Deep Inelastic Scattering: What did HERA teach us, what can be improved?

April 2017- EIC Assessment Committee

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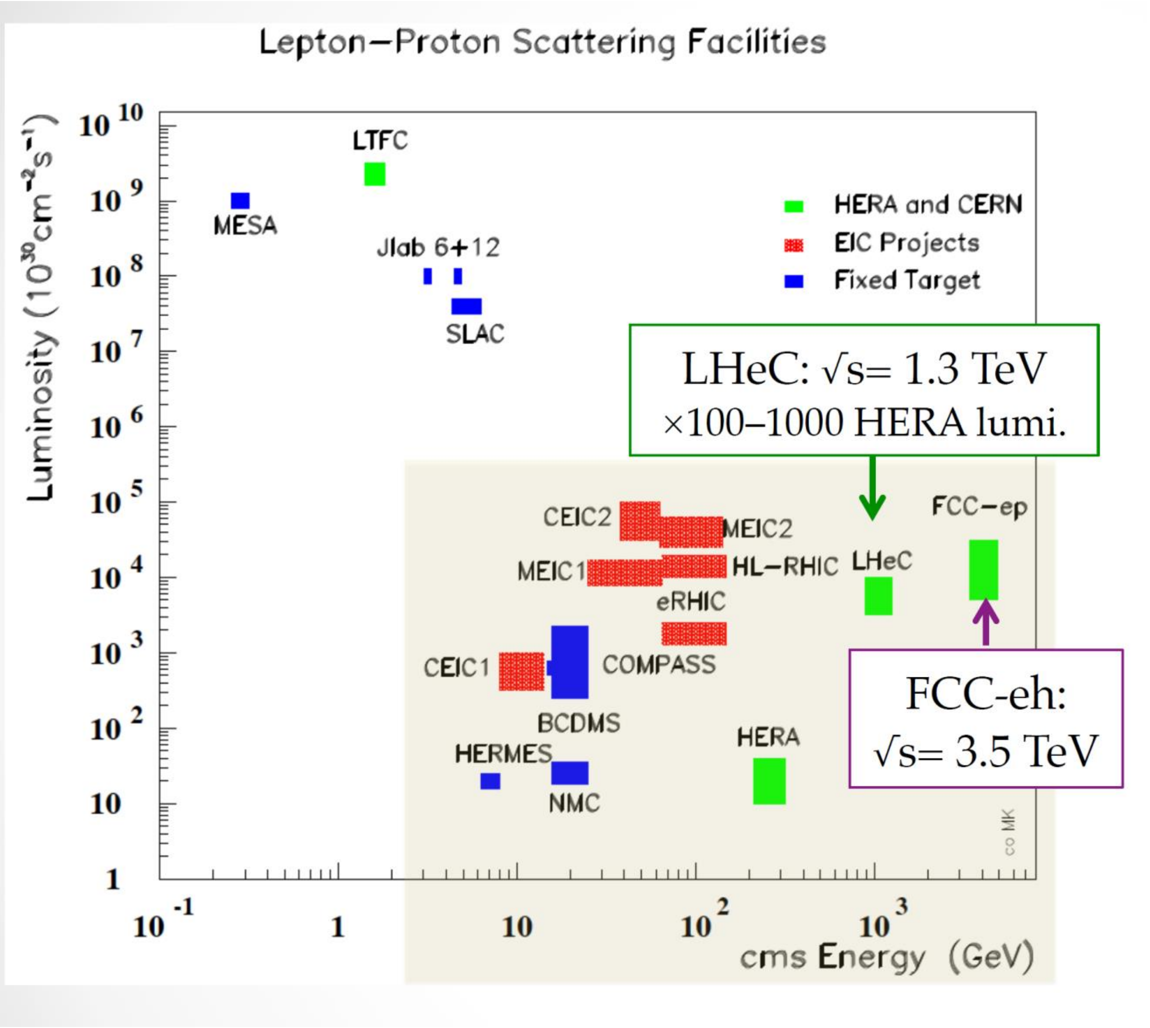
We have the final HERA combined data on inclusive neutral and charge current cross-sections for e^+p and e^-p scattering, for 4 different centre of mass energies.
We also have final data on charm and beauty production.
And there is final inclusive jet, di-jet and tri-jet data from both H1 and ZEUS

This has been used to extract the Parton Distribution Functions HERAPDF
AND is the back-bone of all the other PDFs –CT14, MMHT14, NNPDF3.0, ABM- most of which do not YET have the final HERA data

PDF fitting assumes the validity of conventional QCD DGLAP evolution, which sums $\ln(Q^2)$ diagrams
This has served us very well

But we have always had suspicions that at low-x we should also be re-summing $\ln(1/x)$ diagrams AND that we could be heading into a new kinematic regime of high-density partons in which we need non-linear evolution, which could lead to saturation.

This is the region in which an EIC could tell us more about QCD- especially because of the higher densities in nuclei as opposed to nucleons



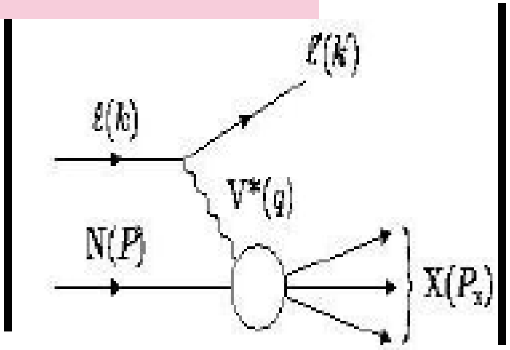
LHeC: √s= 1.3 TeV
×100–1000 HERA lumi.

FCC-eh:
√s= 3.5 TeV

My remarks on what the EIC can do to improve on HERA will be based on my understanding from this plot of likely kinematic range and luminosity

PDFs were first investigated in deep inelastic lepton-nucleon scatterning -DIS

$d\sigma \sim$

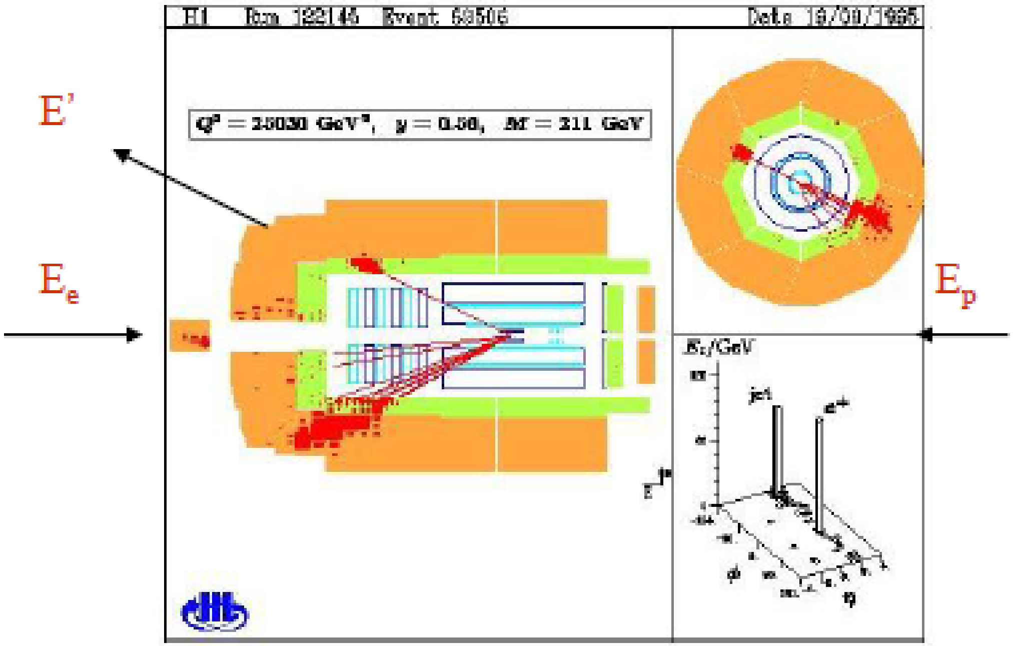


$$\left| \cdots \right|^2 \quad L^{\mu\nu} W_{\mu\nu}$$

Leptonic tensor - calculable

Hadronic tensor- constrained by Lorentz invariance

Candidate from NC sample



$q = k - k'$, $Q^2 = -q^2$ ← This is the scale of the vector boson probe

$s = (p + k)^2$

$x = Q^2 / (2p.q)$

$y = (p.q)/(p.k)$ ← These are 4-vector invariants

$Q^2 = s\ x\ y$

$s = 4\ E_e\ E_p$

$Q^2 = 4\ E_e\ E'\ \sin^2\theta_e/2$

$y = (1 - E'/E_e \cos^2\theta_e/2)$

$x = Q^2/sy$

The kinematic variables are measurable

Schematically,

$$d\sigma \sim \sum_x \Bigg|$$

$$\Bigg|^2 \sim L^{\mu\nu} \cdot W_{\mu\nu}$$

Leptonic tensor, calculable
ELECTROWEAK

Hadronic tensor, constrained
by LORENTZ INVARIANCE

⇒ Charged lepton-neutral current γ, Z

$$\frac{d^2\sigma(\ell^\pm)}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[y_+ F_2^{NC}(x,Q^2) - y^2 F_L^{NC}(x,Q^2) \mp y_- xF_3^{NC}(x,Q^2)\right]$$

$$y_\pm = 1 \pm (1-y)^2$$

Charged lepton- charged current $W^\pm$

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{G_F^2}{4\pi x}\,\frac{M_W{}^4}{(Q^2+M_W{}^2)^2}\left[y_+ F_2^{CC}(x,Q^2) - y^2 F_L^{CC}(x,Q^2) \mp y_- xF_3^{CC}(x,Q^2)\right]$$

F_2 , F_L , xF_3 are STRUCTURE FUNCTIONS

which parameterise our ignorance of the hadronic sector

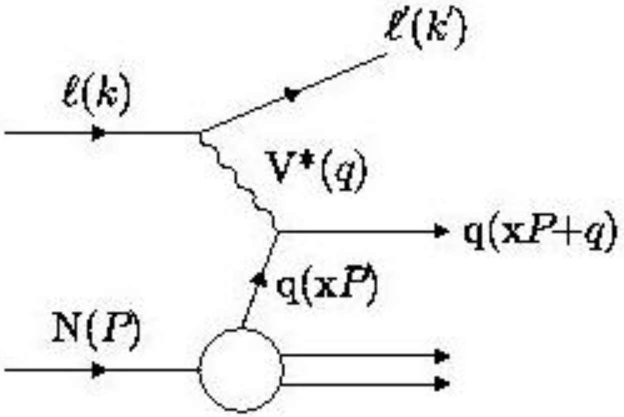
⇒ MEASUREABLE as functions of x, Q^2

Without assumptions as to what goes on in the hadron the double differential cross-section for $e^{\pm}$ N scattering can be written as

$$\frac{d^2\sigma(e\pm N)}{dxdy} = \frac{2\pi\alpha^2 s}{Q^4} [\, Y_+ \mathbf{F_2}(x,Q^2) - y^2 \mathbf{F_L}(x,Q^2) \pm Y\_x\mathbf{F_3}(x,Q^2)], \quad Y\pm = 1 \pm (1-y)^2$$

Leptonic part

hadronic part



F_2, F_L and xF_3 are structure functions

which express the dependence of the cross-section

on the structure of the nucleon (hadron)–

The Quark-Parton Model interprets these structure functions as related to the momentum distributions of point-like quarks or partons within the nucleon AND the measurable kinematic variable $x = Q^2/(2p.q)$ is interpreted as the FRACTIONAL momentum of the incoming nucleon taken by the struck quark .

QCD improves on the QPM by accounting for the quarks interaction with gluons.

We can extract all three structure functions experimentally by looking at the x, y, Q^2 dependence of the double differential cross-section- thus we can check out the parton model predictions

$$(xP+q)^2 = x^2p^2 + q^2 + 2xp.q \sim 0$$

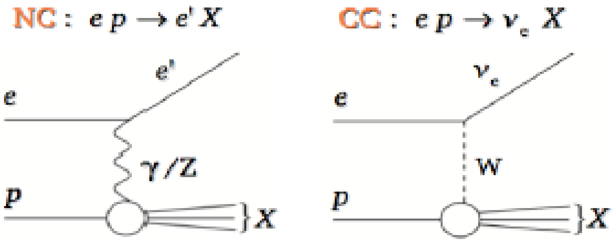
for massless quarks and $p^2 \sim 0$

so

$$x = Q^2/(2p.q)$$

The FRACTIONAL momentum of the incoming nucleon taken by the struck quark is the MEASURABLE quantity x

Deep Inelastic Scattering (DIS) is the best tool to probe proton structure



- Kinematic variables:

$Q^2 = -q^2 = -(k-k')^2$ Virtuality of the exchanged boson

$x = \frac{Q^2}{2p\cdot q}$ Bjorken scaling parameter

$y = \frac{p\cdot q}{p\cdot k}$ Inelasticity parameter

$s = (k+p)^2 = \frac{Q^2}{xy}$ Invariant c.o.m.

Neutral current:

$$\frac{d^2\sigma^{\pm}_{NC}}{dxdQ^2} = \frac{2\alpha\pi^2}{xQ^4}\left(Y_+ F_2 \mp Y_- xF_3 - y^2 F_L\right)$$

$F_2 \propto \sum_i e_i^2 (xq_i + x\bar{q}_i)$ quark distributions

$xF_3 \propto \sum_i (xq_i - x\bar{q}_i)$ valence quarks

$F_L \propto \alpha_s \times g$ gluon at NLO

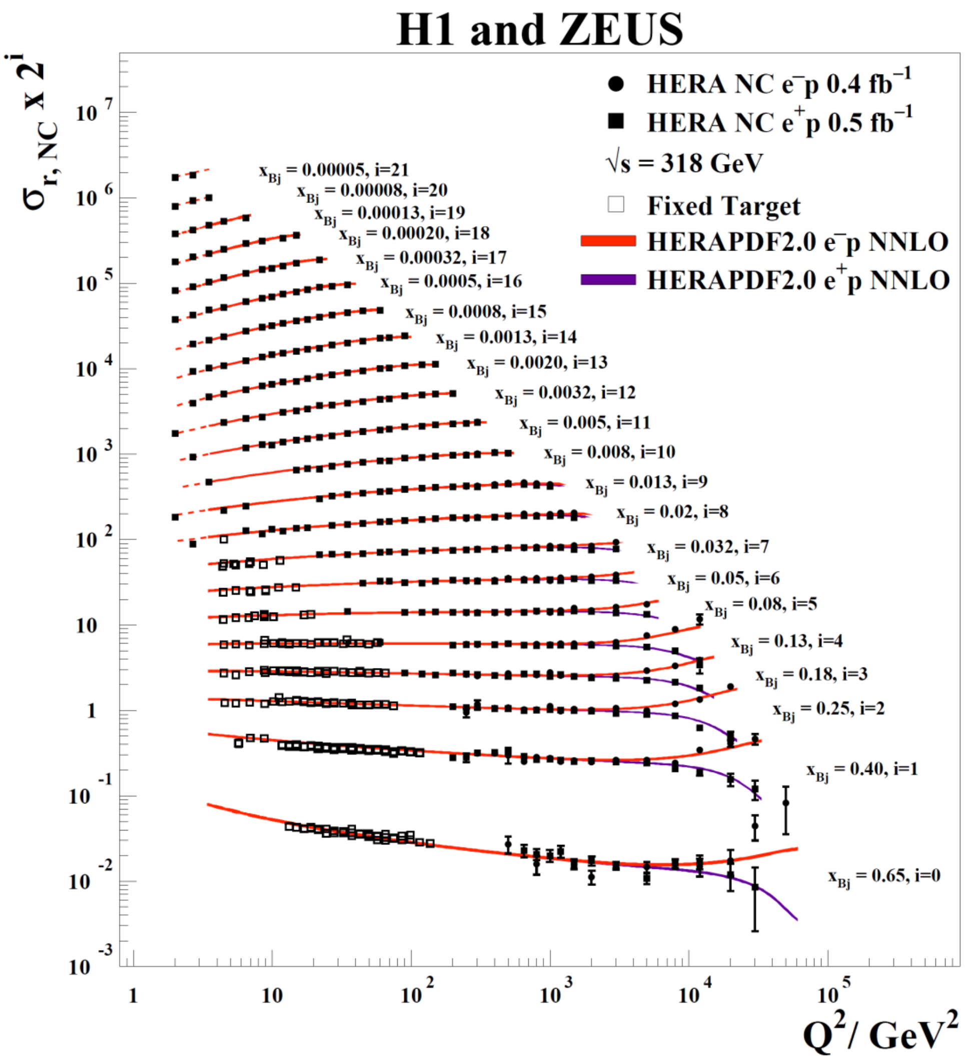
LO expressions

Charged current:

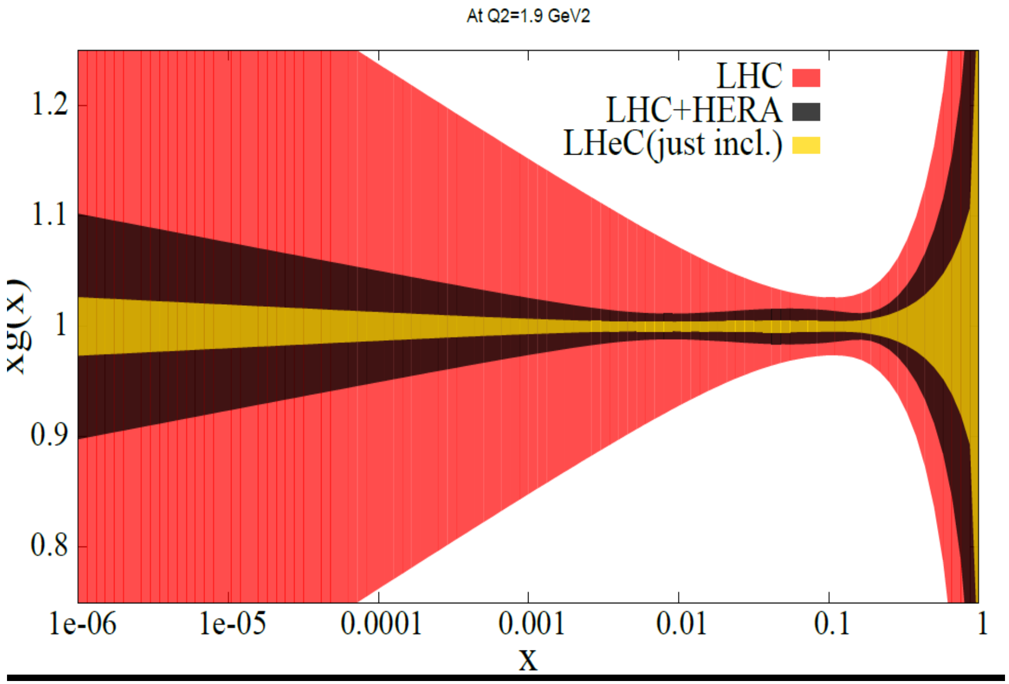
$$\frac{d^2\sigma^{-}_{CC}}{dxdQ^2} = \frac{G_F^2}{2\pi}\frac{M_W^2}{M_W^2+Q^2}\left(u+c+(1-y^2)(\bar{d}+\bar{s})\right)$$

$$\frac{d^2\sigma^{+}_{CC}}{dxdQ^2} = \frac{G_F^2}{2\pi}\frac{M_W^2}{M_W^2+Q^2}\left(\bar{u}+\bar{c}+(1-y^2)(d+s)\right)$$

flavour decomposition



Gluon from the scaling violations: DGLAP equations tell us how the partons evolve



Let's ask the question-
Can we determine PDFs just from the LHC?

NOT with any precision NO !

Present LHC W,Z data and jet data are included and LHC ultimate precision is extrapolated according to our current experience– we are systematics limited already

PDFs come from DIS

Final inclusive data combination from all HERA-1+11 running

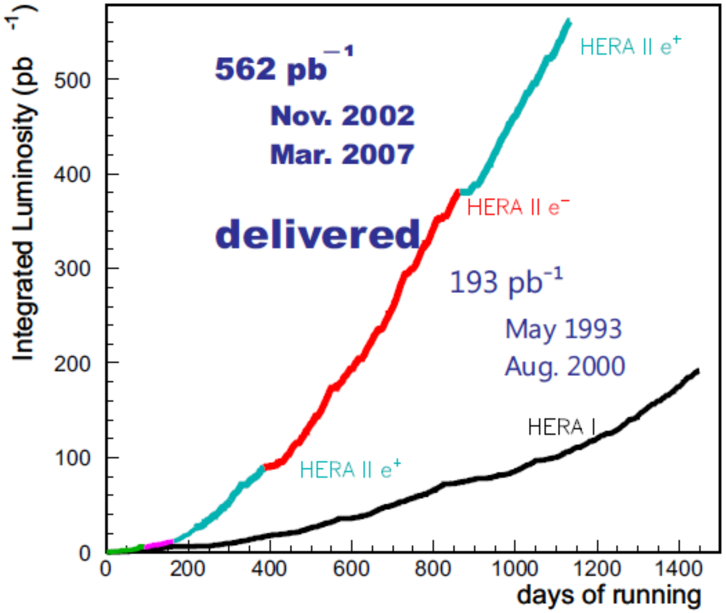
~500pb^{-1} per experiment split ~equally between e$^+$ and e$^-$ beams:arXiV:1506.06042

10 fold increase in e$^-$ compared to HERA-I

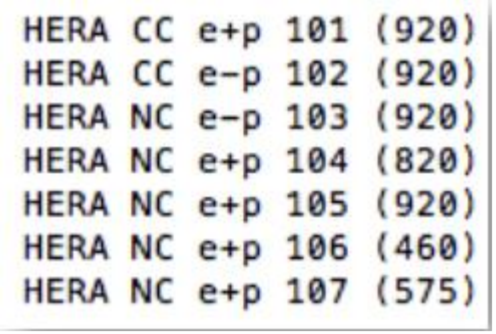
Running at Ep = 920, 820, 575, 460 GeV

$\sqrt{s}$ = 320, 300, 251, 225 GeV

The lower proton beam energies allow a measurement of F_L and thus give more information on the gluon.

