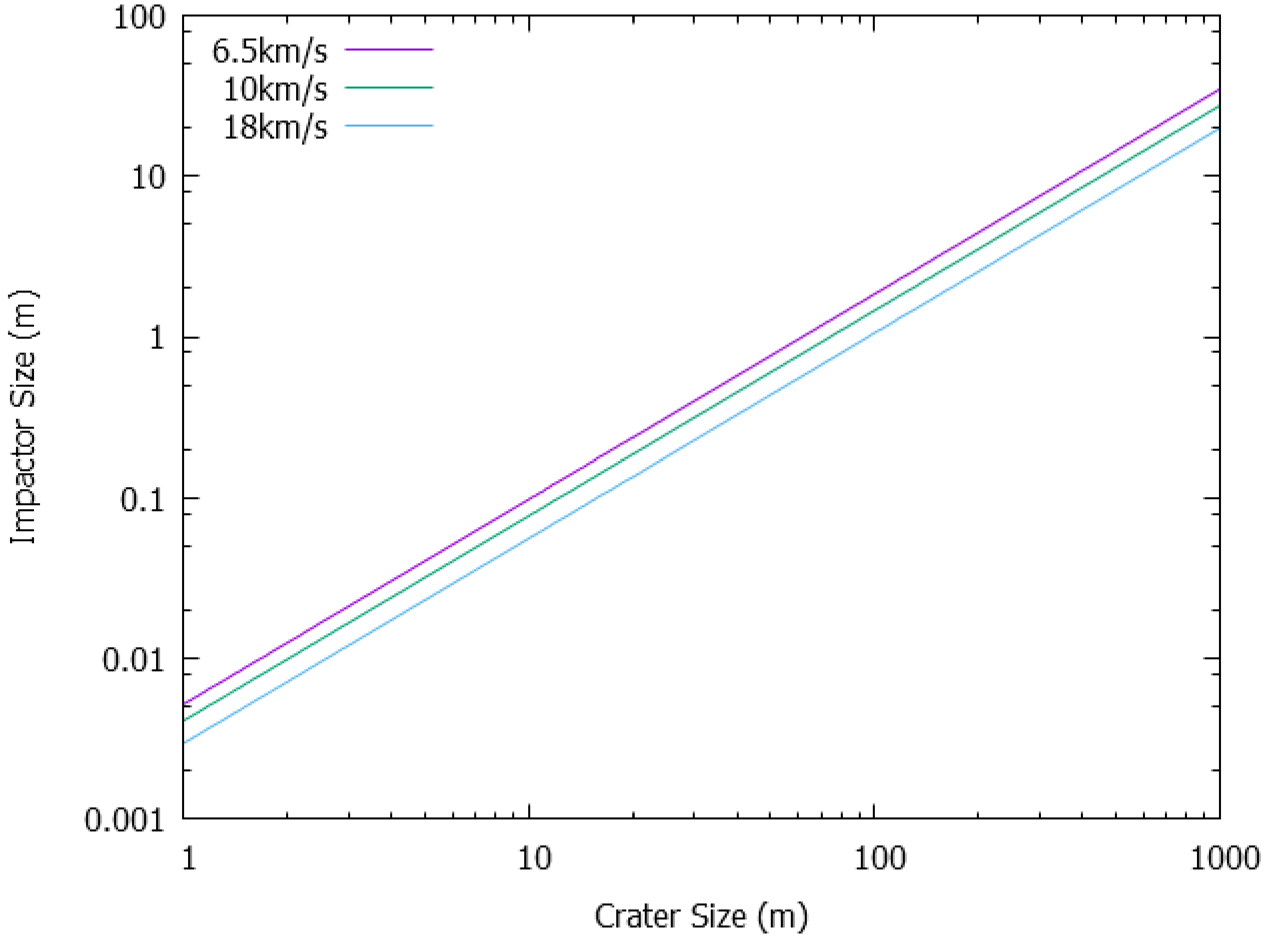


Figure 9-6. The relationship between impactor size and crater size on Mars, for various impact velocities which span most common speeds from the asteroid belt.