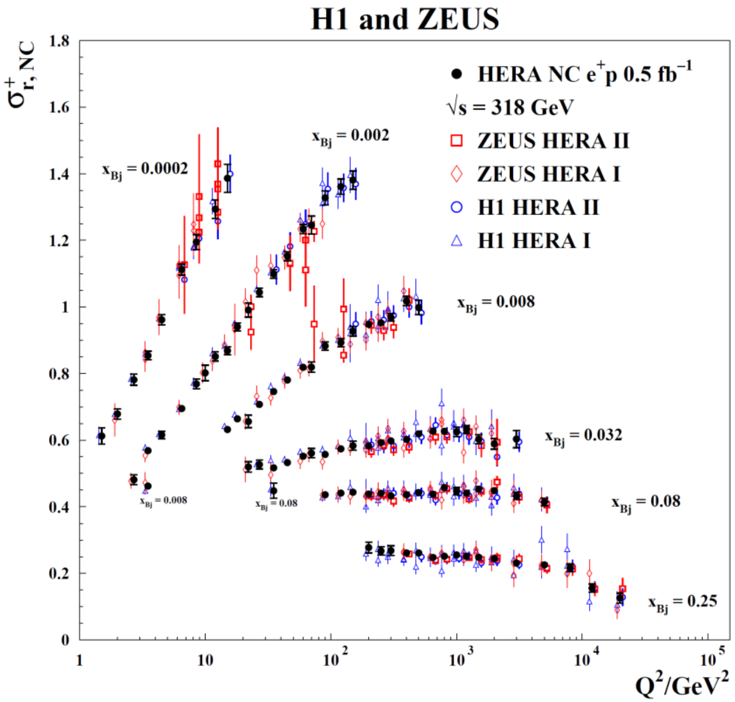


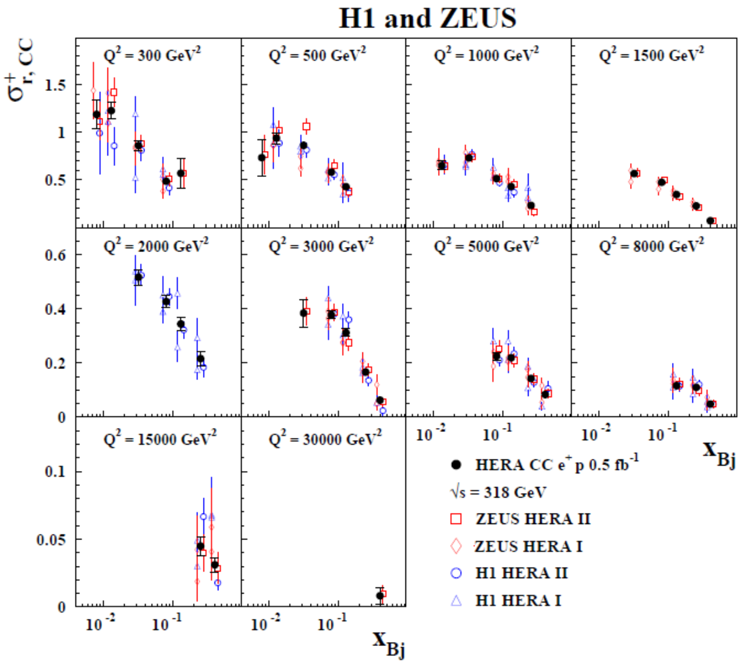
41 input data files to 7 output files with 169 sources of correlated uncertainty


HERA CC e+p 101 (920)
HERA CC e-p 102 (920)
HERA NC e-p 103 (920)
HERA NC e+p 104 (820)
HERA NC e+p 105 (920)
HERA NC e+p 106 (460)
HERA NC e+p 107 (575)

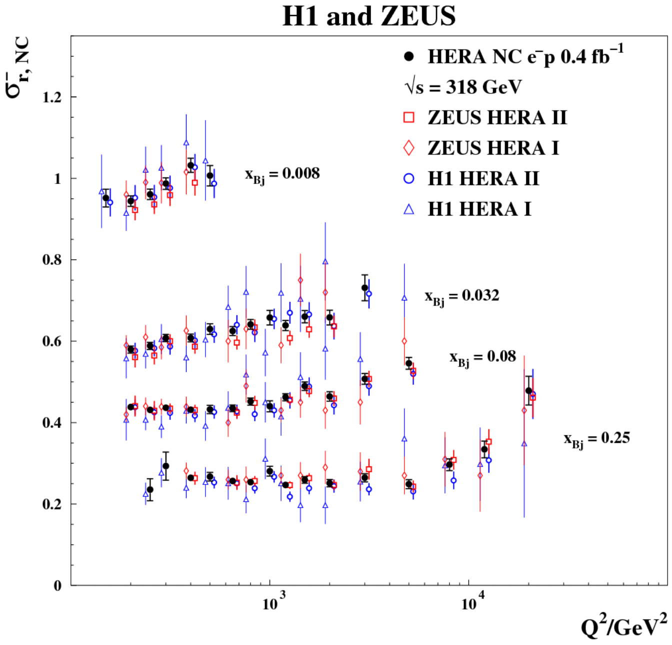
$0.045 < Q^2 < 50000$ GeV^2

$6.\ 10^{-7} < x_{Bj} < 0.65$



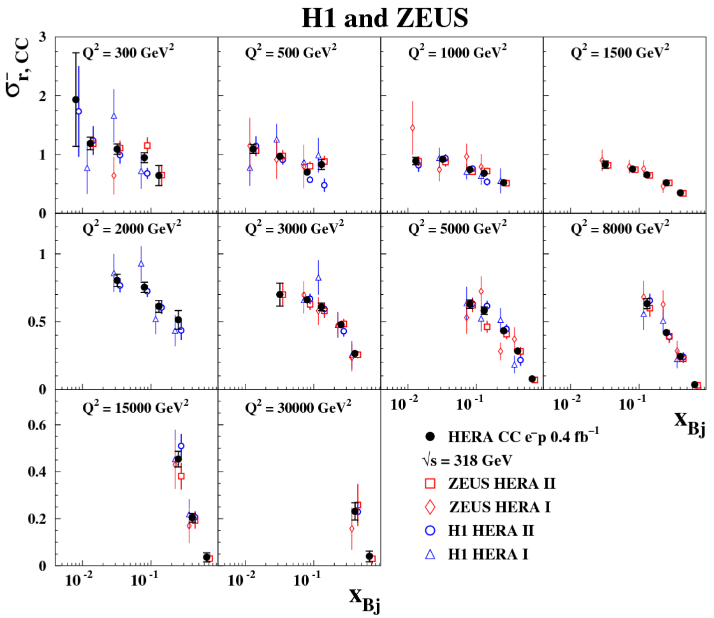


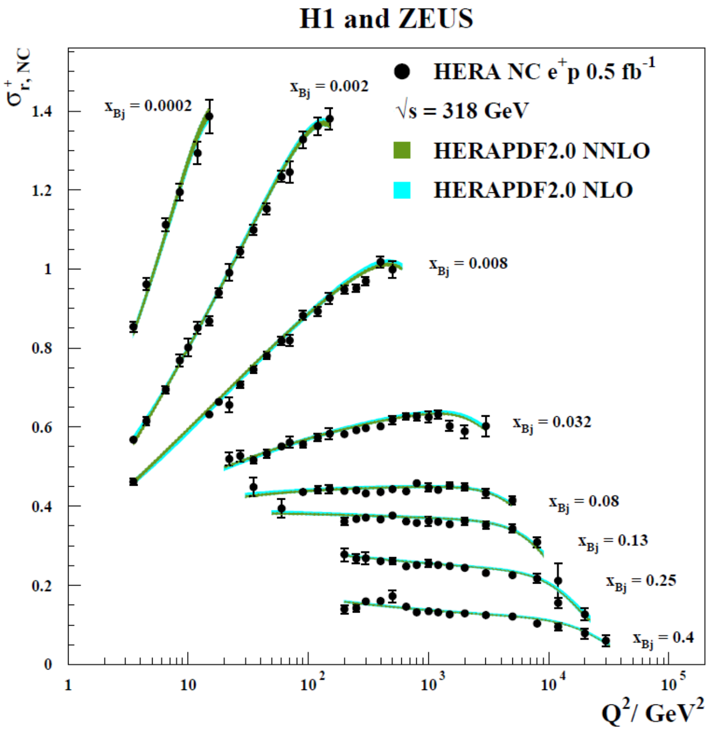
NC and CC e^+ vs H1 and ZEUS inputs

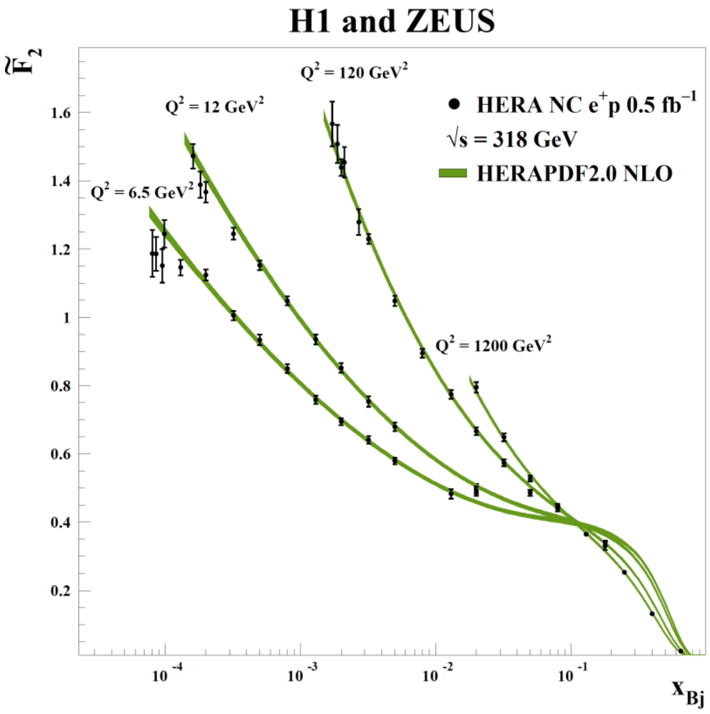


NC and CC e^- vs H1 and ZEUS inputs

10 fold increase in e^- statistics compared to old HERA-1 combination





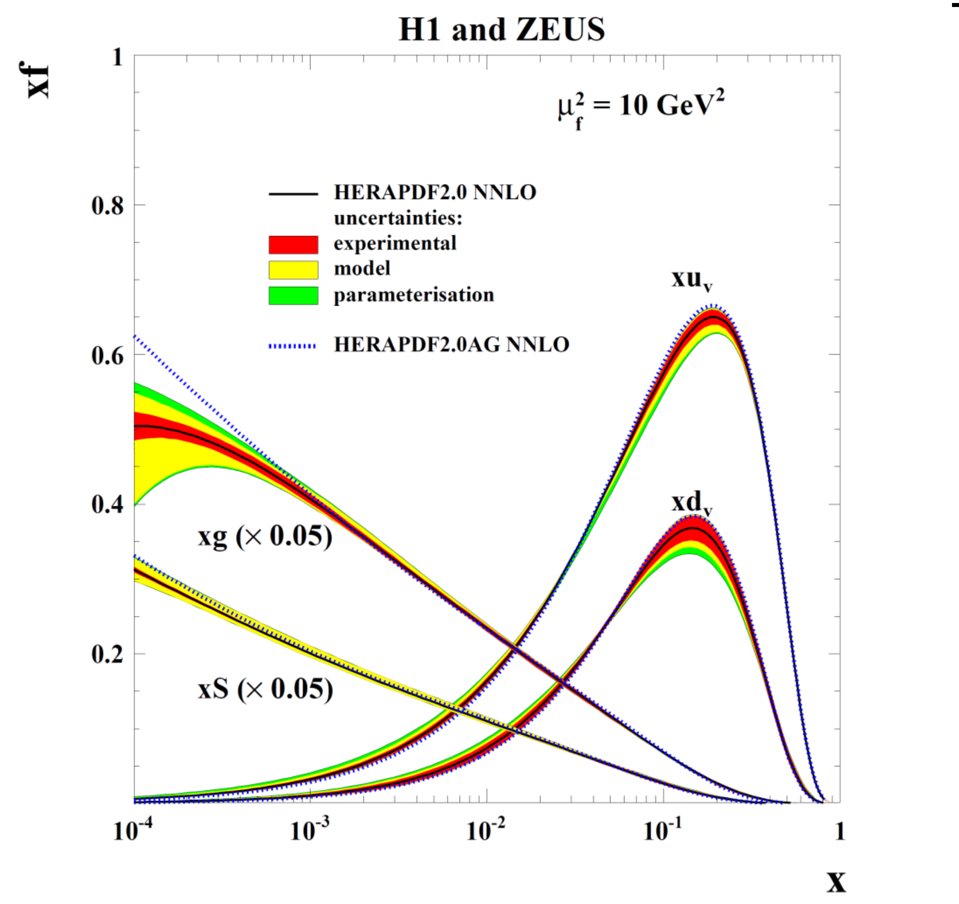


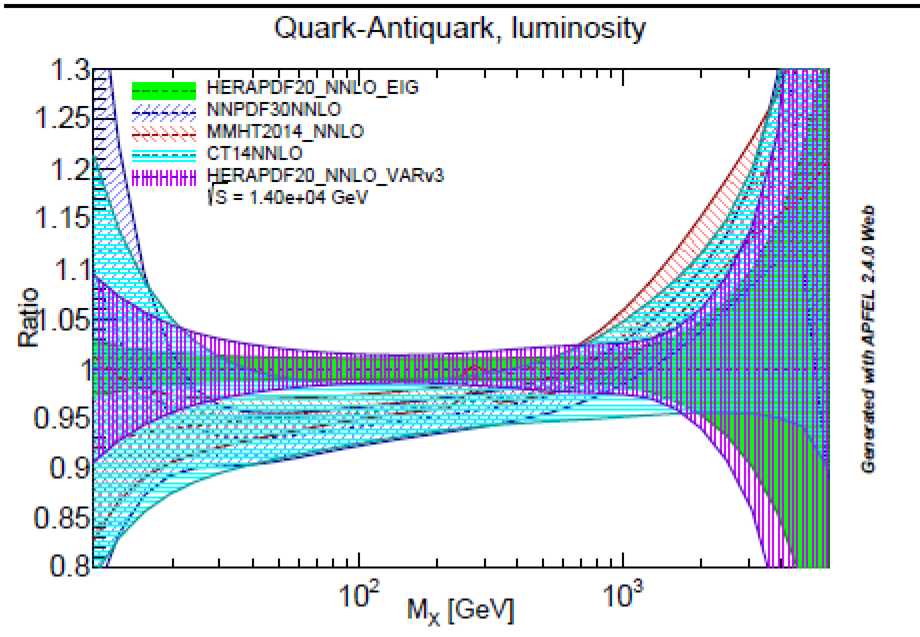
Scaling violations

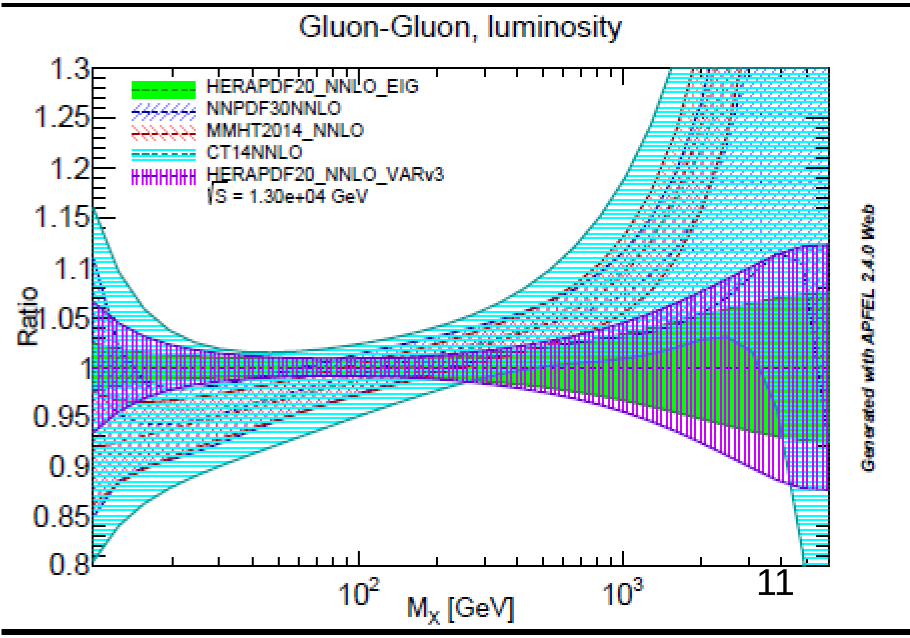
Low-x rise of F_2.. Let’s come back to this..

A full perturbative QCD analysis to determine the parton distributions called the HERAPDF…

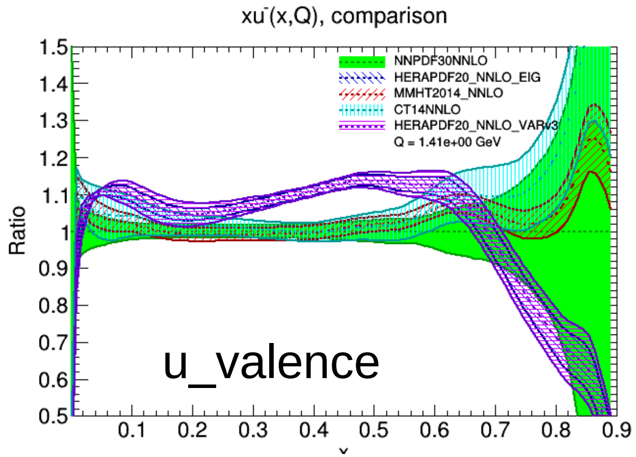
Compare HERAPDF2.0 to other PDFs at NNLO

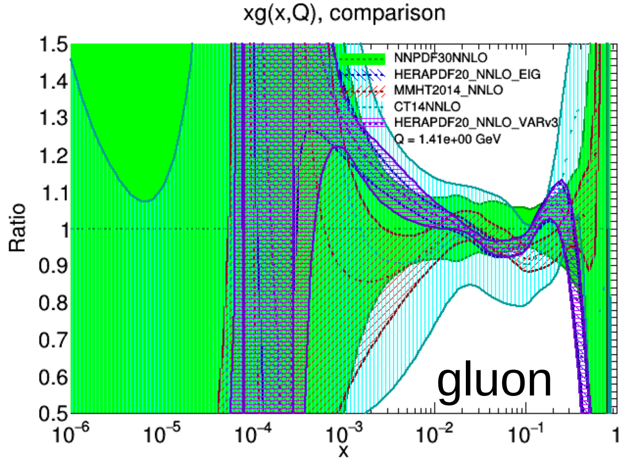


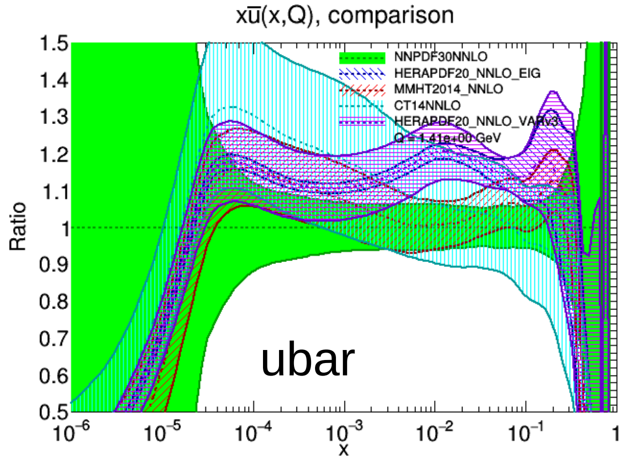




Comparison of q-qbar and gluon-gluon luminosity at 13 TeV show consequences for LHC







We don't know low-x PDFs well for $x < 10^{-4}$ for the Sea, for $x < 10^{-3}$ for the gluon.

This is a matter of kinematic reach -and EIC does not extend this– but it is also a matter of paying it due respect- and EIC could do that

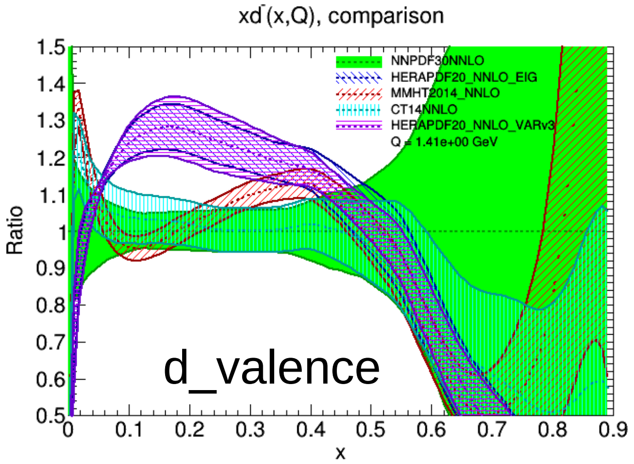
We also do not know PDFs well at high-x

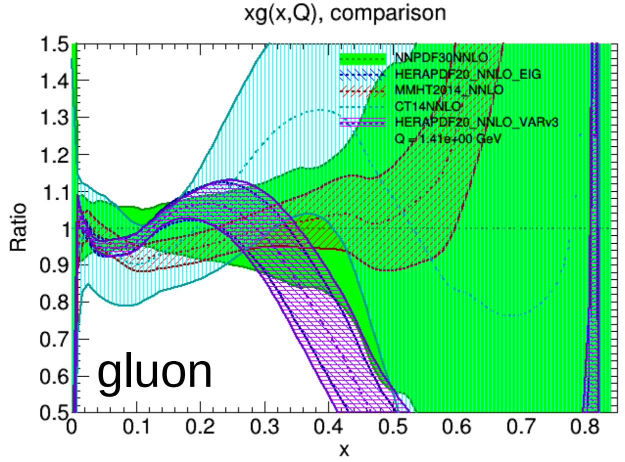
$x > 0.8$ for u_valence

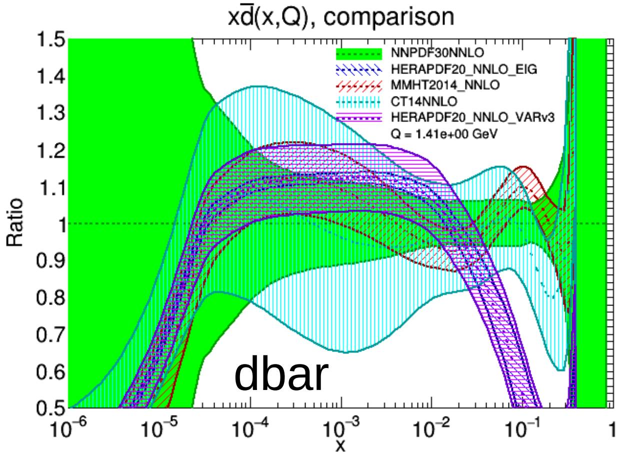
$x > 0.6$ for d_valence

$x > 0.5$ for gluon

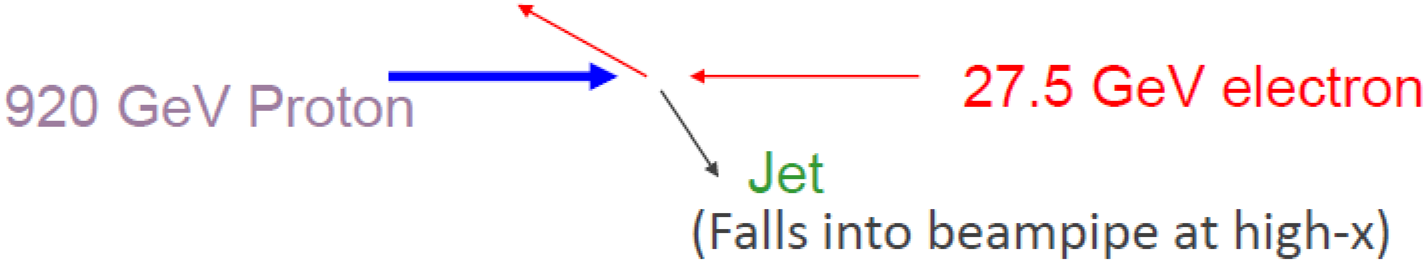
EIC could help here.







One reason that the HERA kinematic region did not allow us to measure well at high-x is that jets fall into the beam-pipe at high-x



Can EIC do better than HERA at high-x?

There are several advantages:

- Much higher luminosity (2 to 3 orders of magnitude)
- Run deuterons (measure neutrons)—get d_valence
- Access to lower angle jets (large crossing angle for the beams)
- Better flavor tagging.

Also at least one disadvantage:

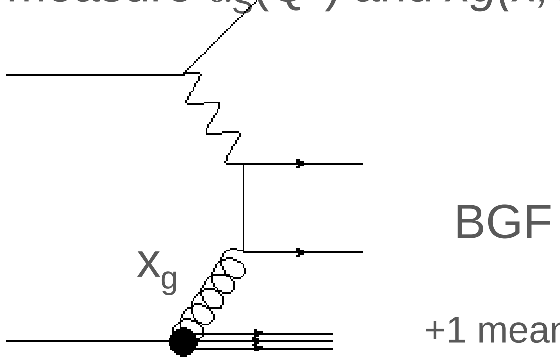
- Lower energies mean lower energy jets—worse calorimetric resolution.