This graph agrees well with that derived in the document. Also comparing it to the figures from [RD9]

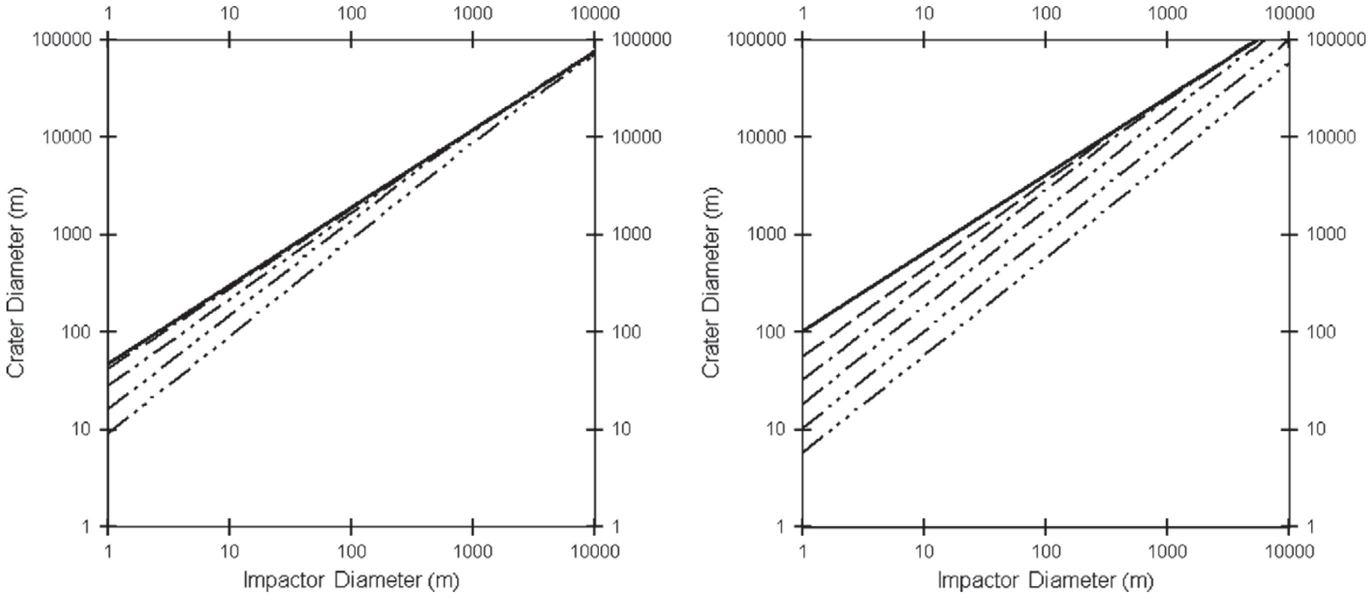


Figure 9-7. Fig3 from [RD9] Giving impactor to crater size on the Moon (left) with 17.5km/s impact, and Asteroid 433 Eros on the right and 5km/s.

The numbers are a comparable order of magnitude to [RD9], which given the different configurations gives confidence in the code.

9.4 Mass Ejected

[RD10] to which the ejection mass curves has been fitted, gave the total mass ejected over the Mars Escape velocity for three impact velocities 7.5,13.1 and 20.0 km/s, for a 10km sized impactor.

The model formed was a fit to all three velocity models, to form dm/dv_E the mass ejected as a function of ejection velocity. The fit formed was:

$$\frac{dm}{dv_E} = \exp\left(\left(2.66\times10^{-4}\left(\frac{s}{m}\right)v_I + 22.4\right) - \left(2.25\times10^{-8}\left(\frac{s}{m}\right)^2 v_I + 3.64\times10^{-4}\left(\frac{s}{m}\right)\right)v_E\right)\left(\frac{kg}{m/s}\right)$$