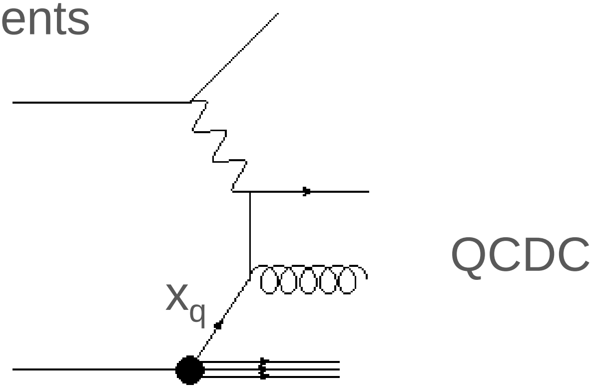
(at high-x, Q^2~10 GeV^2: essentially x is measured by jet energy)

Jets could be important for improving the gluon PDF and measuring $\alpha_S(M_Z)$

Jet studies in the Hadron Final state gives us more information

- You can measure $\alpha_S(Q^2)$ and $xg(x,Q^2)$ from 2+1 jet events





+1 means proton remnant

$$\sigma_{2+1} \sim \alpha_s \left[A\, x_g\, g\left(x_g,\, Q^2\right) + B\, x_q\, q\left(x_q,\, Q^2\right) \right]$$

(glue) + (quark)

This helps to break the $\alpha_S(Q^2)$ / gluon PDF correlation
Use more information that depends directly on the gluon -- jet cross-sections

To get x $g(x,Q^2)$

- Assume α_S is known
- Choose kinematic region BGF > QCDC (i.e. low x, Q^2)

To get $\alpha_S(Q^2)$

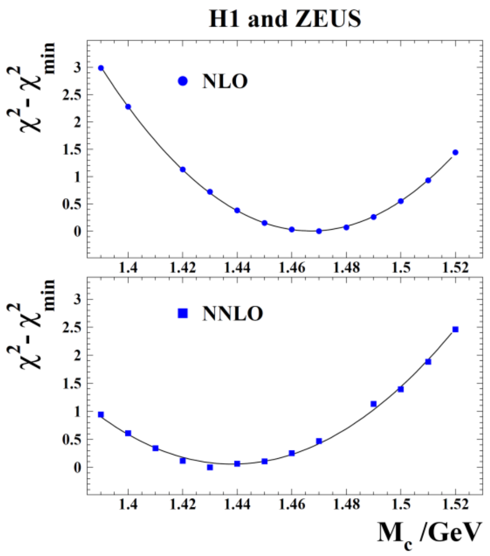
- Choose kinematic region where PDFs $xq(x)$, $x\,g(x)$ are well known. (i.e. $x_g > 10^{-2}$, $x_q > 10^{-3} – 10^{-2}$ and $\sigma_{BGF} \sim \sigma_{QCDC}$

In practice make a simultaneous fit for $\alpha_S(M_Z)$ and the PDFs- where the gluon PDF has the strongest correlation to $\alpha_S(M_Z)$ – use both inclusive data and jet production data to do this.

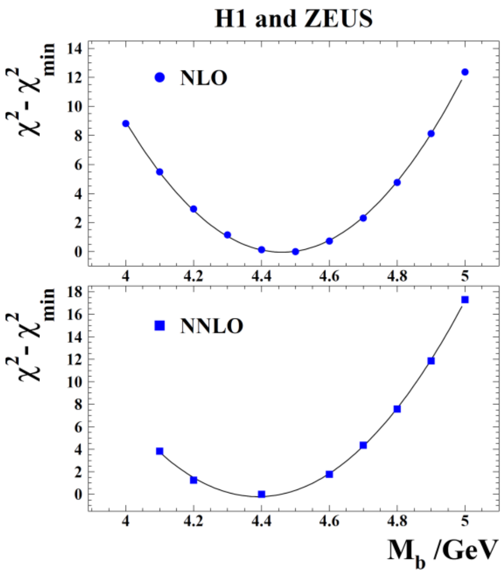
Adding more data to HERAPDF2.0: heavy flavour data and jet data

HERAPDF2.0Jets is based on inclusive + charm + jet data
The fits with and without jet data and charm data are very compatible for fixed $\alpha_S(M_Z)$

The main effect of heavy flavour data is to determine the optimal values of the charm and beauty mass parameters and their variation



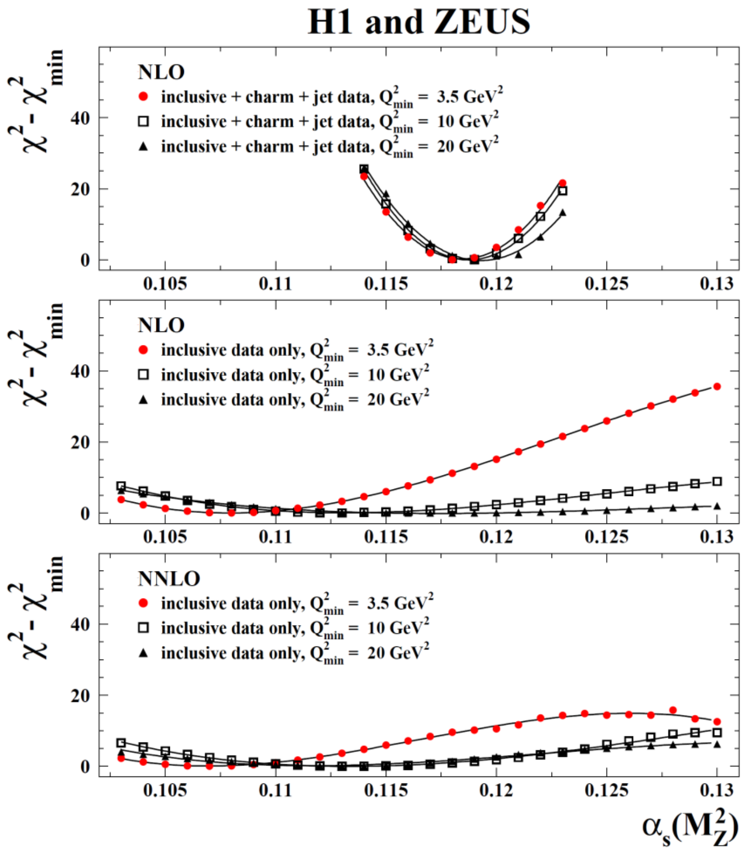
Similarly for beauty data



The main effect of jet data is to allow a determination of $\alpha_S(M_Z)$ at NLO. Inclusive data alone cannot give a reliable determination.

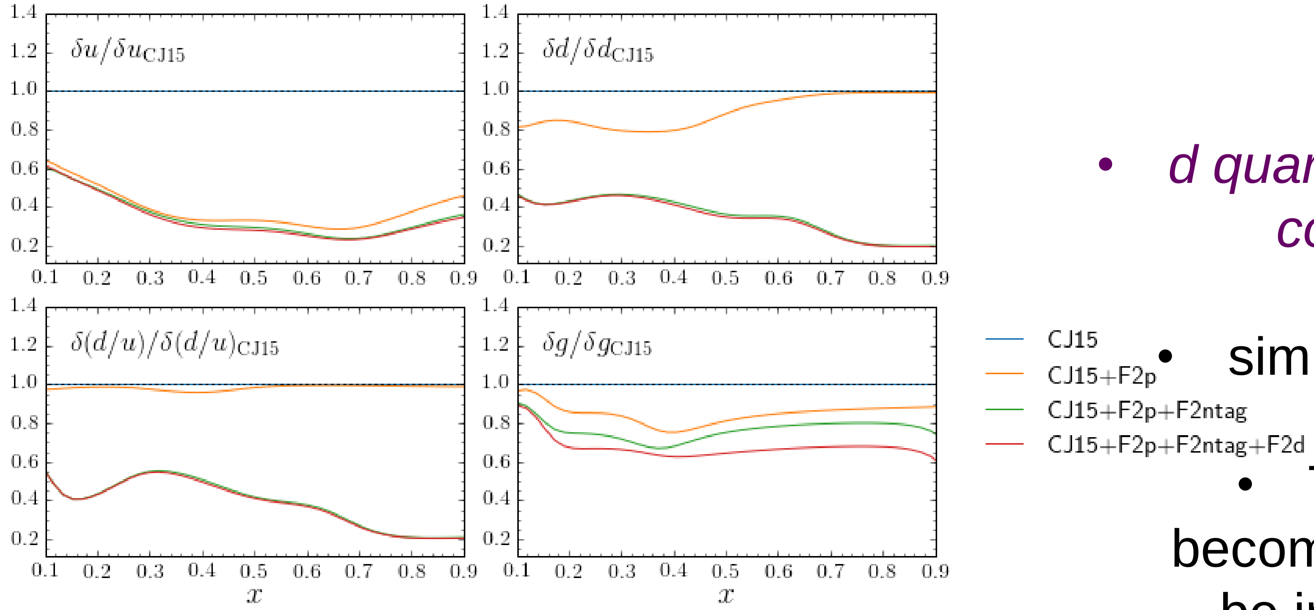
When jet data are added one can make a simultaneous fit for PDF parameters and $\alpha_S(M_Z)$ at NLO---

NNLO calculation still not available



$\alpha_S(M_Z) = 0.1183 \pm 0.0009_{(exp)} \pm 0.0005_{(model/param)} \pm 0.0012_{(had)}$

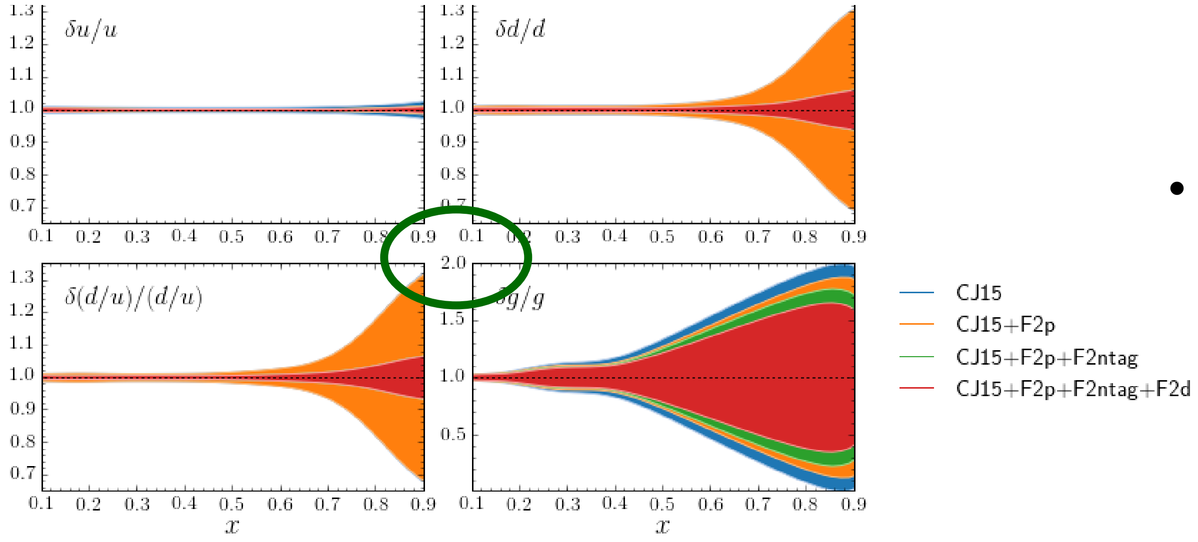
Improving high-xPDFs with100/fb luminosity



- *d quark precision will become comparable to current u!!*

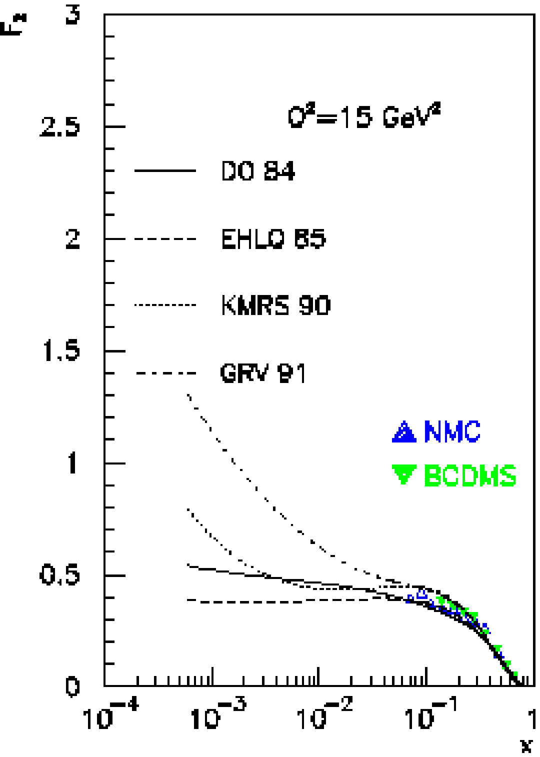
- similar improvement in g(x)

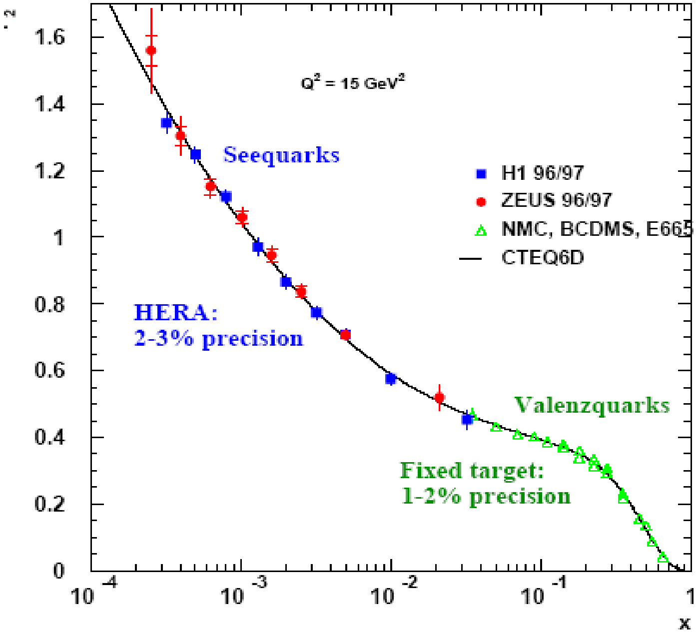
- The u quark uncertainty becomes less than ~1%; may be important for large mass BSM new particles.

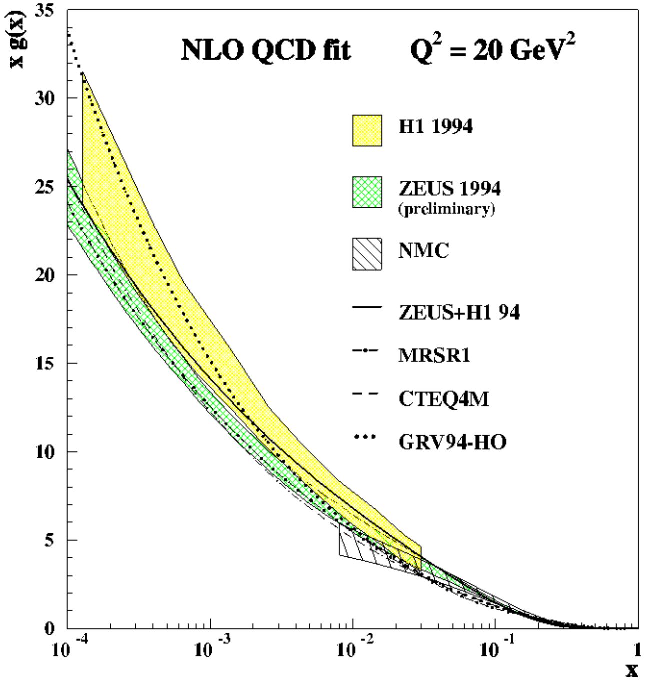


- With d quark nailed by F_2^n, fitting F_2^d data will explore details of nuclear effects

Let us look at low-x physics at HERA– because the connection to EIC is strongest







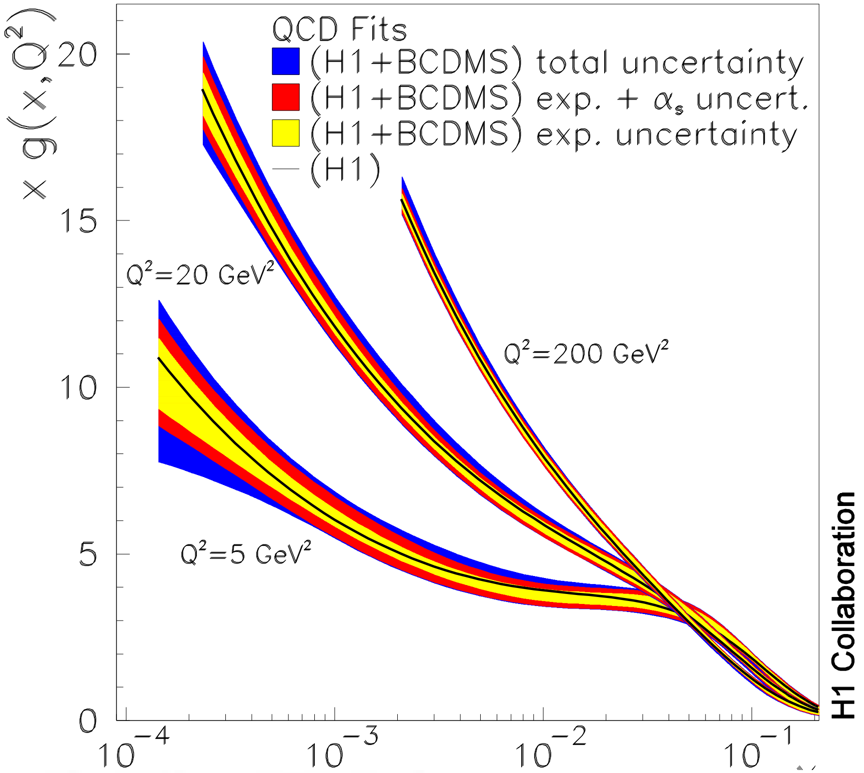
Before the HERA measurements most of the predictions for low-x behaviour of the structure functions and the gluon PDF were wrong – most theoreticians expected it to flatten out. It actually rises steeply

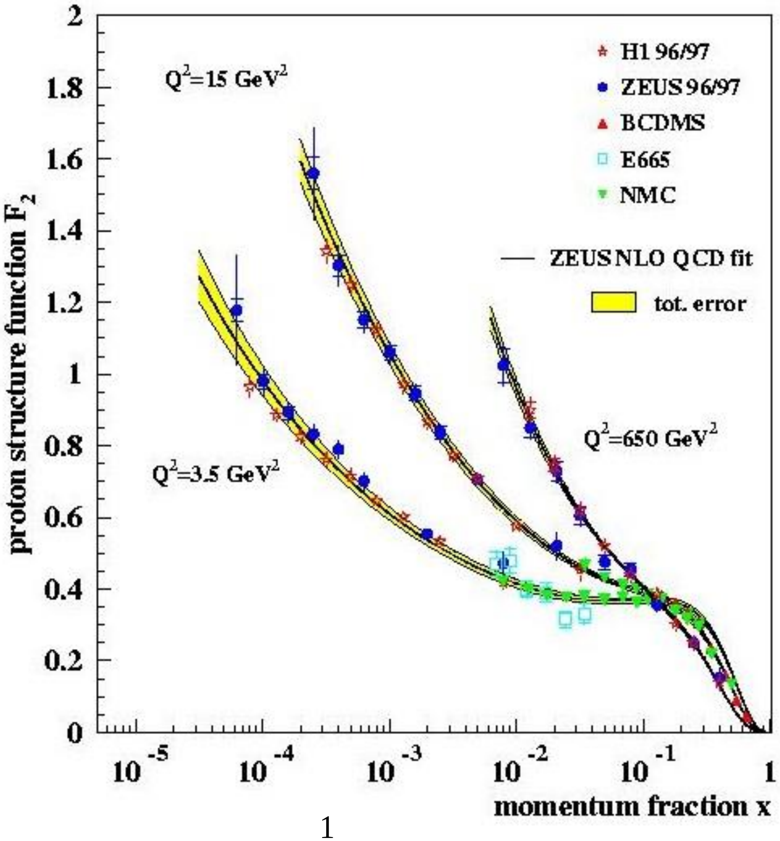
AND YET—DGLAP does predict the rise that we saw!

Now it seems that the conventional DGLAP formalism works TOO WELL !

(we think there **should be** $\ln(^1/_x)$ corrections and/or non-linear high density corrections for $x < 5$ x 10^{-3})

Low-x





$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\left[\Sigma_q Pgq(z)q(y,Q^2) + Pgg(z)g(y,Q^2)\right]$$

At small x,

small z=x/y

$$Pgq \to \frac{C_F}{z}; \quad Pgg \to \frac{2C_A}{z}$$

Gluon splitting functions become singular

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 \frac{dy}{y}\frac{6}{z}g(y,Q^2)$$

$xg(x,Q^2) \sim x^{-\lambda g}$

$$\lambda_g = \left(\frac{12\ln(t/t_0)}{\beta_0\ln(1/x)}\right)^{\frac{1}{2}},$$

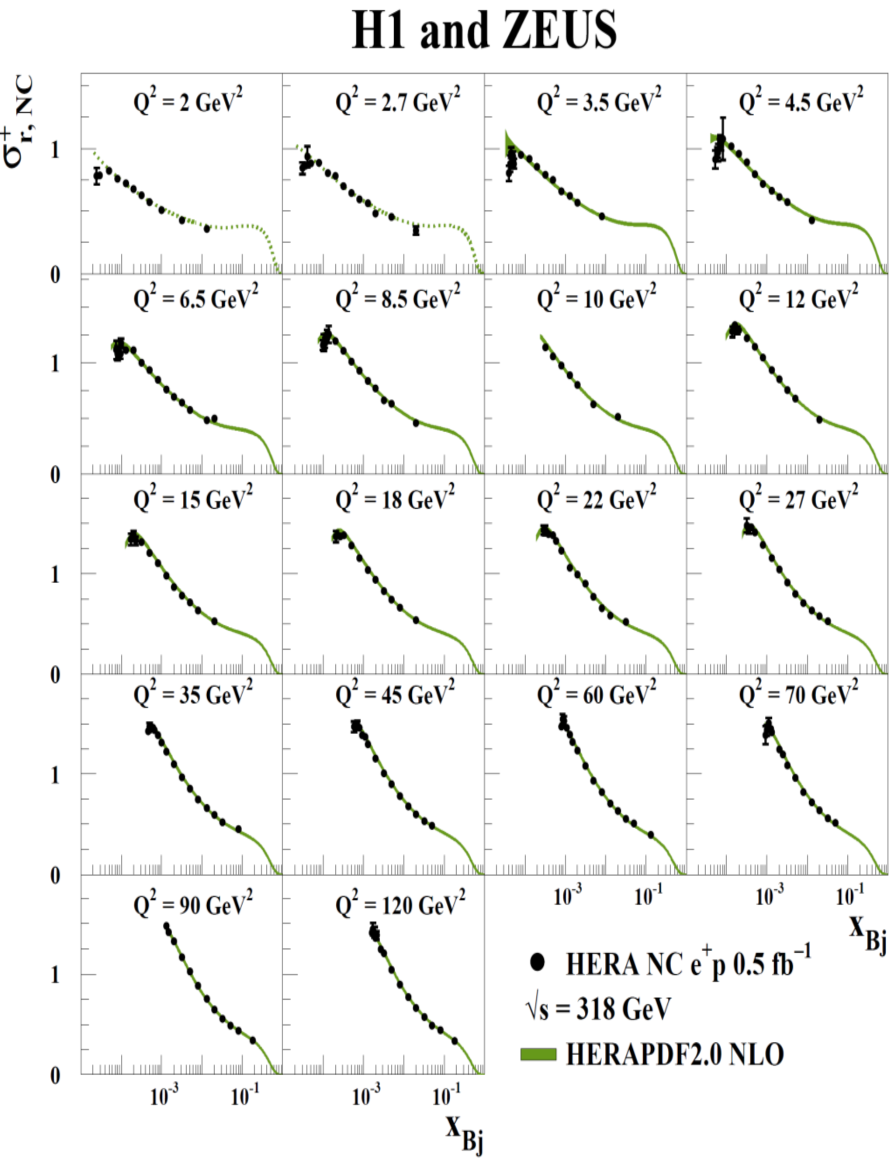
$t = \ln Q^2/\Lambda^2$

$\alpha_s \sim 1/\ln Q^2/\Lambda^2$

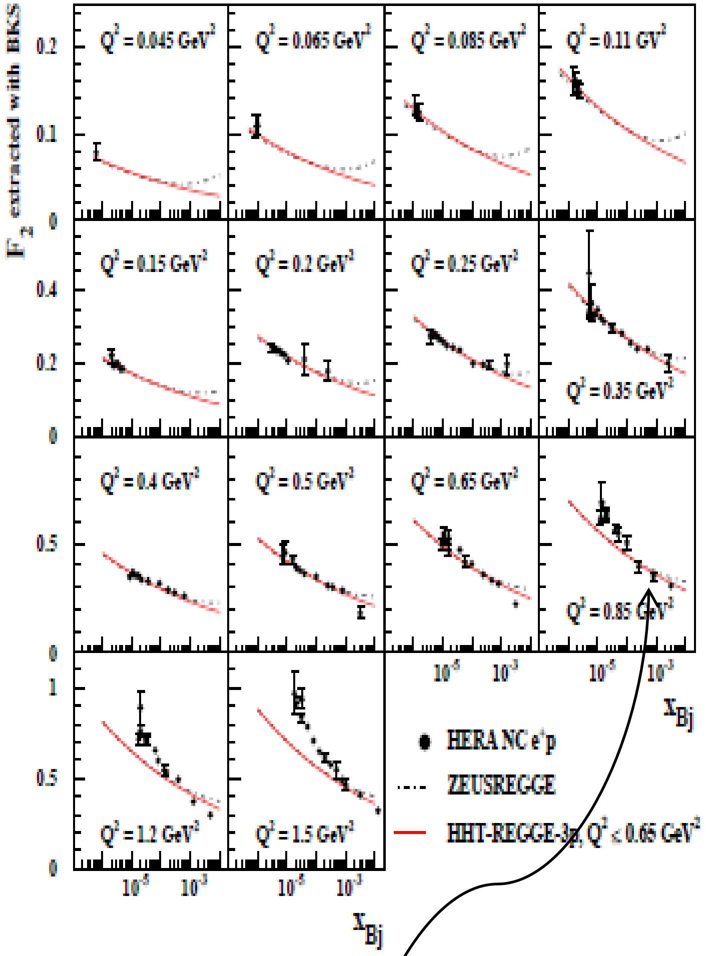
A flat gluon at low Q^2 becomes very steep **AFTER Q^2 evolution** AND F_2 becomes gluon dominated