Now this can be either numerically or analytically evaluated from the mars escape velocity up to infinity. The analytic expression is:

$$m = \int_{\underline{v}}^{\infty} dv_E \frac{dm}{dv_E} = \frac{1}{2.25\times10^{-8}\left(\frac{s}{m}\right)^2 v_I + 3.64\times10^{-4}\left(\frac{s}{m}\right)} \left.\frac{dm}{dv_E}\right|_{v_E=\underline{v}}$$

Performing this integral for the 3 impact velocities gives

Impact Speed	Mass ejected over Mars Escape velocity [RD10]	Numerical	Analytic
7.5km/s	4.7e12 kg	5.07e12kg	5.07e12kg
13.1km/s	1.0e13kg	9.65e12kg	9.65e12kg
20.0km/s	2.33e13kg	2.24e13kg	2.24e13kg

Clearly the integral is reasonable, above at some impact velocities, and below at others. It does however give confidence that the form chosen is accurate to a few percent.

The other area that the Mass Ejected models is its dependence on the impactor size, which is modelled to scale as the cube of the size (as mass~volume~size^3). This is shown in Figure 9-8, and it scales as expected.

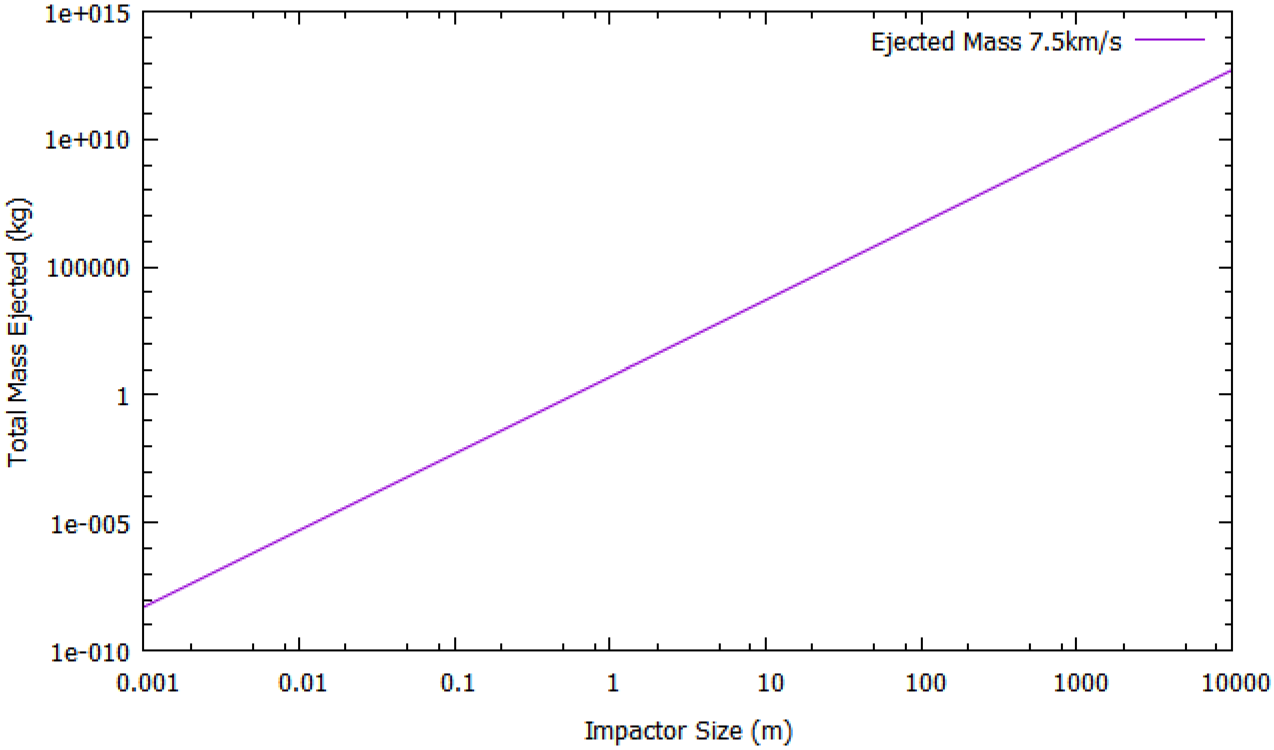


Figure 9-8. Mass ejected as a function of impactor size, this scales as the cube of size as expected. Shown for an impact velocity of 7.5km/s where the mass over the Mars escape velocity is looked at.