$F_2(x,Q^2) \sim x^{-\lambda s}$, $\lambda s = \lambda g - \varepsilon$

The point is that steepness should set in AFTER evolution, so at higher Q^2



NEW Low Q^2 plot from 1704.03187



So it was a surprise to see F_2 steep at small x even for low Q^2, Q^2 <~5 GeV^2 and even more of a surprise to see it steep down to $Q^2 \sim 1$ GeV^2

Should perturbative QCD work? α_s is becoming large - α_s at $Q^2 \sim 1$ GeV^2 is ~ 0.4

There is another reason why the application of conventional DGLAP at low x is questionable:

The splitting functions, $P(x) = P^0(x) + P^1(x)\,\alpha_s(Q^2) + P^2(x)\,\alpha_s^2(Q^2)$

have contributions,

$$P^n(x) = \frac{1}{x}\left[a_n \ln^n\left(\frac{1}{x}\right) + b_n \ln^{n-1}\left(\frac{1}{x}\right)\right]$$

dominant at small x

Their contribution to the PDF comes from,

$$\frac{\partial q(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P(x)\, q(y, Q^2)$$

→ and thus give rise to contributions to the PDF of the form,

$$\alpha_s^P(Q^2)\,(\ln Q^2)^q \left(\ln \frac{1}{x}\right)^r$$

Leading log(Q^2):

conventionally in LO DGLAP: p = q ≥ r ≥ 0 LL(Q^2)

NLO: p = q +1 ≥ r ≥ 0 NLL(Q^2)

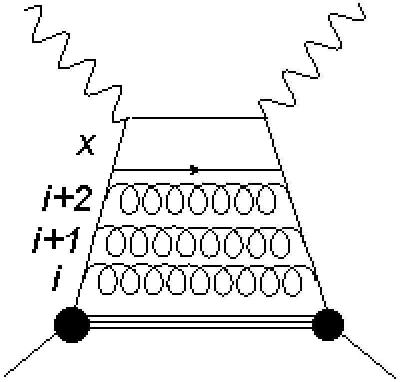
But if ln($^1/_x$) is large, we should also consider, Leading log(1/x):

p = r ≥ q ≥ 1 LL(1/x)

p = r+1 ≥ q ≥ 1 NLL(1/x)

This is what is meant by BFKL summation.

Diagrammatically,



Leading $\log Q^2$ → *strong* p_t ordering

$$Q^2 >> p_{t_i}^2 >> p_{t_{i-1}}^2 \dots >> p_{t_1}^2$$

and at small *x* we also have strong ordering in *x*

$$x << x_i << x_{i-1} \dots << x_1$$

$$\Rightarrow \text{leading } \ln(1/x)$$

→ double leading logs $\alpha_s \ln Q^2 \ln(1/x)$ at small x (double asymptotic scaling)
But why not sum up $\alpha_s \ln(1/x)$ independent of Q^2?

→ Diagrams ordered in *x*, but *not* in p_t
BFKL formalism

→

$$x\,g(x, Q^2) \sim x^{-\lambda}$$

$$\lambda = \frac{\alpha_s}{\pi} C_A \ln 2 \simeq 0.5$$

for $\alpha_s \sim 0.25$ (low Q^2)

→ A singular gluon behaviour even at low-ish Q^2
→ Is this the reason for the steep behaviour of F_2 at low-x ?
IS there a "BFKL Pomeron"- (for relation to the Pomeron see later)

However we all know that this steep behaviour was modified once NLO BFKL calculations were made. It has proved very difficult to get 'smoking gun' evidence for anything beyond DGLAP

Furthermore if the **gluon density becomes large** there maybe **non-linear effects**

Gluon recombination g g → g

$\sigma \sim \alpha_s^2 \rho^2/Q^2$

may compete with gluon evolution g → g g

$\sigma \sim \alpha_s \rho$

where ρ is the gluon density

~

Non-linear evolution equations – GLR

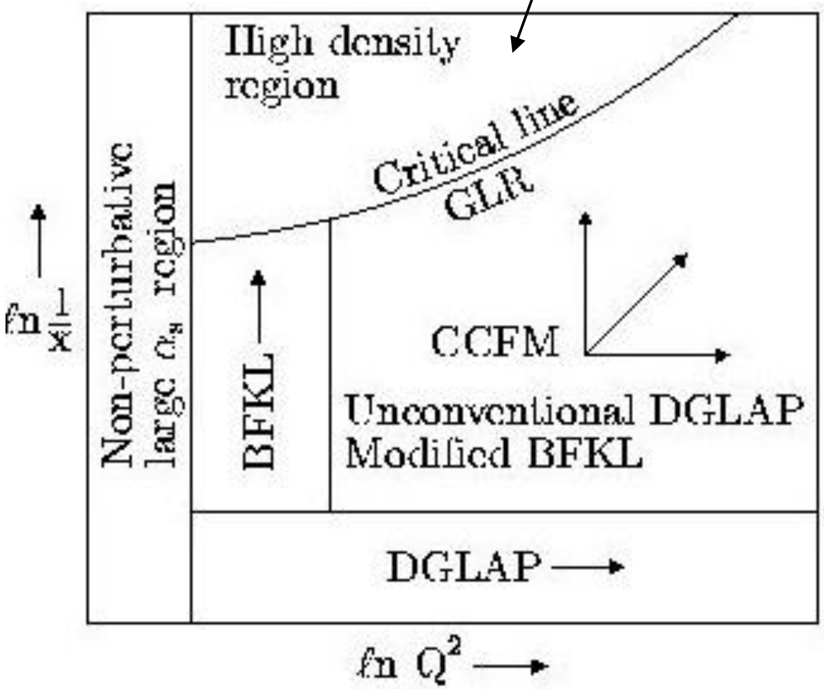
$$\frac{d^2xg(x,Q2)}{dlnQ2dln1/x} = \underbrace{\frac{3\alpha_s}{\pi} xg(x,Q2)}_{\alpha_s \rho} - \underbrace{\frac{\alpha_s^2 \, 81}{16Q^2R^2} [xg(x,Q2)]^2}_{\alpha_s^2 \rho^2/Q^2}$$

The non-linear term slows down the evolution of $xg(x,Q^2)$ and thus tames the rise at small x

The gluon density may even saturate

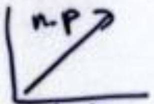
(-respecting the Froissart bound)

Colour Glass Condensate, JIMWLK, BK etc. etc. At higher Q^2 the region moves to lower and lower x

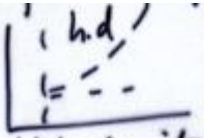


Extending the conventional DGLAP equations across the x, Q2 plane.

Plenty of debate about the positions of these lines!



non-perturbative region may subsume BFKL



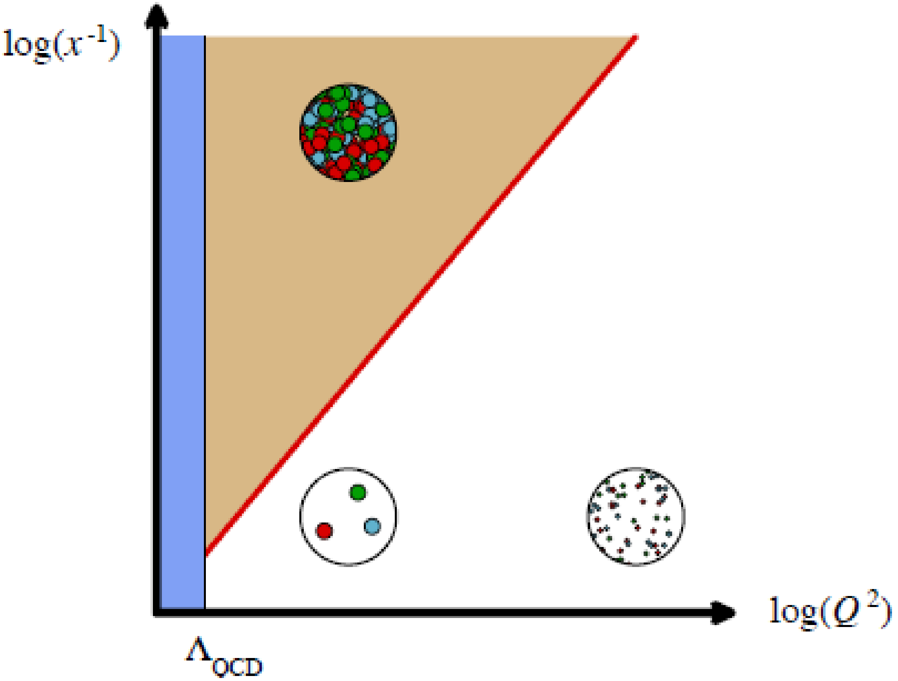
high density region may subsume BFKL

There are various reasons to worry that conventional $\ln(Q^2)$ summations – as embodied in the DGLAP equations may be inadequate

It was a surprise to see F_2 steep at small x - even for very low Q^2, $Q^2 \sim 1$ GeV^2

1. Should perturbative QCD work? α_s is becoming large - α_s at $Q^2 \sim 1$ GeV^2 is ~ 0.4
2. There hasn't been enough lever arm in Q^2 for evolution, but even the starting distribution is steep- **the HUGE rise at low-x makes us think**
3. there **should be** $\ln(1/x)$ resummation (BFKL) as well as the traditional $\ln(Q^2)$ DGLAP resummation- BFKL predicted $F_2(x,Q^2) \sim x^{-\lambda s}$, with λ_s=0.5, even at low Q^2
4. and/or there should be non-linear high density corrections for x < 5 10 $^{-3}$
5. In nuclei these could be enhanced by $A^{1/3}$

Colour Glass Condensate, JIMWLK, BK etc. At higher Q^2 the region moves to lower and lower x



Extending the conventional DGLAP equations across the x, Q2 plane.

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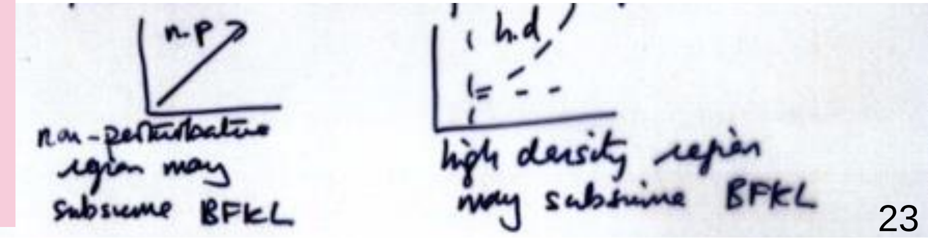


non-perturbative region may subsume BFKL

high density region may subsume BFKL

Does the data *need* unconventional explanations?

- *ln($^1/_x$)* terms in the splitting factors
- CCFM
- modified BFKL

Afficionados claim χ^2 improvements over conventional NLLO DGLAP..

But, one seems to be able to use DGLAP by absorbing unconventional behaviour in the boundary conditions i.e. the unknown shapes of the non-perturbative parton distributions at Q_0^2

We measure, $F_2 \sim x\,q$

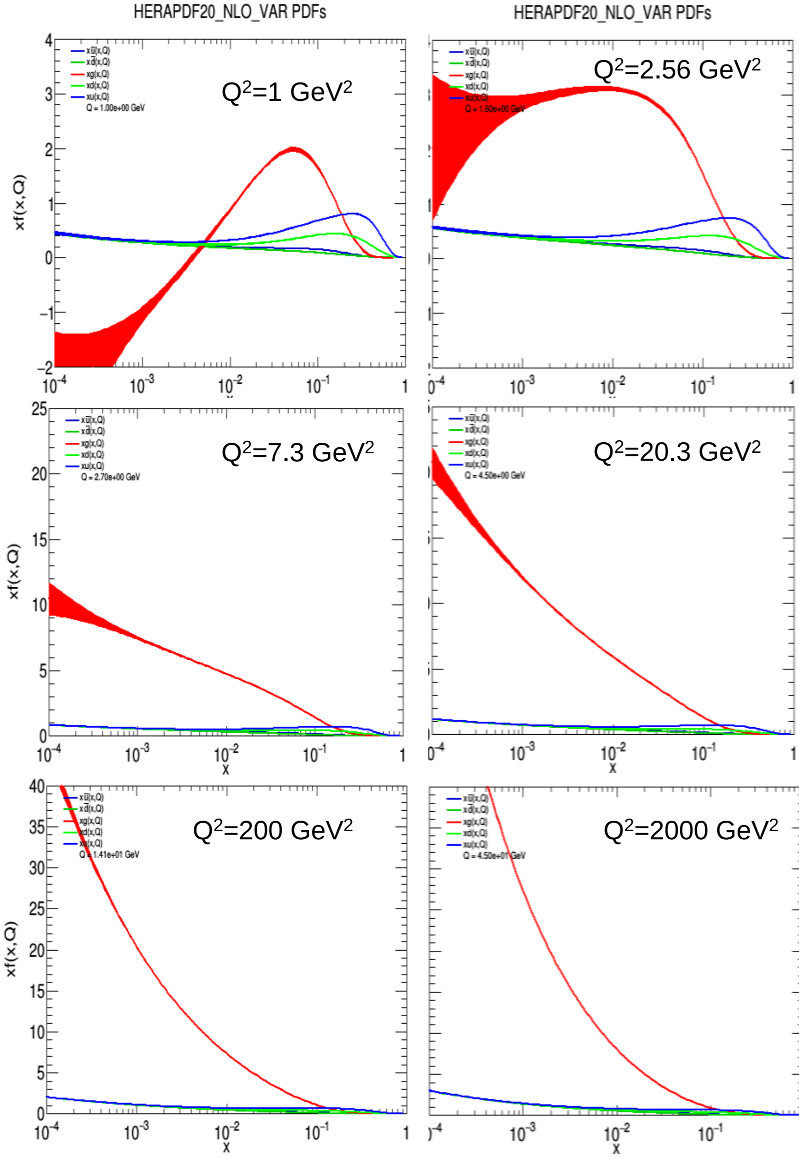
$$\frac{dF_2}{d\ln Q^2} \sim P_{qg} \cdot x\,g$$

we can explain behaviour of $\frac{dF_2}{d\ln Q^2}$ by:

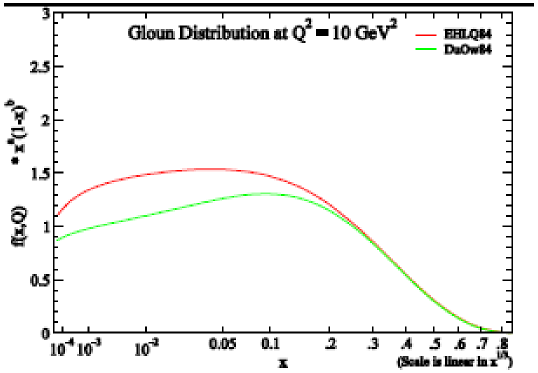
unusual P_{qg} → eg *ln($^1/_x$)*, BFKL

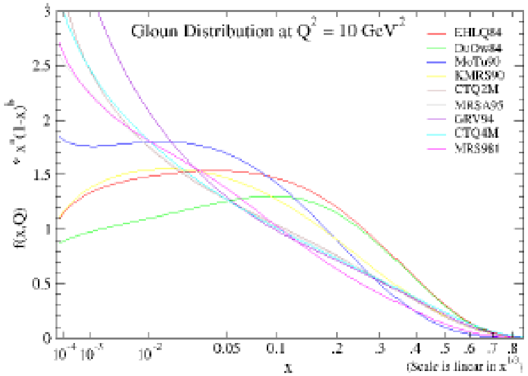
OR unusual $x\,g(x,Q_0^2)$ → “valence-like” gluon etc.

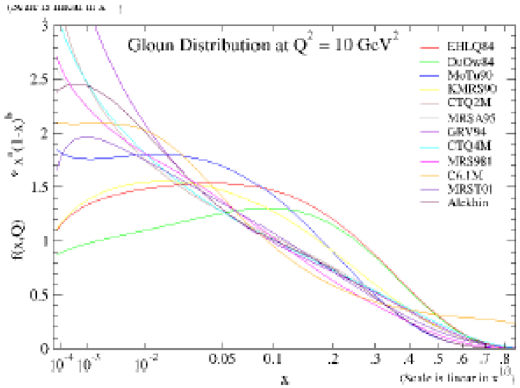
→ need to measure other gluon sensitive quantities at low x: F_L



Conventional NLO-DGLAP needs a valence-like gluon but a singular sea at lowQ^2
This does not get better at NNLO





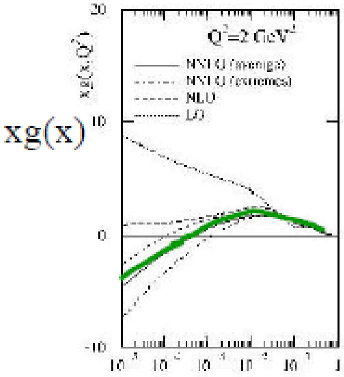


To recap what has happened…

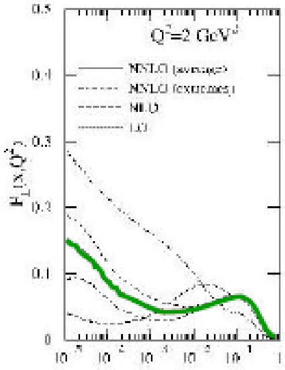
When HERA data were first published the gluon went from being flat to being steep at low-x

BUT when HERA data proved to still be steep at very low-Q^2 the DGLAP fits produced gluons which turn over again at low-x. the gluon evolves very fast- in order to evolve fast upwards it also evolves fast downwards – and this has consequences for the measurable structure function F_L

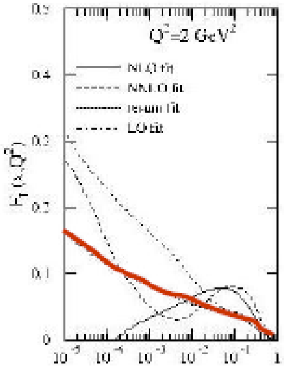
$Q^2 = 2\text{GeV}^2$



The negative gluon predicted at low x, low Q^2 from NLO DGLAP remains at NNLO (worse)



The corresponding F_L is NOT negative at $Q^2 \sim 2$ GeV^2 – but has peculiar shape



Including ln(1/x) resummation in the calculation of the splitting functions (BFKL `inspired') can improve the shape - and the χ^2 of the global fit improves

This indicates that you might want to go beyond DGLAP but it is not overwhelming

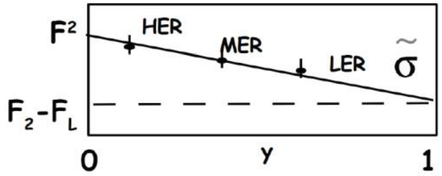
Longitudinal Structure Function

Longitudinal structure function FL is a pure QCD effect:

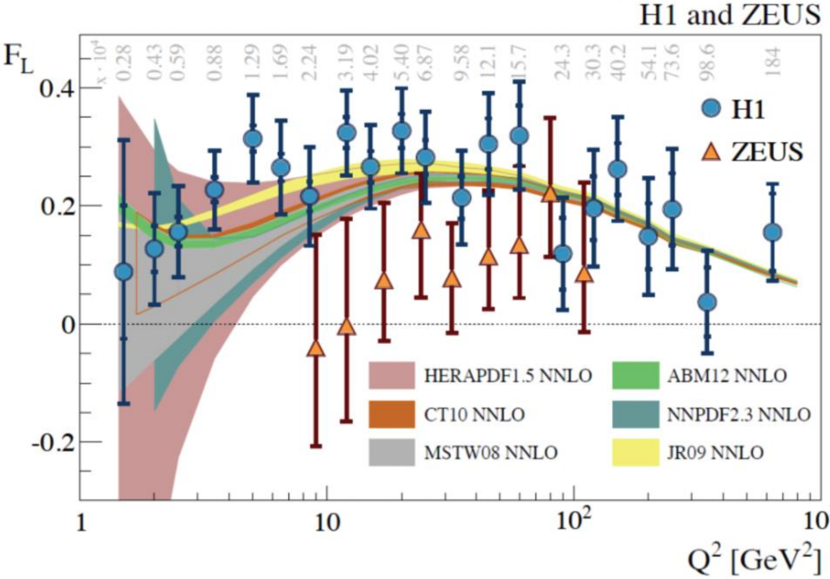
—> an independent way to probe sensitivity to gluon

$$F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3}\left[\frac{16}{3}\underbrace{F_2}_{\text{quarks radiating a gluon}} + 8\sum_q \underbrace{e_q^2\left(1-\frac{x}{z}\right)zg(z)}_{\text{gluons splitting into quarks}}\right]$$

Direct measurement of FL at HERA required differential cross sections at same x and Q^2 but different y —> different beam energies: Ep= 460, 575, 920 GeV

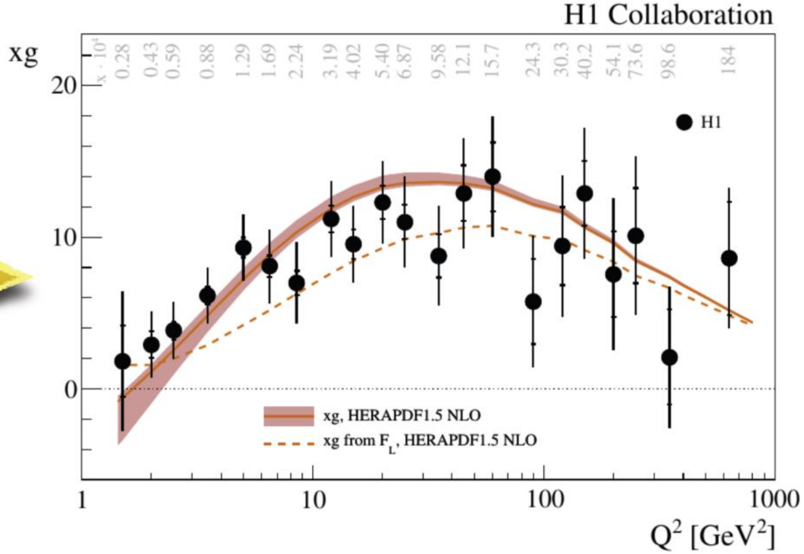


$$\sigma_{NC}(x, Q^2, y) \propto F_2(x, Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x, Q^2)$$



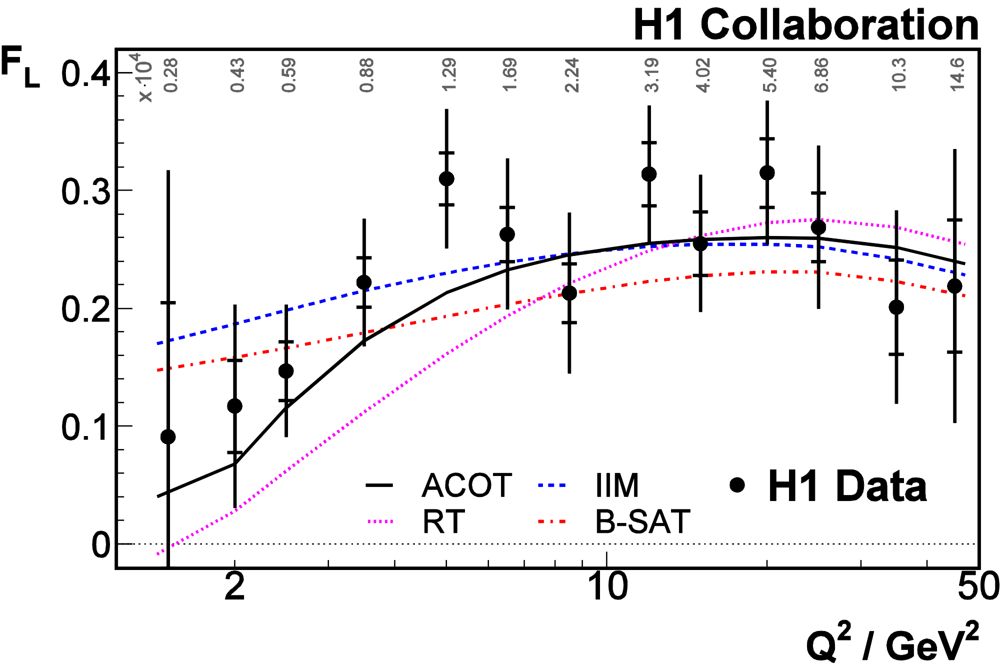
Consistency of H1 and ZEUS FL was checked accounting for corr. unc: χ2/ndf = 11/8 (p-value = 20%)

$$xg(x, Q^2) \approx 1.77 \frac{3\pi}{2\alpha_S(Q^2)} F_L(ax, Q^2)$$



Eur. Phys. J. C 74 (2014) 2814 [arXiv:1312.4821]

These are the final data on FL from ZEUS and H1



A slightly earlier version illustrating some saturation models and different DGLAP predictions – (ACOT and RT differ mostly in their treatment of FL to O(α_s) and O(α_s^2) respectively.)