9.5 Mars To the Moon

The point of the impact on Mars in the simulation is unknown, and modelled as uniform. Similarly the phasing of the moons orbit is unknown, and modelled as uniform. However the angle between the ejection point and the moon (measured from the centre of Mars) is important. Now the uniformity of the distribution means this point will be uniform over a sphere, a uniform distribution is given by:

$$d\varphi \, d\cos\theta$$

Where θ is the angle between the point of ejection and the position of the moon. Hence $\cos\theta$ is modelled with a uniform distribution.

However what is important is how much material from the ejection is transferred to the moon, this depends on the ejection velocity, and this has been shown to be dependent on the angle between moon and ejection. Thus knowledge of the velocity is needed. In the calculation of the relation between θ and velocity, the velocity was used to calculate the angle. However for the simulation the reverse is needed, for a given angle what ejection velocity is required. This is what is implemented via a search.

The validity of that search is tested here. The relation between velocity and angle is easily calculated, but between angle and velocity requires a slow search. Both though produce (angle,velocity) pairs, and these can be compared. This is the test performed, and the result shown in Figure 9-9 and Figure 9-10 – where agreement can be seen.

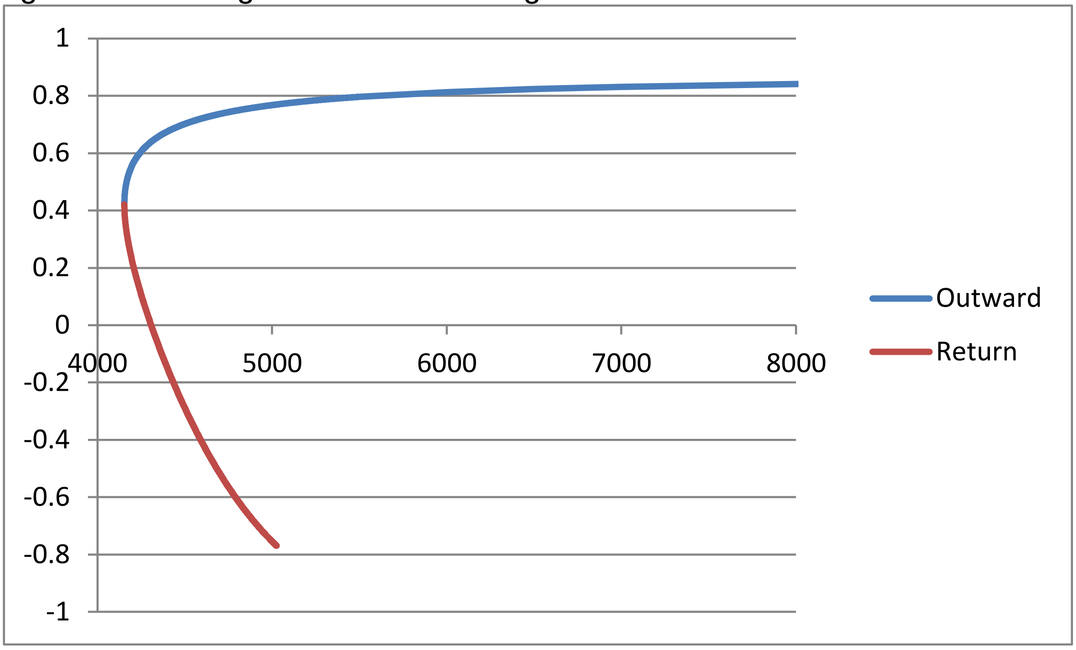


Figure 9-9. Excel calculation of the angle between ejection and moon as a function of ejection speed. This is performed for a 45° ejection cone.