It is not possible to tell if models beyond DGLAP such as saturation are needed.

EIC could help here if you bear the following in mind:

- High luminosity at all proton beam energies– HERA did not do this
- Well spread energies- maximize range in y^2– You can do better than HERA
- Ability to measure LOW energy electrons (sub-GeV if possible)
- High resolution electron calorimetry
- Control the background- mostly photo-production

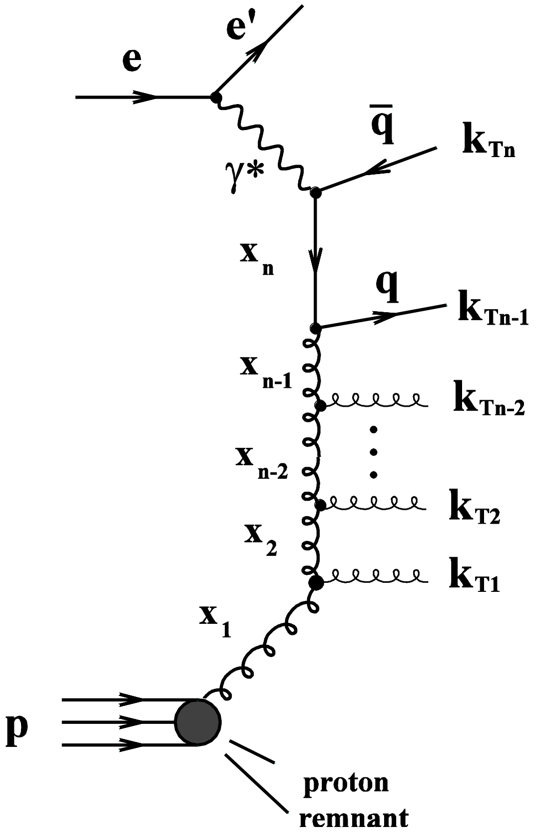
- taggers down the rear beam-line

-distinguish right and wrong sign electron candidates even at low angles and low energies

-needs excellent tracking and minimum inactive material

No smoking gun for something new at low-x…so let's look more exclusively

Now let's look at forward jets

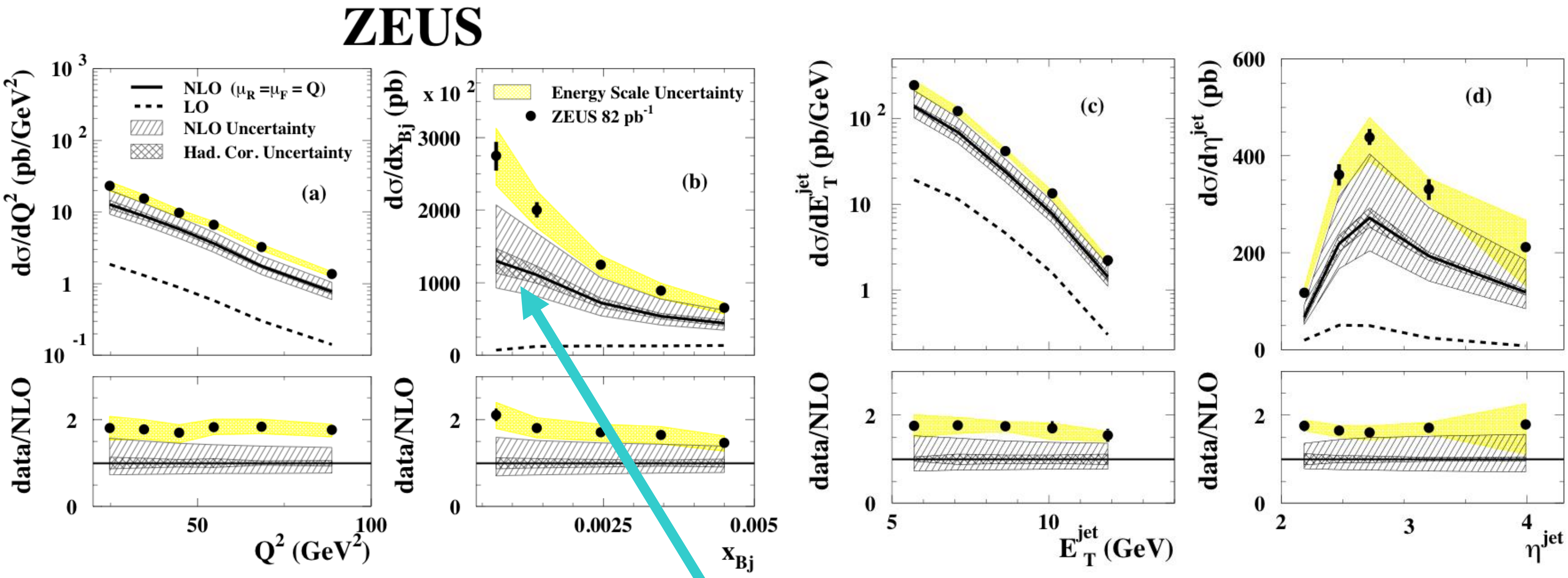


Look at the hadron final states..lack of pt ordering has its consequences. Forward jets with x_j » x and $k_{tj}{}^2 \sim Q^2$ are suppressed for DGLAP evolution but not for k_t disordered BFKL evolution

But this has served to highlight the fact that the conventional calculations of jet production were not very well developed. There has been much progress on more sophisticated calculations e.g DISENT, NLOJET ++, rather than ad-hoc Monte-Carlo calculations (LEPTO-MEPS, ARIADNE CDM …)

Forward Jets

DISENT vs data

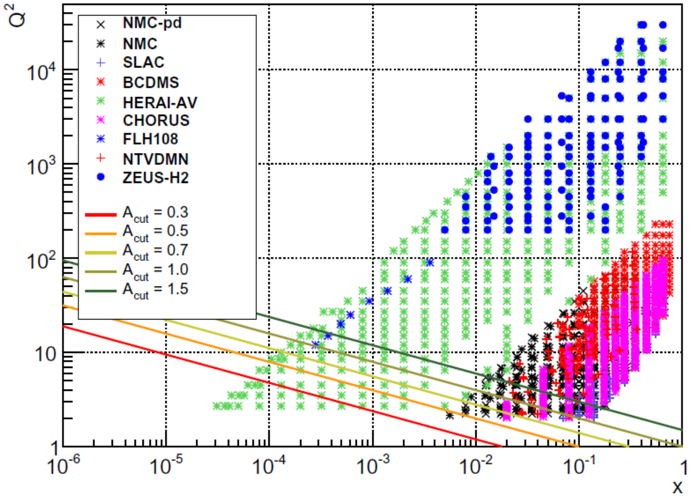


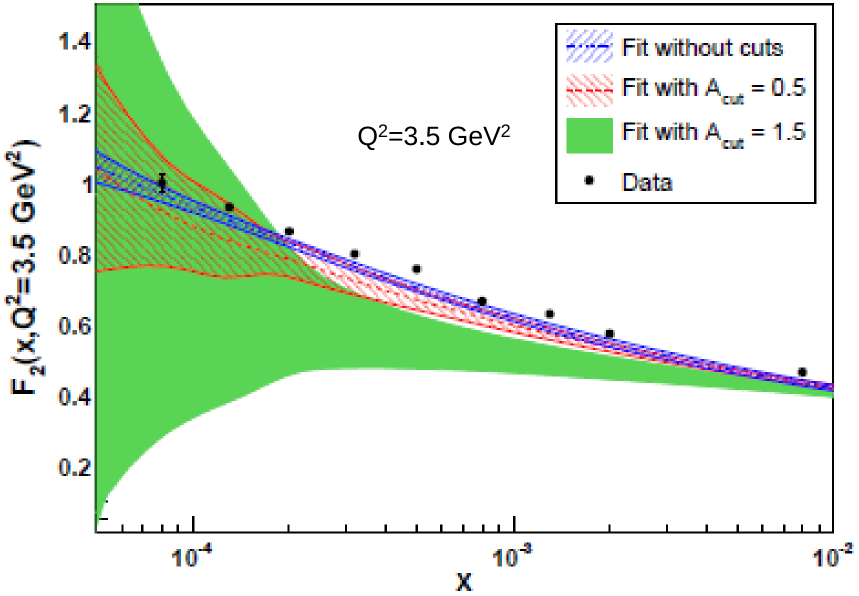
NLO below data, especially at small x_{Bj} but theoretical uncertainty is large

Comparison to LO and NLO conventional calculations

But this is hardly overwhelming it could be because we are missing NNLO…or it could be due to the need for virtual photon structure

Are there clever ways of looking at the inclusive data to uncover hints of something beyond DGLAP?





Caola *et. Al.,* arXiv:1007.5405 observe that the combined HERA-I data shows tension as cuts are made to cut out low-x,Q^2 data.
Cut $Q^2 > A_{cut}\ x^{-0.3}$
Such a cut is 'saturation inspired' : at low x the region moves to higher and higher Q^2

If all is well then a fit done with harder cuts should be compatible with fits done without cuts (though obviously the uncertainties grow larger) when evolved backwards

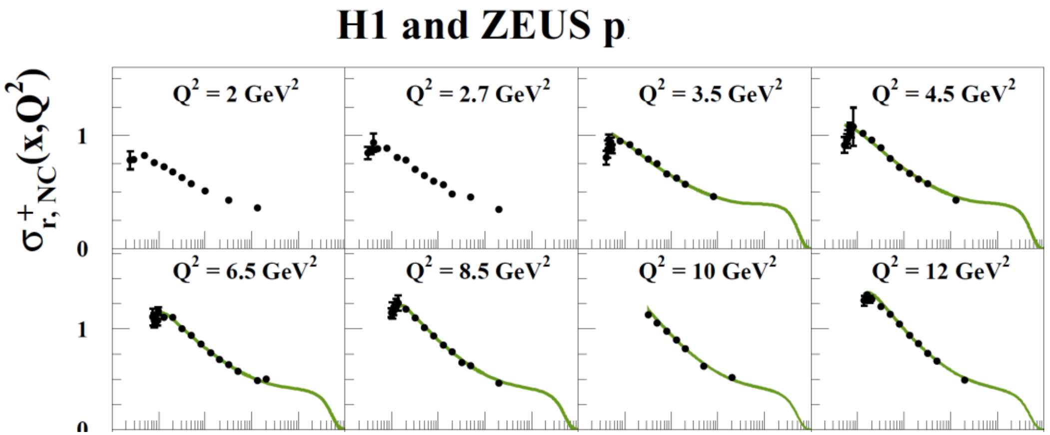
The fit with harder cuts undershoots the data, thus the this fit wanted more evolution of F_2 between Q^2=3.5 GeV^2 and Q^2=10 GeV^2 than is seen in the data. The fit was DGLAP at NLO.

NNLO gives even more evolution, which is not what is needed

Ln(1/x) resummation gives less evolution, this could help

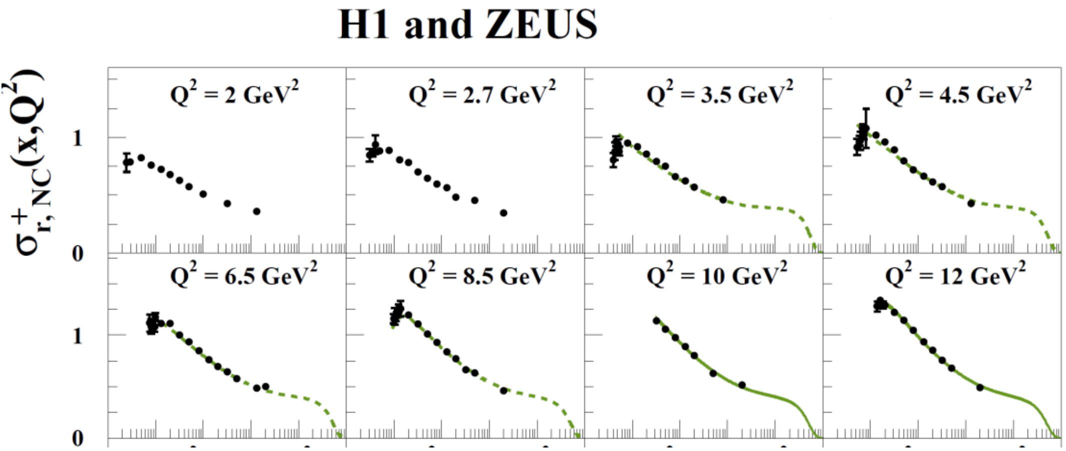
Saturation could also lead to less evolution

Look at the final HERA combined data arXiv:1506.06042



Let's take a look at the lowest Q^2 bins for a fit which includes data down to Q^2= 3.5 GeV^2

The NLO fit compromises between fitting the high-y turnover and fitting the data at slightly higher x 0.0001< x <0.001

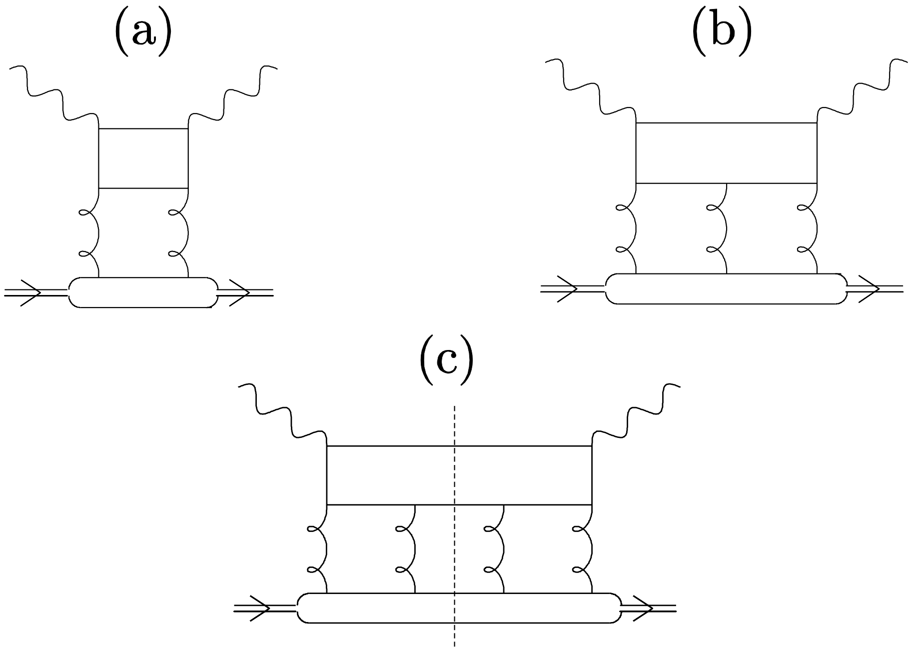


Let's take a look at the lowest Q^2 bins for a fit which includes data down to Q^2= 10 GeV^2 (this cuts out most of the same events as a cut Q^2> 1.0 x $^{-0.3)}$ Freed from having to describe these bins the fit undershoots the region 0.0001 to 0.001 more severely . The discrepancies with the fit are systematically worse at lower-x and lower Q^2.

This is similar to the observations of Caola et al on the HERA-I data

These are the NLO fits but NNLO is not better

One approach: (arXiv:1604.02299) consider adding higher twist terms at low-x



Their origin COULD be connected with recombination of gluon ladders- a non-linear evolution effect.

Bartels, Golec-Biernat, Kowalski suggest that such higher twist terms would cancel between σ_L and σ_T in F_2, but remain strong in F_L

Try the simplest of possible modification to the structure functions

F_2 and F_L as calculated from HERAPDF2.0 formalism

$$F_{2,L} = F_{2,L} (1 + A_{2,L}^{HT}/Q^2)$$

Such a modification of F_L is favoured, whereas for F_2 it is not.

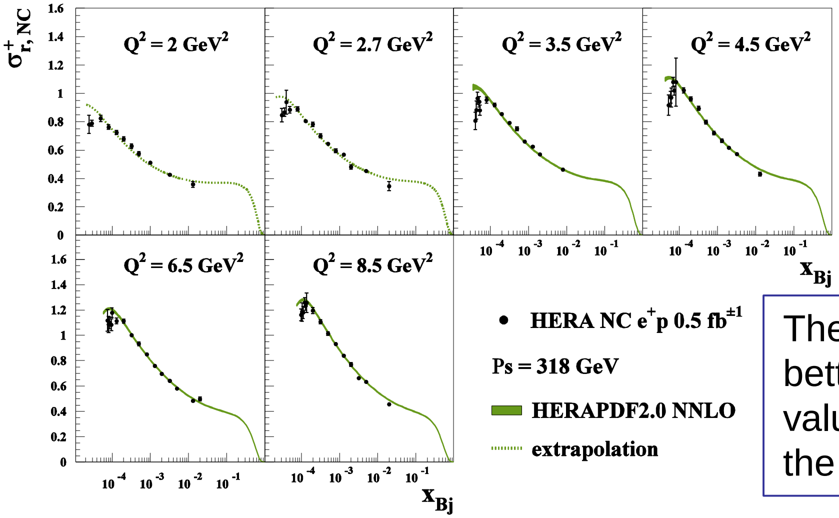
If A_L^{HT} is added $A_L^{HT} = 5.5 \pm 0.6$ GeV^2 and $\Delta\chi2$ =-47

So now let’s look at why the Higher Twist fits do so well

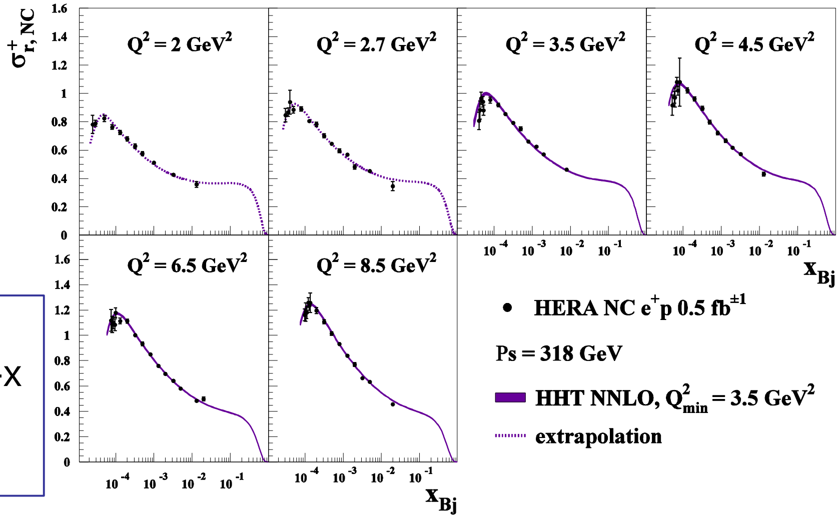
It is because they describe the turn over of the cross section at low x, Q2 much better

$$\sigma_{red} = F_2 – y^2/Y_+ F_L$$

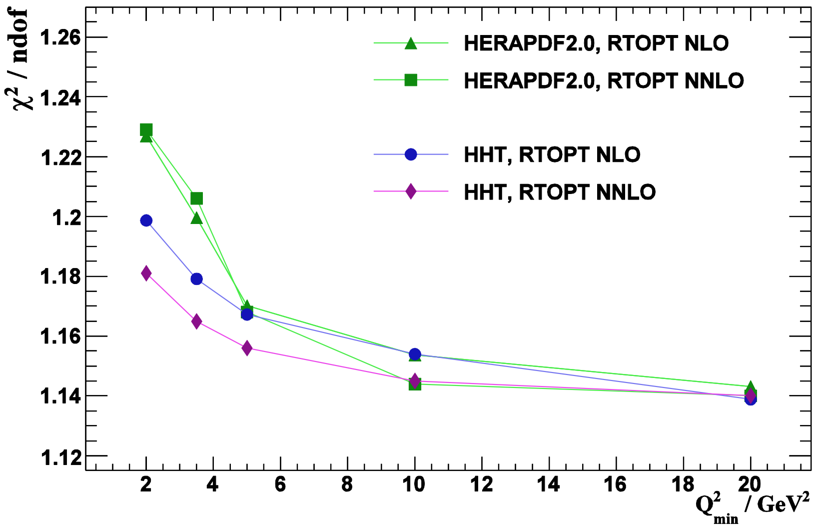
The data clearly wants a larger F_L and this is what the higher twist term provides



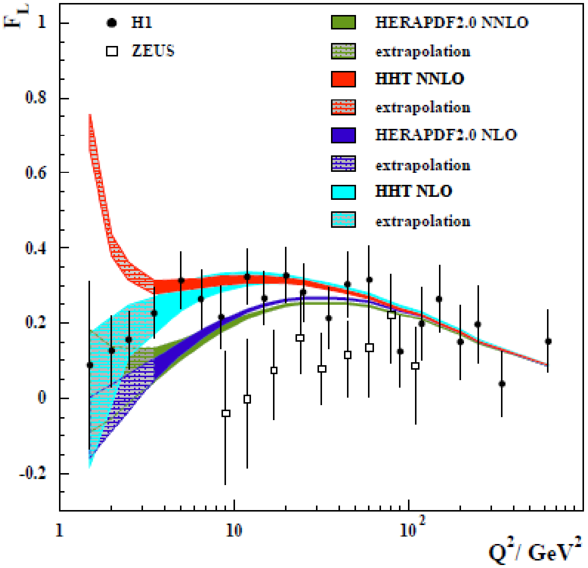
The fit is also better for low-x values above the turn over



You can also see that NNLO does better than NLO

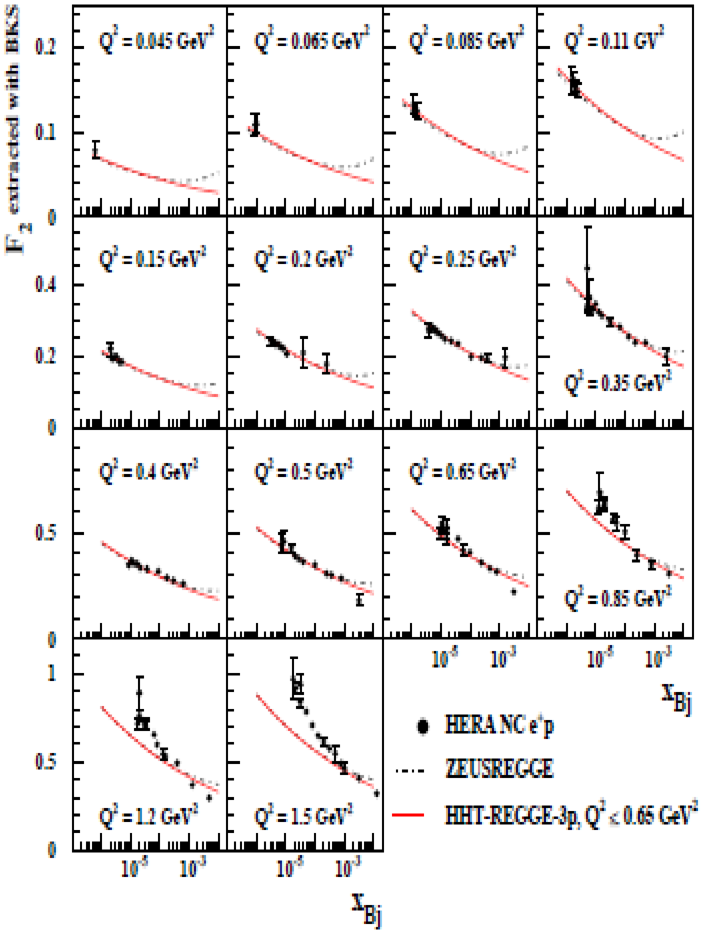


And here is what FL itself looks like. Clearly one cannot push this too low in Q^2



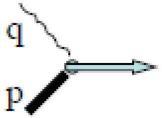
But there are other approaches to looking for effects beyond DGLAP, consider the transition to the non-perturbative regime

Linear DGLAP evolution doesn't work for Q^2 < 1 GeV2, WHAT does? – REGGE ideas?



arXiv:1704.03187

Small x is high W^2, $x=Q^2/2p.q$ Q^2/W^2



$p_x{}^2 = W^2$

$\sigma(\gamma^*p) \sim (W^2)^{\alpha-1}$ – Regge prediction for high energy cross-sections

α is the intercept of the Regge trajectory
α=1.08 for the SOFT POMERON

Such energy dependence is well established from the SLOW RISE of all hadron-hadron cross-sections - including $\sigma(\gamma p) \sim (W^2)^{0.08}$ for real photon- proton scattering

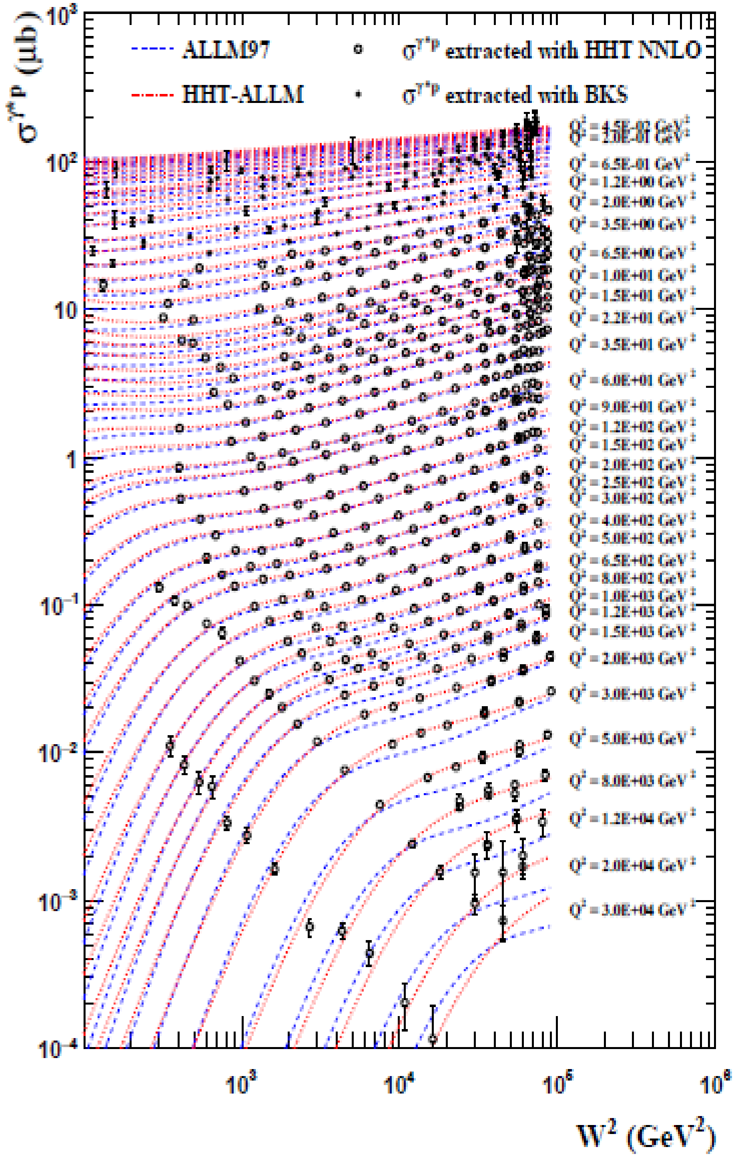
For virtual photons, at small x

$$\sigma(\gamma^*p) = \frac{4\pi^2\alpha\ F_2}{Q^2}$$

$\rightarrow$ $\sigma \sim (W^2)^{\alpha-1} \rightarrow F_2 \sim x^{1-\alpha} = x^{-\lambda}$

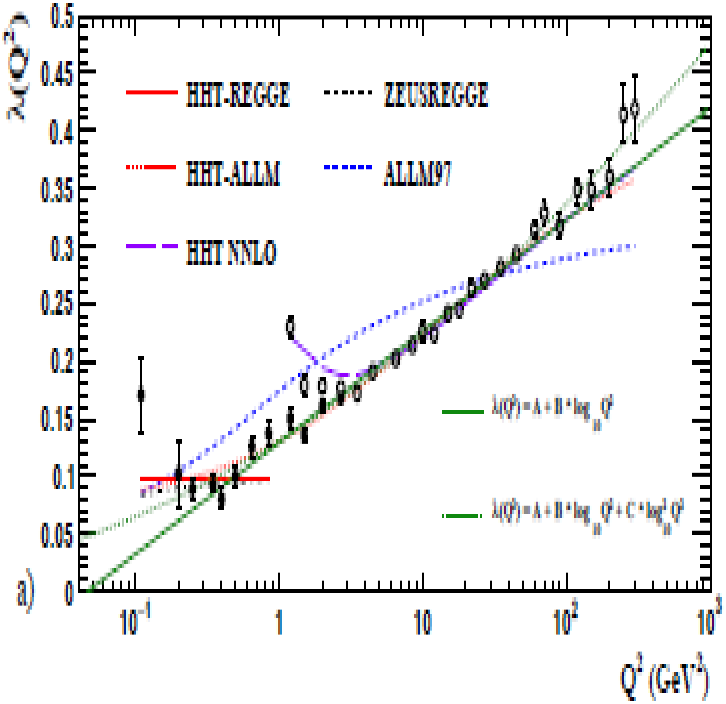
so a SOFT POMERON would imply $\lambda = 0.08$ gives only a very gentle rise of F_2 at small x

For $Q^2 > 1$ GeV^2 we have observed a much stronger rise…..



gentle rise

much steeper rise



The slope of F_2 at small x , $F_2 \sim x^{-\lambda}$, is equivalent to a rise of $\sigma(\gamma^* p) \sim (W^2)^{\lambda}$ which is only gentle for $Q^2 < 1$ GeV^2

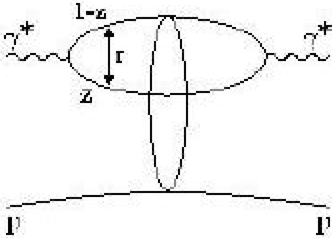
As well as the soft Pomeron, $\alpha - 1 = \lambda = 0.08$ (REGGE) should we consider

- a QCD POMERON, $\alpha(Q^2) - 1 = \lambda(Q^2)$- where this is the λ introduced on slide 18 (NNLO-DGLAP)
- a BFKL POMERON, $\alpha - 1 = \lambda \sim 0.5$
- a mixture of HARD and SOFT Pomerons to explain the transition $Q^2 = 0$ to high Q^2? (ALLM)

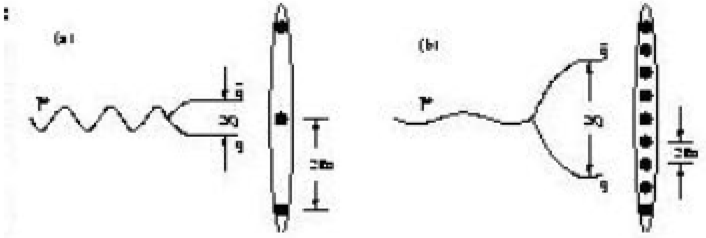
What about the Froissart bound ? – the rise MUST be tamed – non-linear effects?

Dipole models provide another way to model the transition $Q^2=0$ to high Q^2

At low x, γ^* to$q\bar{q}$ and the LONG LIVED ($q\bar{q}$) dipole scatters from the proton



The dipole-proton cross section depends on the relative size of the dipole r~1/Q to the separation of gluons in the target R_0



$\sigma = \sigma_0(1 - \exp(-r^2/2R_0(x)^2))$, $R_0(x)^2 \sim (x/x_0)^{\lambda} \sim 1/xg(x)$

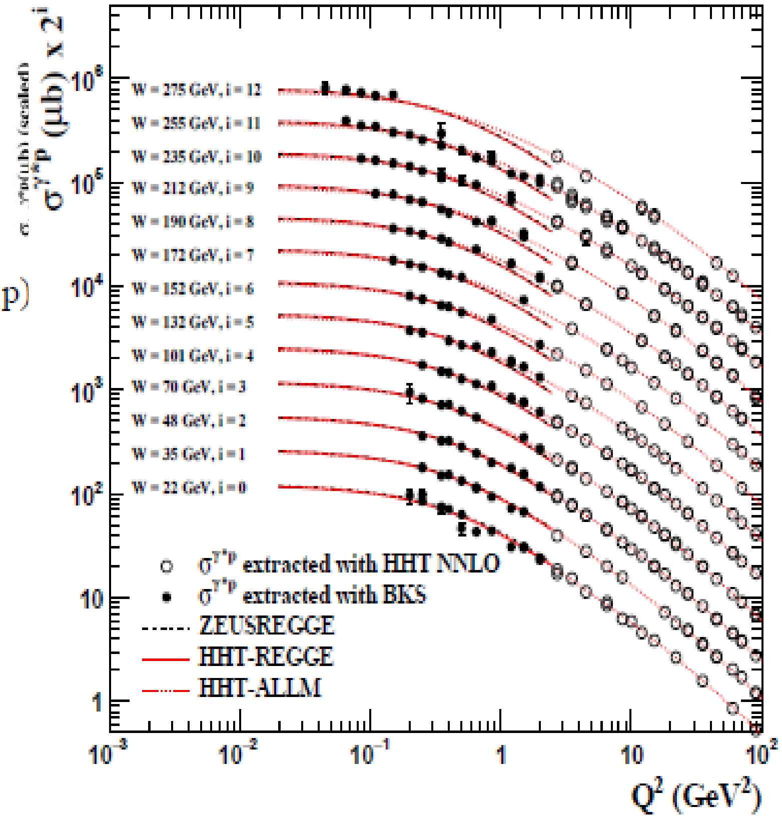
r/R_0 small → large Q^2, x
$\sigma \sim r^2 \sim 1/Q^2$, F2 flat

Bjorken scaling

r/R_0 large → small Q2, x
$\sigma \sim \sigma_0$ → saturation of the dipole cross-section

GBW dipole model

$\sigma(\gamma^*p)$



But $\sigma(\gamma^*p) = \frac{4\pi\alpha^2}{Q^2} F_2$ is general (at small x)

$\sigma(\gamma p)$ is finite for real photons , $Q^2=0$. At high Q^2, F_2 ~flat (weak $\ln Q^2$ breaking) and $\sigma(\gamma^*p) \sim 1/Q^2$

More sophisticated Dipole models have been developed in the context of non-linear evolution models with and without saturation. They often predict geometric scaling.

τ is a new scaling variable, applicable at small x

It can be used to define a `saturation scale' , $Q^2_s = 1/R_0^2(x) \sim \quad x^{-\lambda} \quad \sim x\, g(x)$, gluon density

- such that saturation extends to higher Q^2 as x decreases
- And INDEED, for x<0.01, $\sigma(\gamma^* p)$ depends only on τ, not on x, Q^2 separately

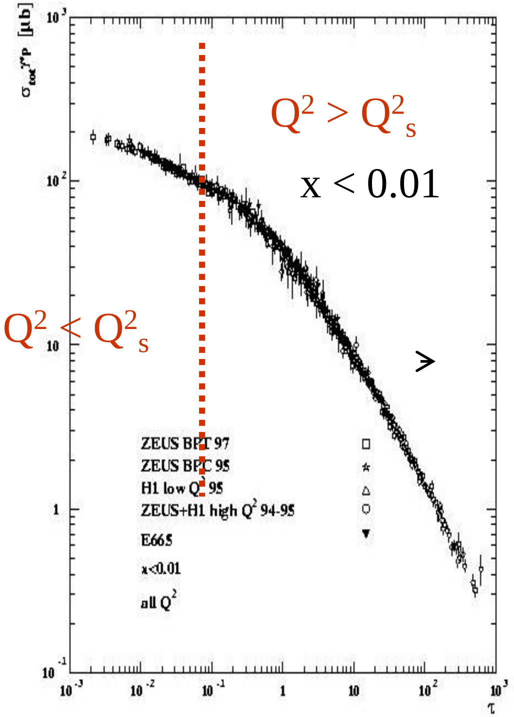
$\sigma(\gamma^* p) = \sigma_0\,(1 - \exp(-1/\tau))$

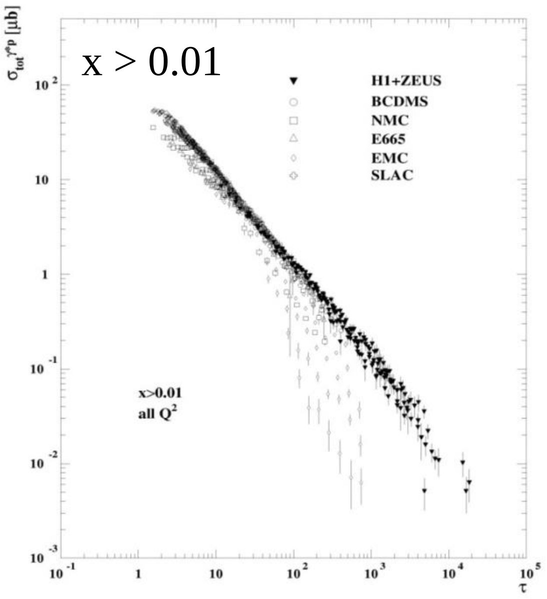
Involves only

$\tau \quad = Q^2 R_0^2(x)$

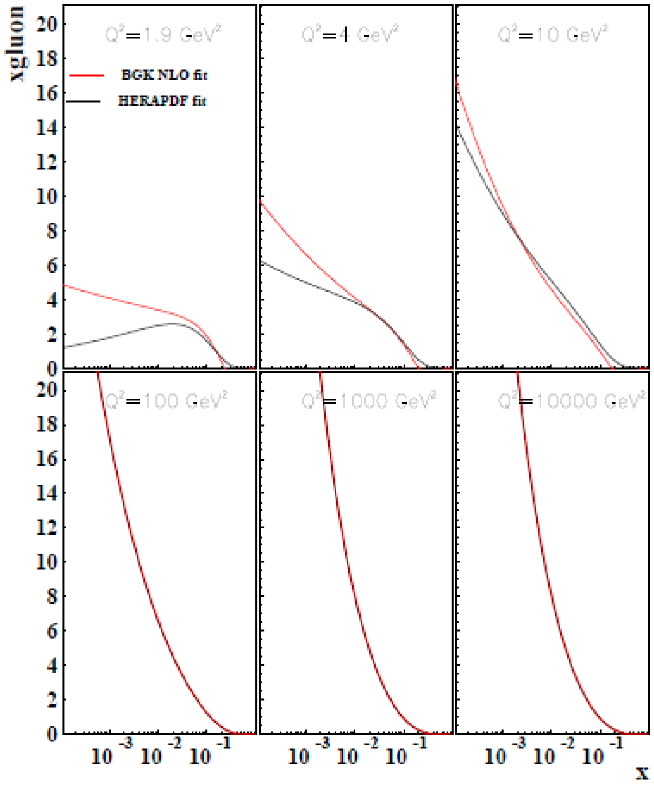
$\tau = Q^2/Q_0^2\,(x/x_0)^\lambda$

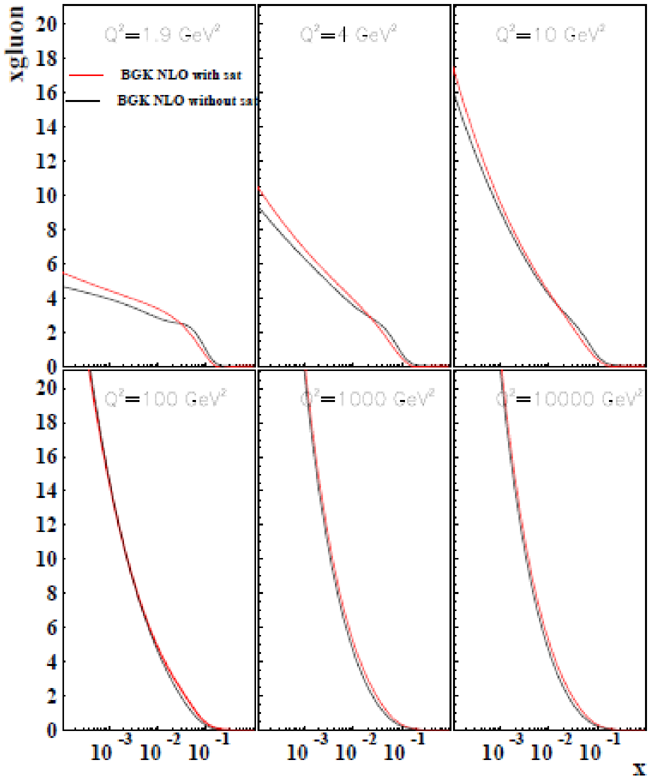
It is often said that geometric scaling has established evidence for saturation. However it is possible to get geometric scaling over quite a large kinematic range from DGLAP/BFKL ‘double asymptotic scaling’





A very recent arXiV:1611.10100 attempt to establish saturation in the dipole picture uses the BGK (Bartels, Golec-Biernat, Kowalski) which combines a dipole model with DGLAP evolution of the gluon density





The BGK model gives a more reasonable shape to the low Q2 gluon, which is enhanced somewhat if saturation is included

> Two-staged analysis approach

- **Fit of the high-Q^2 regime: 3.5 – O(250) GeV^2**

  → Test of the validity of the dipole model in the regime still well described by DGLAP evolution

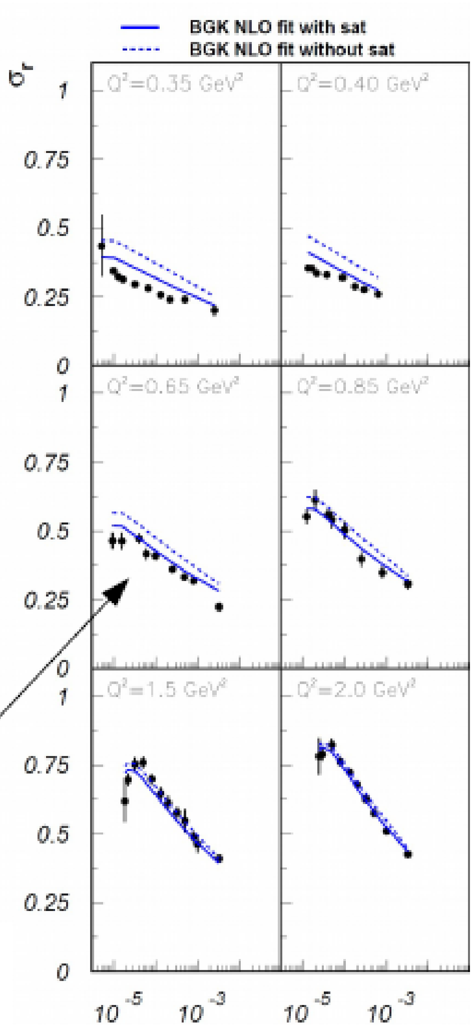
  → Test soft and hard gluon models

  → Test impact of valence quarks on the fits (previous data not sensitive enough)

- **Extend the fit to lower Q^2: 0.35 - 3.5 GeV^2**

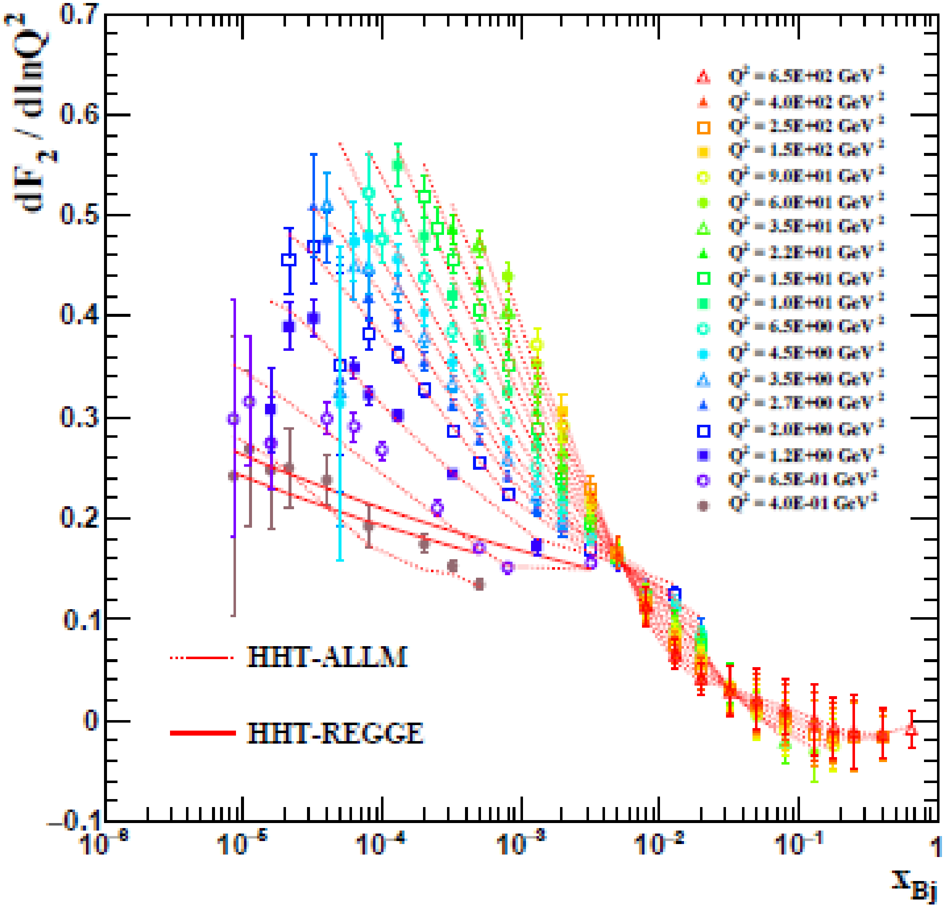
  → Test whether dipole models can extend the perturbative regime also when confronted with much more precise data

  → Are there signs of saturation?