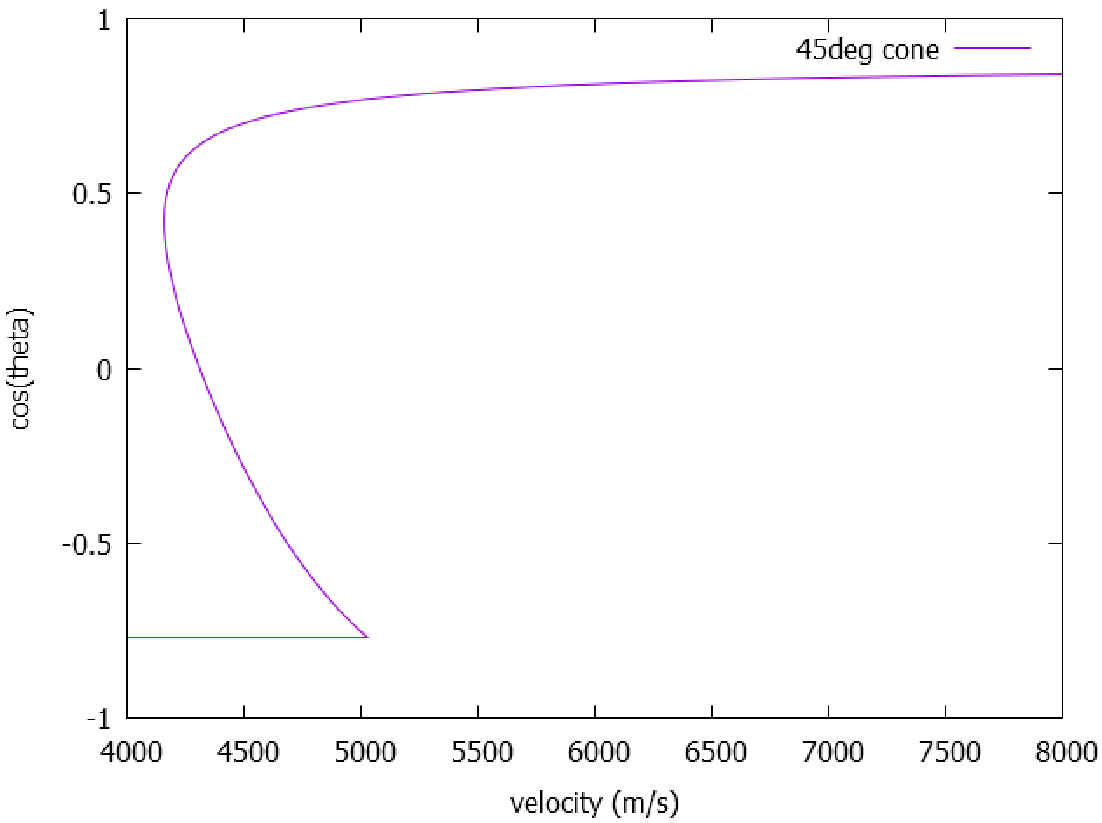


Figure 9-10. Numerical code that from an ejection angle, calculates the velocity. Clearly it is very similar to Figure 9-10.

9.6 $\frac{d\cos\theta}{dv}$

The Mars to Moon function also constructs the derivative of the curve of cos θ to velocity. That this derivative is correct can be checked my numerically integrating it. This is performed via

$$v - \text{const} = \int d\cos\theta \frac{1}{d\cos\theta/dv}$$

Where the offset arises, as the derivative has no knowledge of the starting place for velocity. The integral has been performed numerically, and the comparison to velocity is shown in Figure 9-11.