Maybe

Parting remarks



arXiv:1704.03187

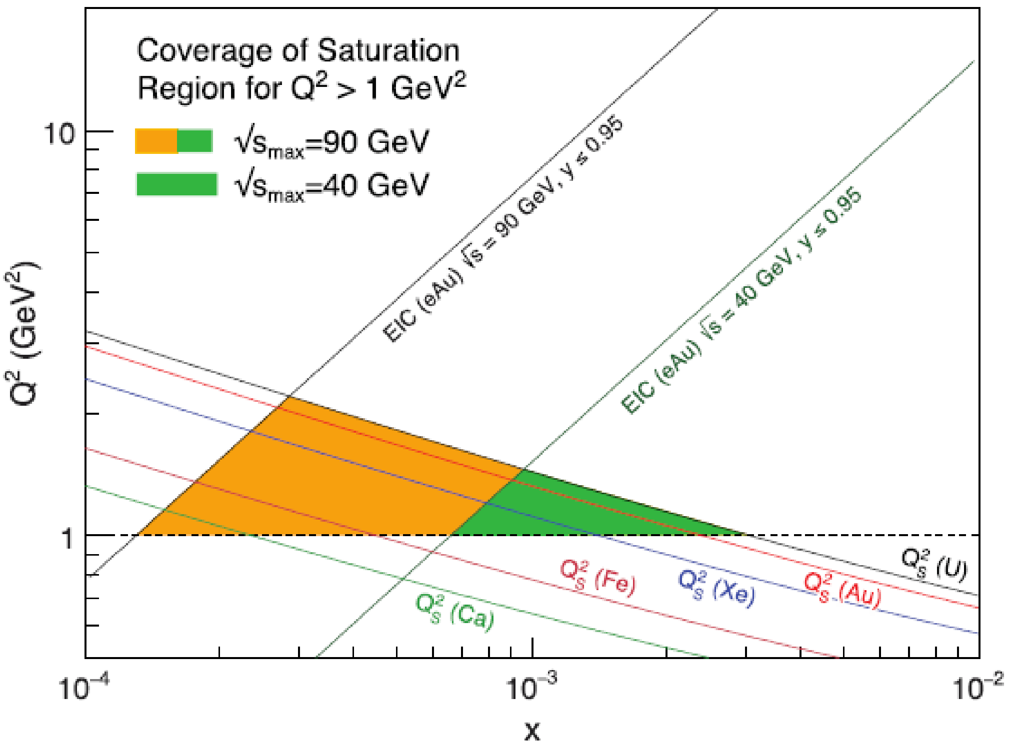
And here's a recently produced plot on dF$_2$/dlnQ^2. At LO and x <~0.005 this quantity is directly related to the gluon PDF.

At very low x and Q ^{2} the turnovers could indicate saturation– a new state of high-density gluons- but one is also falling into the non-perturbative region. At HERA this is not definitive.

To really probe the high density region there are two ways:

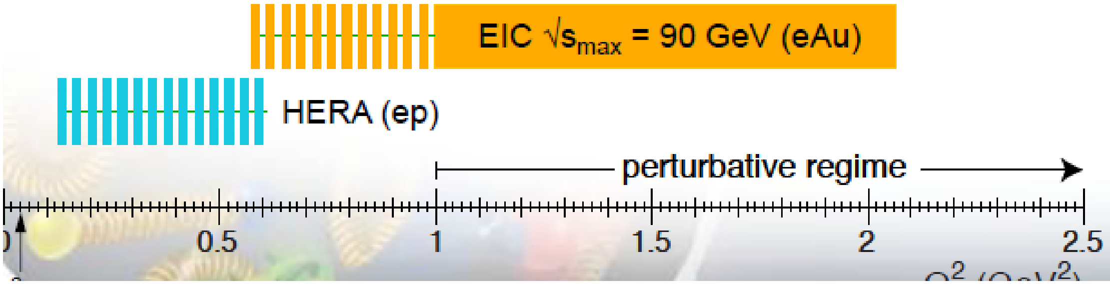
- **A machine with lower x reach for higher Q^2 – the LHeC**
- **A machine with higher-density reach due to the use of nuclei -- the EIC**

Gluon Saturation: √s and A Matter



eA at EIC:

- Need to push into regime where comparison with our current understanding can be made ⇒ $Q^2 > 1$ GeV/c^2
- Require sufficient lever arm in *Q*, *x* to study evolution
- CGC predicts characteristic *A* dependence ⇒ requires large *A* lever arm

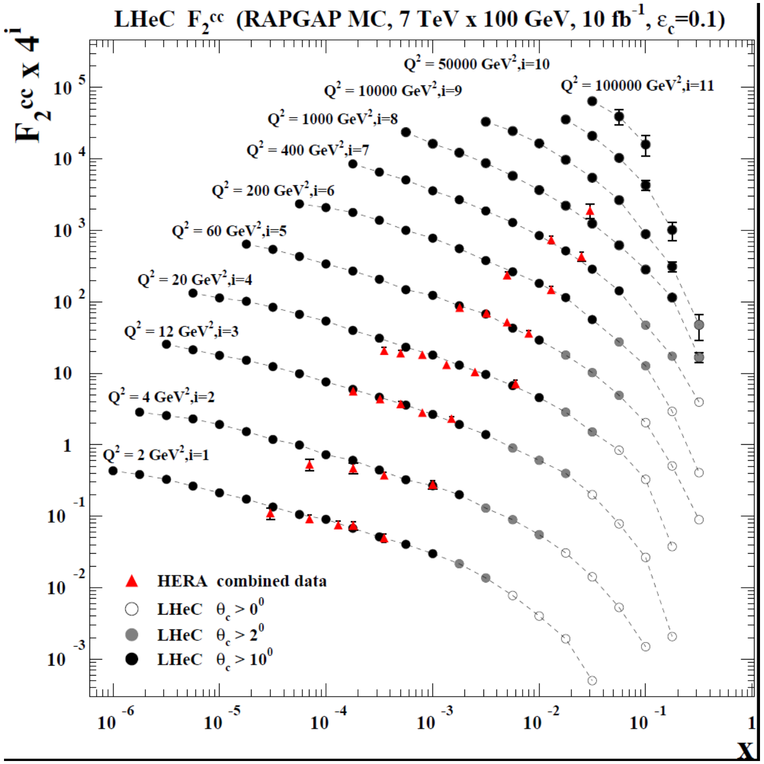


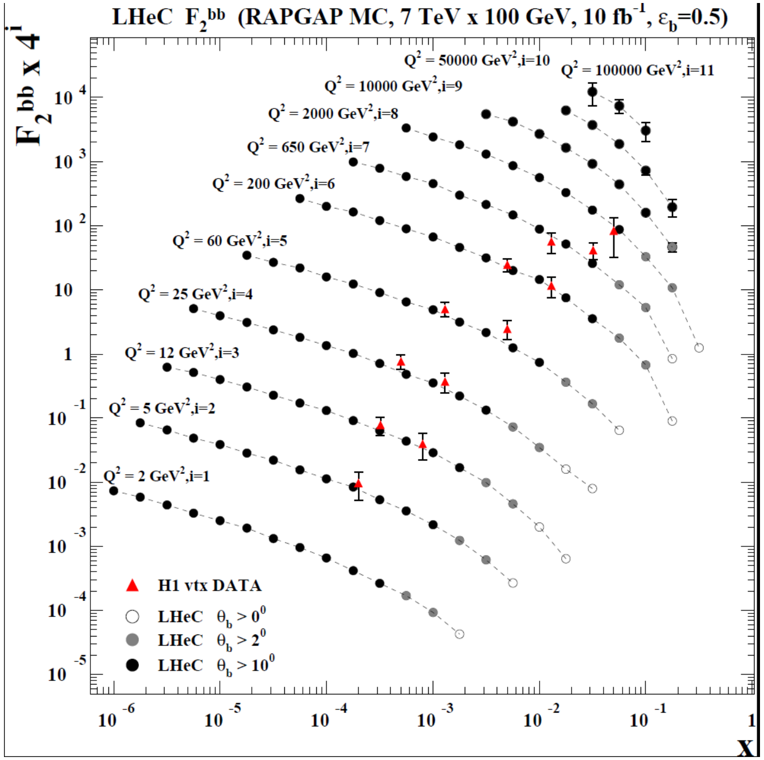
- Need to push for highest energy and heaviest ions

Back-up

EIC may also be able to improve the measurements of heavy quarks

Compare the potential of the LHeC for the measurement of $F2^{c\text{-}cbar}$ and $F2^{b\text{-}bbar}$ with what is currently available from HERA





The range will not be extended so much by EIC BUT there is the possibility of

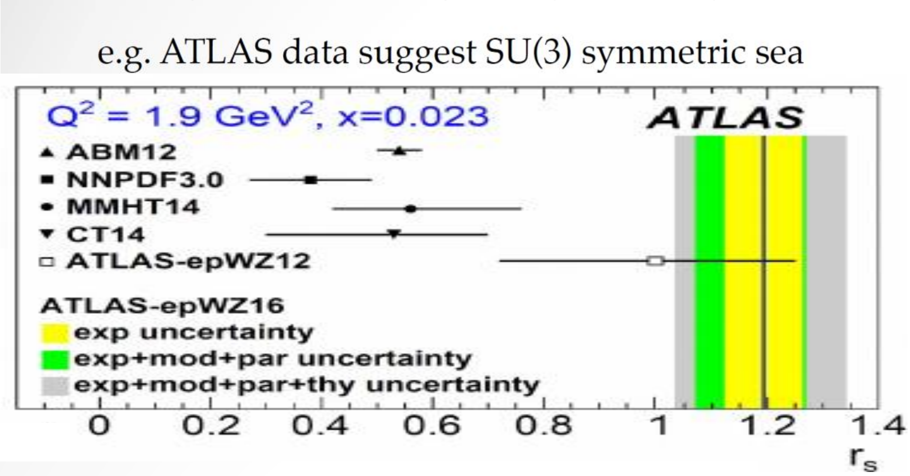
Higher luminosity and better detectors

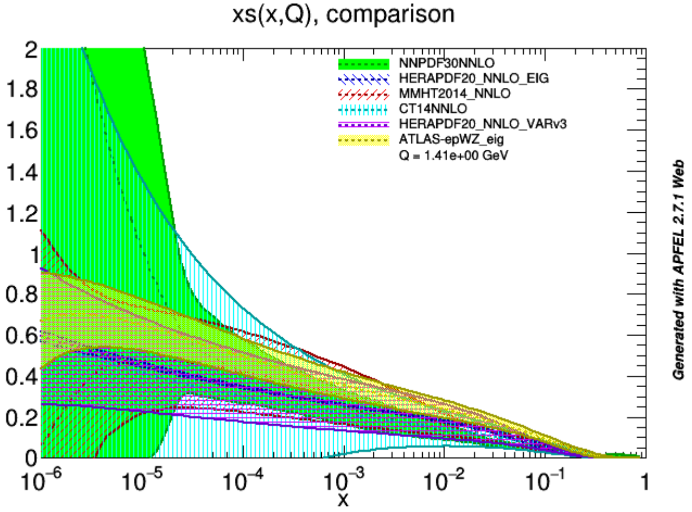
Strange quarks could also be studied for the first time

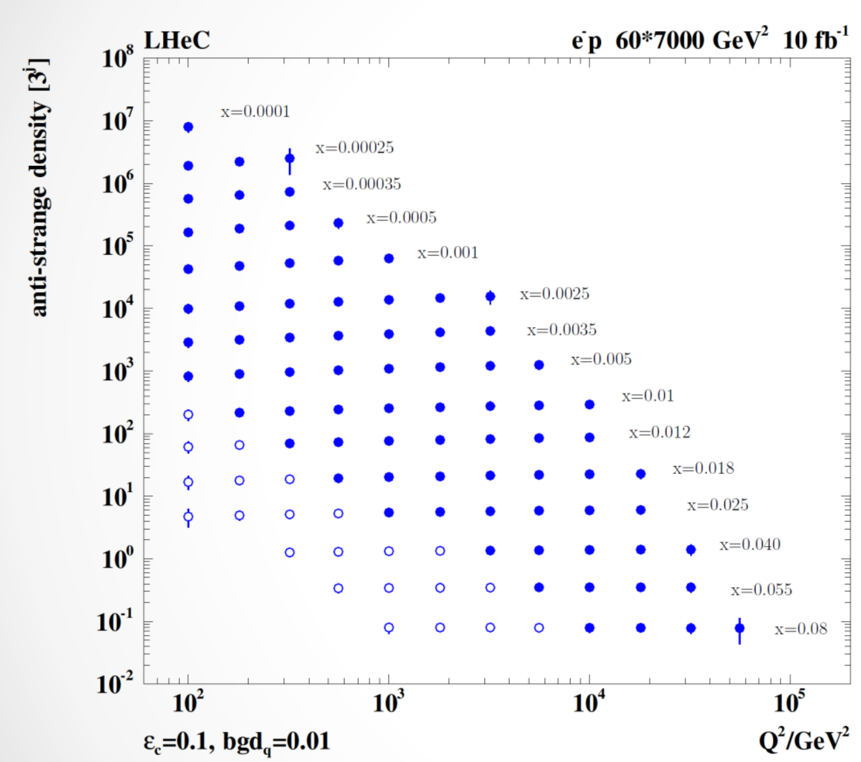
The strange PDF is not well known

Is it suppressed compared to other light quarks?

Is there strange-antistrange asymmetry?


e.g. ATLAS data suggest SU(3) symmetric sea

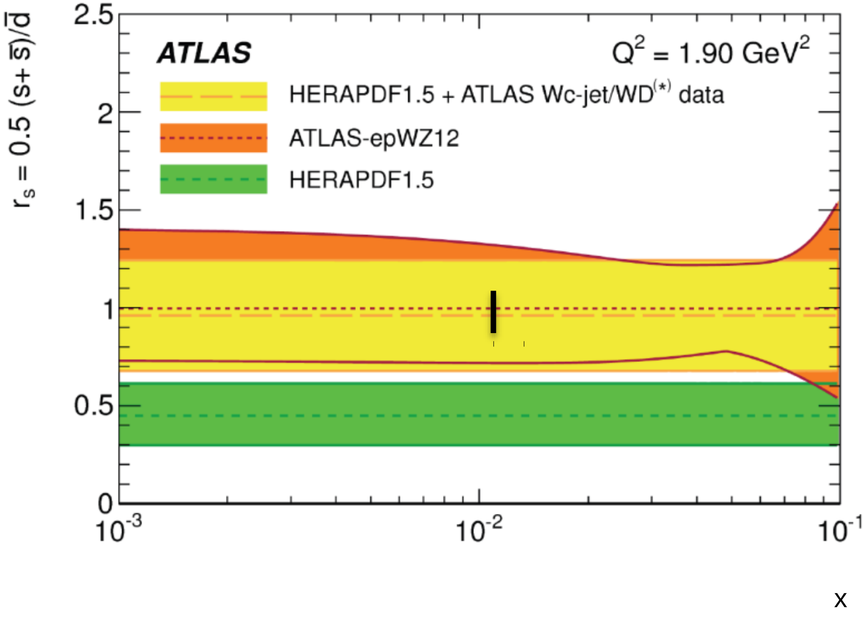


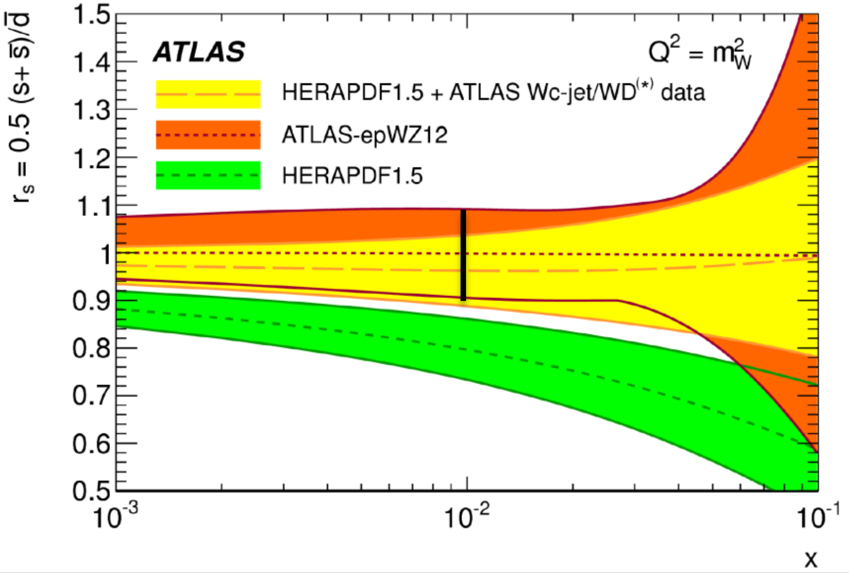


EIC could give direct sensitivity to strange through charm tagging in CC events.

Results are shown for the LHeC for 10% charm tagging efficiency, 1% light quark background in impact parameter.

Again the range may not be so wide but the possibility of a good measurement is there.



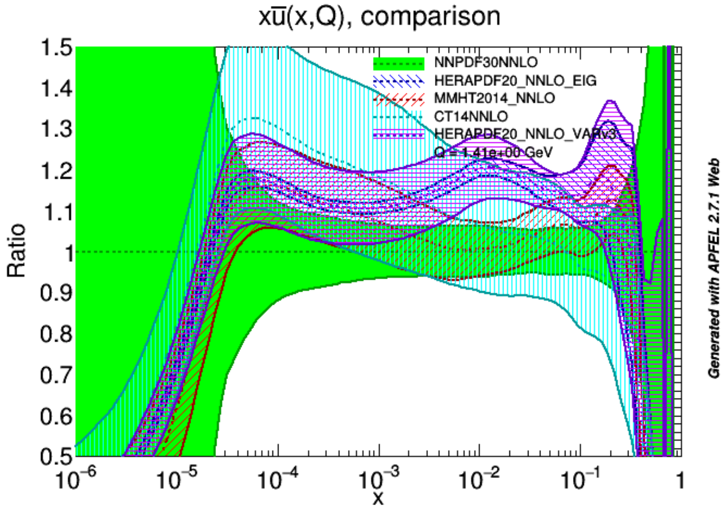


Let us first examine WHY?

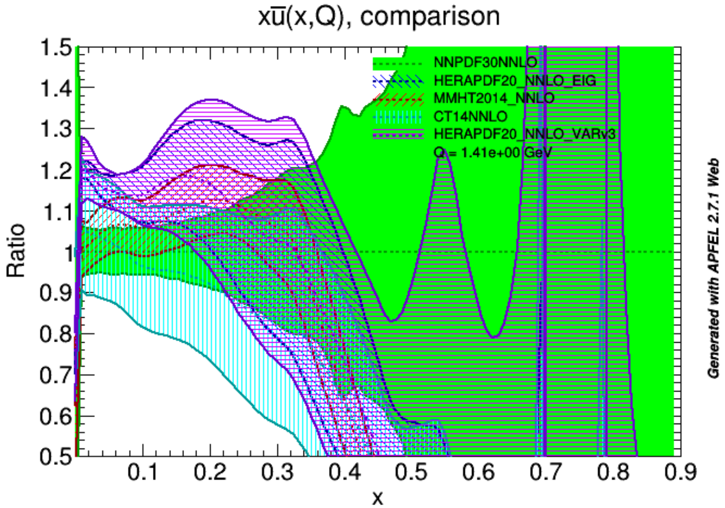
For illustration, these are plots of the strangeness fraction in the proton r_s from ATLAS analyses in which it is equal to the light quarks and in the HERAPDF1.5 in which it is ~0.5 of the light quarks.

This fraction is shown at the starting scale Q^2_0 ~2 GeV^2 and at $Q^2 = M_W{}^2$
NOTE the difference in scale.
PDF uncertainties decrease as Q^2 increases because the PDFs depend LESS on the parametrisation at the starting scale and MORE on the known QCD evolution.

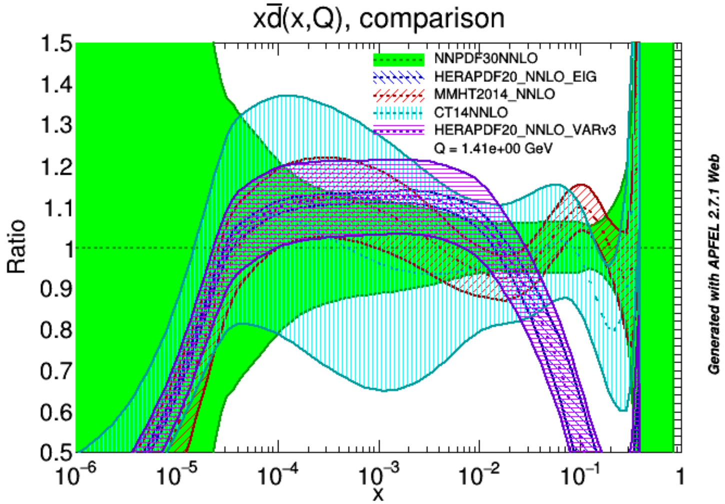
On each plot is shown a hypothetical measurement with ±10% accuracy.
Clearly this could distinguish the rs predictions if performed at Q^2_0, but not if performed at high scale.
At high scale we have to have much more accurate measurements.



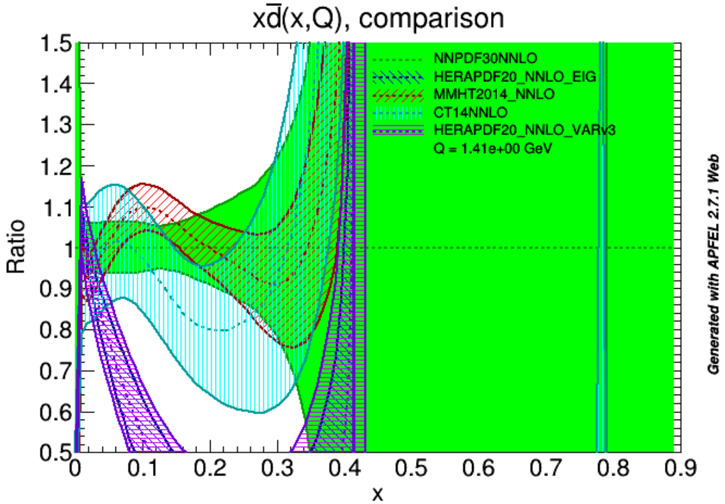
$x\bar{u}(x,Q)$, comparison



$x\bar{u}(x,Q)$, comparison

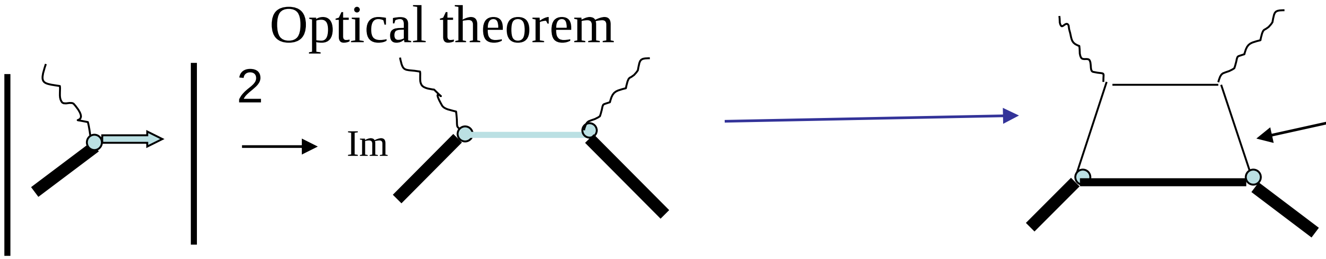


$x\bar{d}(x,Q)$, comparison

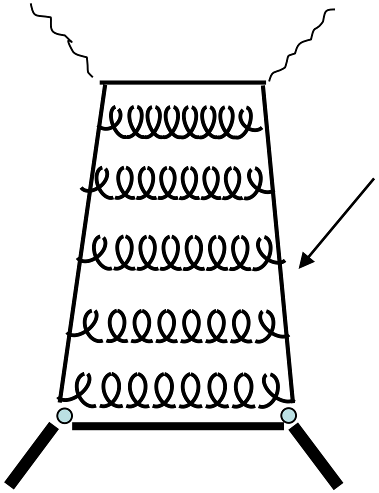


$x\bar{d}(x,Q)$, comparison

Need to extend the formalism?