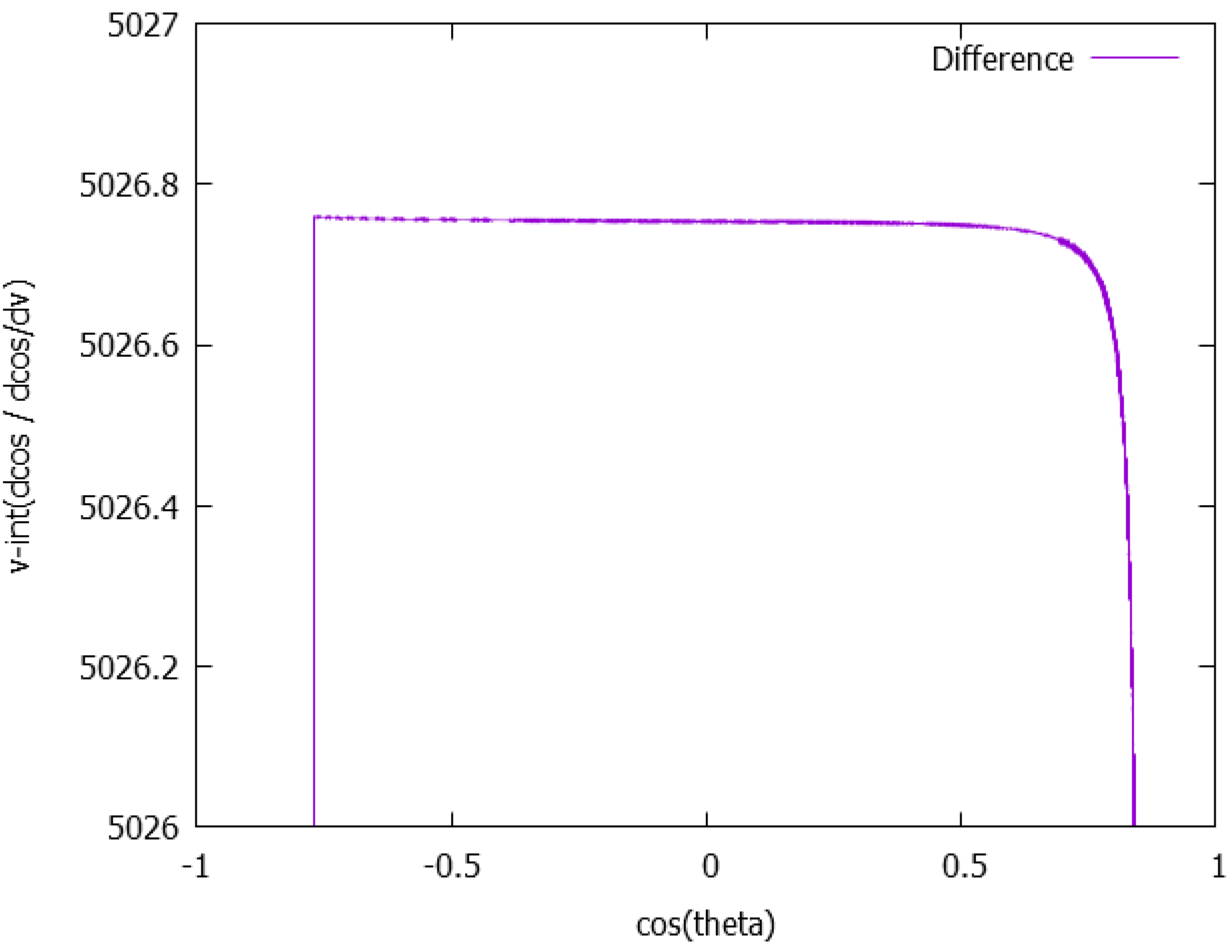


Figure 9-11. The comparison between velocity and $\int d\cos\theta \frac{1}{d\cos\theta/dv}$. This deviates from a constant at ~0.8 as $\frac{d\cos\theta}{dv} \to 0$ at that point, which generates errors in the approximate integral.

The comparison is clearly good over most angles, where the difference is very close to the expected constant. It deviates at cosθ=0.8, this is where the $\frac{d\cos\theta}{dv}$ curve goes to zero – and so expected numerical error arise in the approximate integral. Hence the derivate is shown to be accurate.

10. CONCLUSION

This paper describes the simulation needed to model transfer of material for Mars to a Martian Moon. The development of the model is described, how it is coded, and how it has been tested.

END OF DOCUMENT