The handbag diagram- QPM



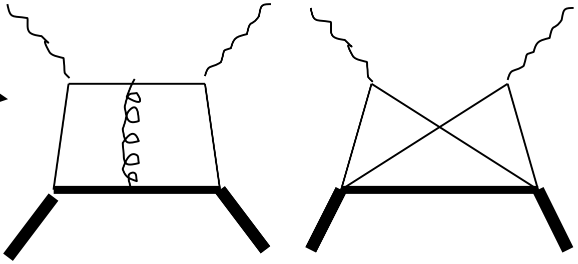
QCD at LL(Q^2)

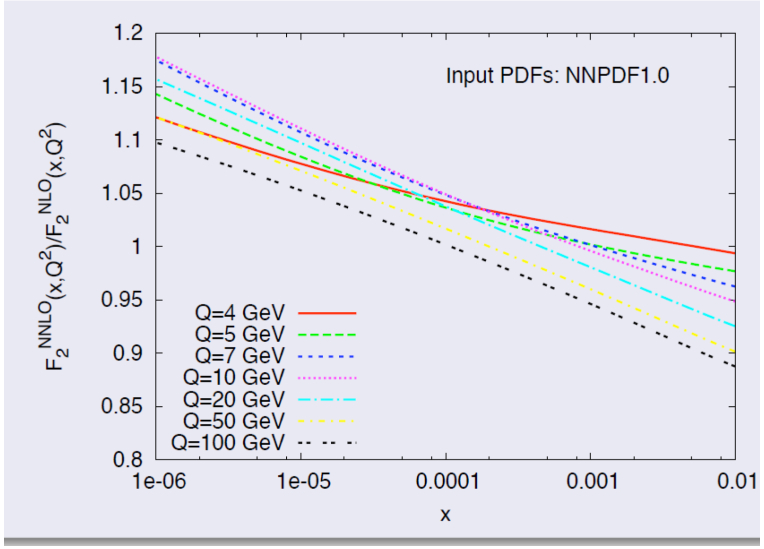
Ordered gluon ladders ($\alpha_s^{\,n} \ln Q^{2\,n}$)

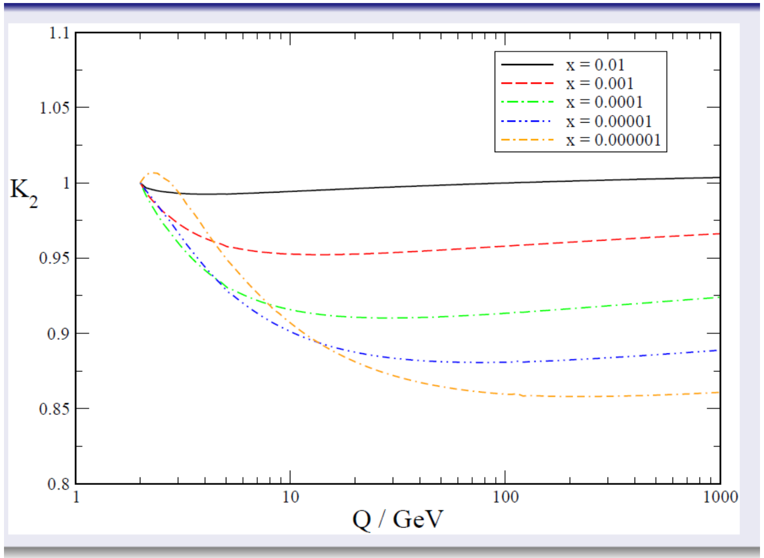
NLL(Q^2) one rung disordered $\alpha_s^{\,n} \ln Q^{2\,n-1}$

BUT what about completely disordered Ladders?
at small x there may be a need for BFKL ln(1/x) resummation?

And what about Higher twist diagrams ?
Are they always subdominant?
Important at high x, low Q^2

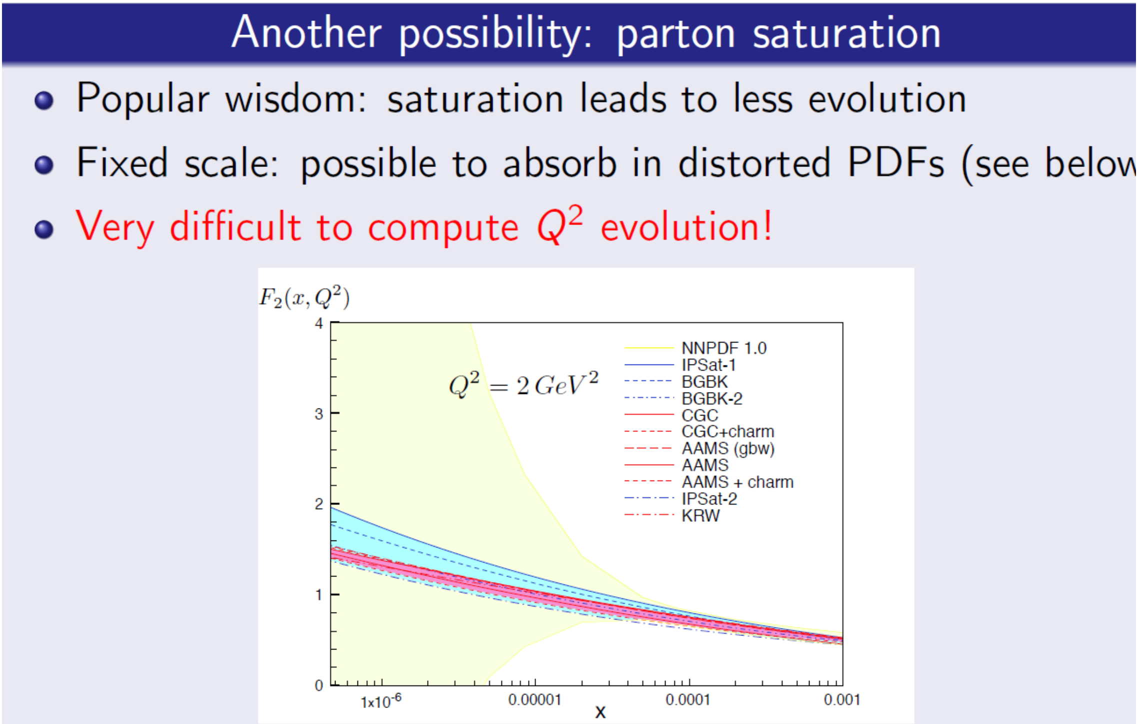






NNLO gives even more evolution, which is not what is needed

Ln(1/x) resummation gives less evolution, this could help



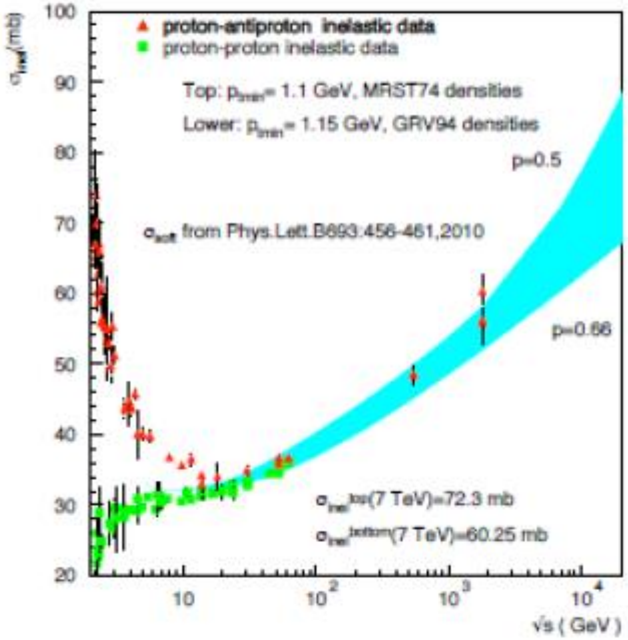
Another possibility: parton saturation

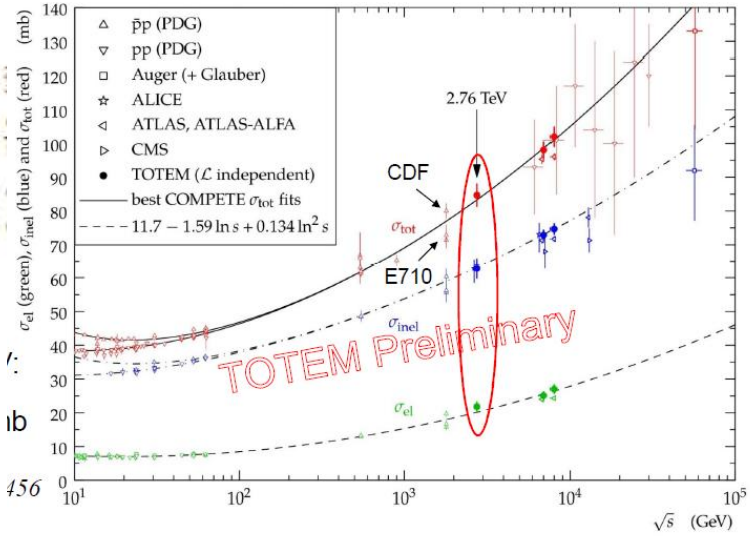
- Popular wisdom: saturation leads to less evolution
- Fixed scale: possible to absorb in distorted PDFs (see below
- Very difficult to compute Q^2 evolution!

Do we understand the rise of hadron-hadron cross-sections at all?

Could there always have been a hard Pomeron- is this why the effective Pomeron intercept is 1.08 rather than 1.00?

Does the hard Pomeron mix in more strongly at higher energies? What about the at the LHC?



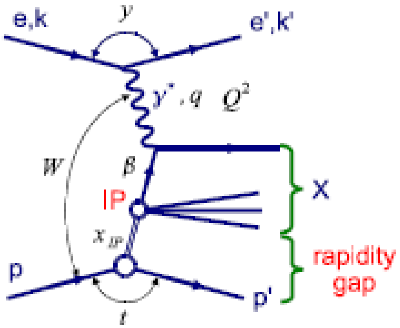


Pre ATLAS prediction with uncertainty from assumptions on mixing in of hard Pomeron

If anything TOTEM result looks even steeper

What about the Froissart bound?

The Pomeron also makes less indirect appearances in HERA data in diffractive events, which comprise ~10% of the total.



The proton stays more or less intact, and a Pomeron, with fraction X_P of the proton's momentum, is hit by the exchanged boson.

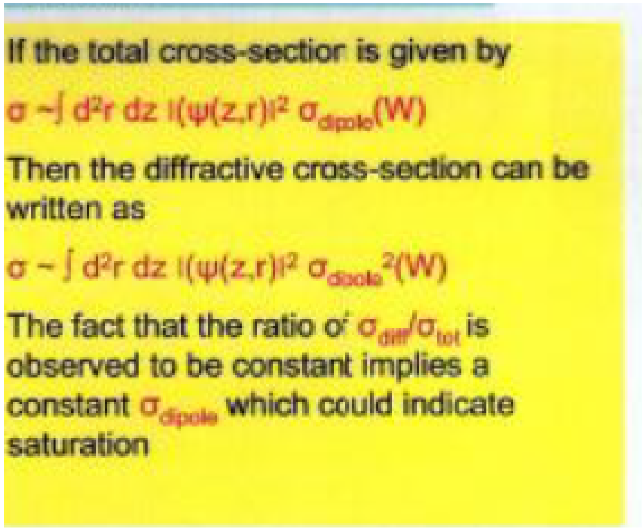
One can picture partons within the Pomeron, having fraction beta of the Pomeron momentum

One can define diffractive structure functions, which broadly factorize in to a Pomeron flux (function of x_P, t) and a Pomeron structure function (function of beta, Q2).

The Pomeron flux has been used to measure Pomeron Regge intercept – which seems marginally harder than that of the soft Pomeron

The Pomeron structure functions indicate a large component of hard gluons

But this is not the only view of difraction. These data have also been interpreted in terms of dipole models



If the total cross-sectior is given by

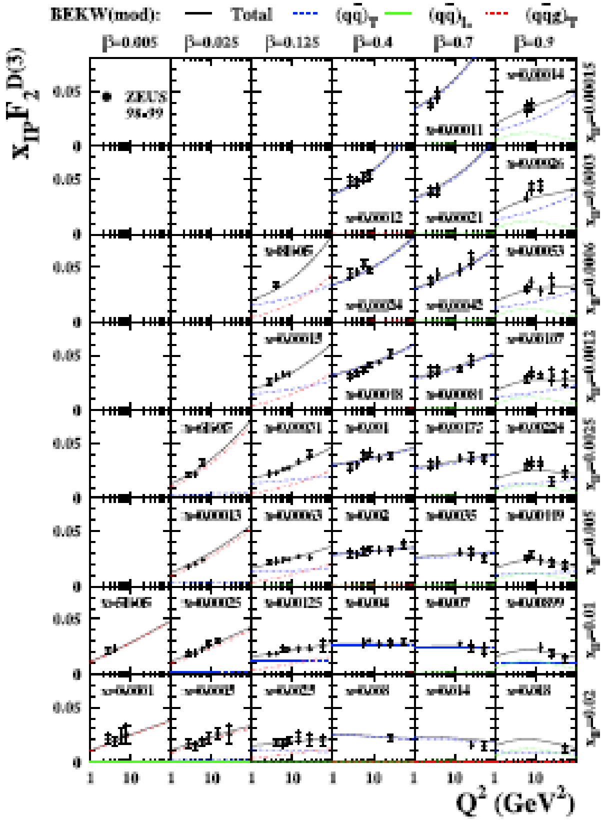
$\sigma \sim \int d^2r\, dz\, |(\psi(z,r)|^2\, \sigma_{dipole}(W)$

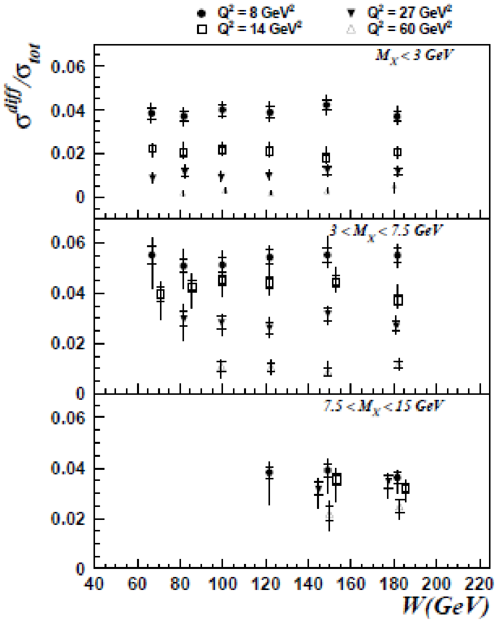
Then the diffractive cross-section can be written as

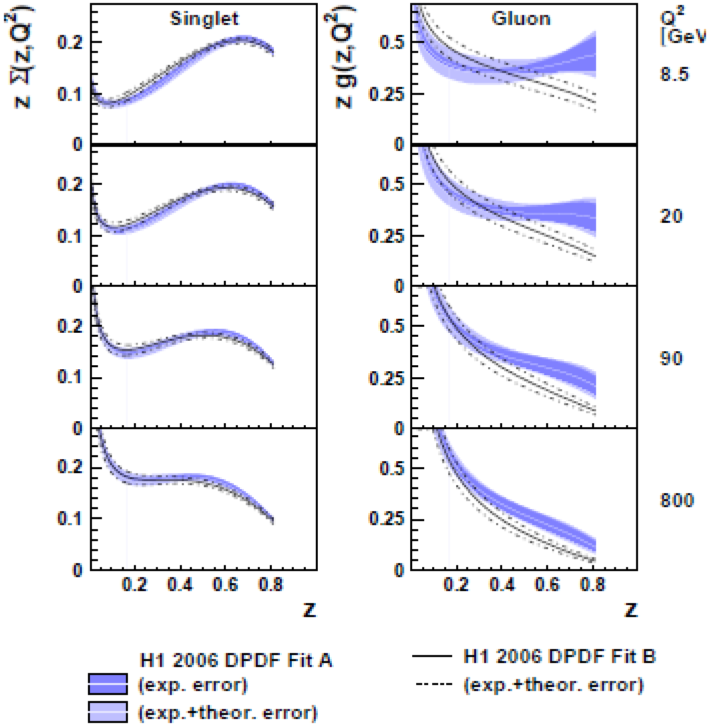
$\sigma \sim \int d^2r\, dz\, |(\psi(z,r)|^2\, \sigma_{dipole}^2(W)$

The fact that the ratio o' $\sigma_{diff}/\sigma_{tot}$ is observed to be constant implies a constant σ_{dipole} which could indicate saturation

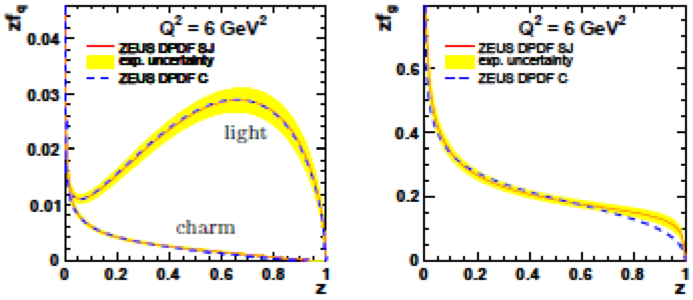
CDM fits to ZEUS diffractive data







In all fits performed, the DPDFs are dominated by the gluon density, which extends to large values of z and accounts for typically 60 % (ZEUS (Chekanov *et al.*, 2010a)) to 70% (H1 (Aktas *et al.*, 2006e)) of the total longitudinal momentum of the diffractive exchange.



DPDF approach does, however, appear to undergo an infrared breakdown at larger Q^2 scales than is the case in inclusive QCD fits (Aktas *et al.*, 2006e; Chekanov *et al.*, 2010a). Whilst this may provide further evidence for saturation effects in DDIS (Frankfurt *et al.*, 2001a), it may also be a consequence of enhanced higher twist contributions or a less quickly convergent QCD order expansion in the diffractive than the inclusive case.

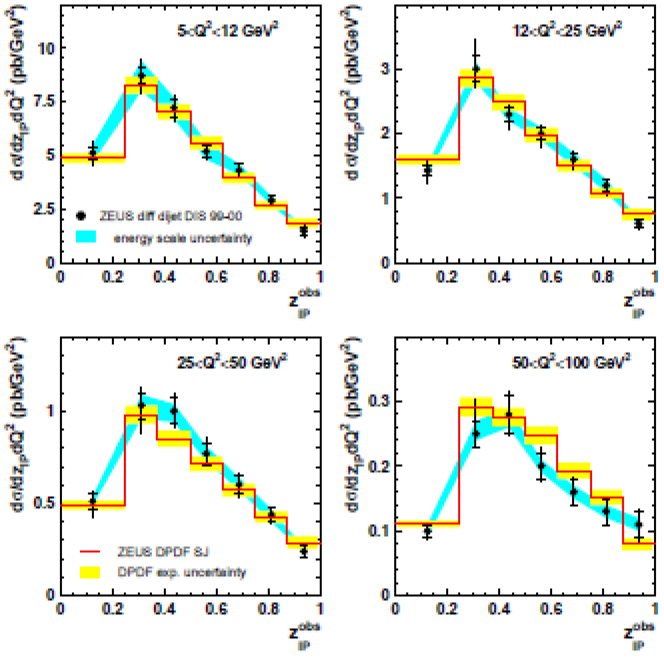


FIG. 59 Comparison of diffractive DIS dijet cross sections, measured double differentially in $z_{\mathbb{P}}$ and Q^2, with NLO QCD predictions based on DPDFs. In this case, the data shown were used together with inclusive diffraction measurements in the fit, to improve the high β sensitivity. From (Chekanov *et al.*, 2010a).

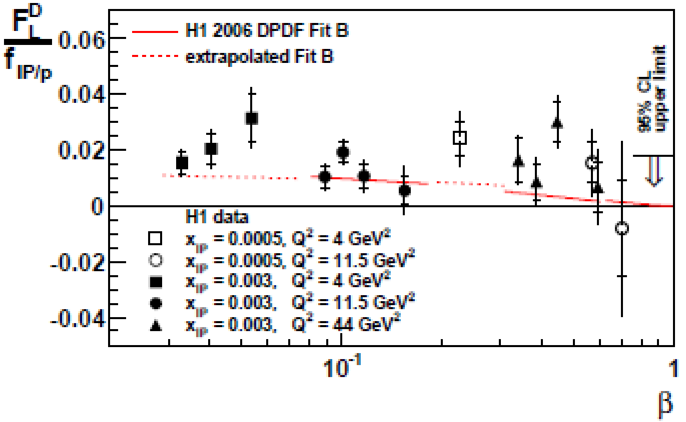


FIG. 61 Summary of H1 measurements of the diffractive longitudinal structure function F_L^D, exploiting data taken with variations in the proton beam energy. To allow comparisons between measurements at different $x_{\mathbb{P}}$ values, the data are normalised by a diffractive flux factor $f_{\mathbb{P}/p}$ (see Section V.C.4). The data are compared with an NLO QCD prediction based on DPDFs. From (Aaron *et al.*, 2012g).

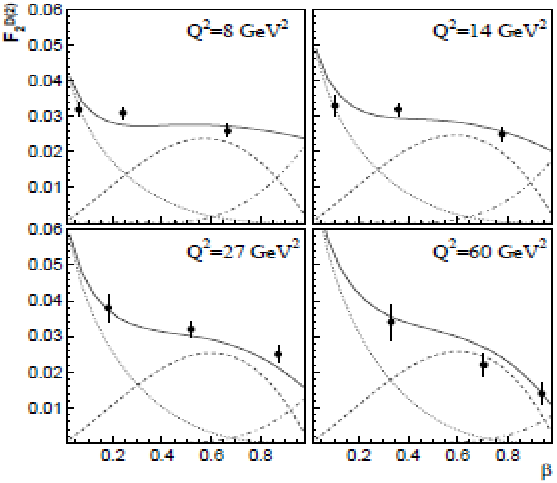
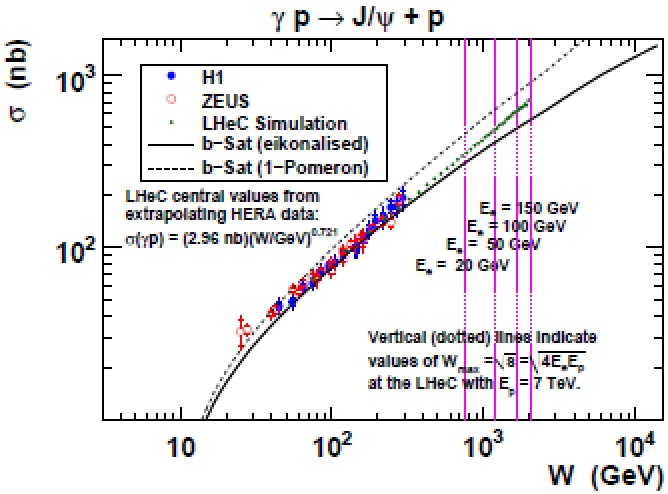


FIG. 63 An example decomposition of the β dependence of ZEUS DDIS data (Breitweg *et al.*, 1999e) into different dipole terms, according to the parametrisation in (Golec-Biernat and Wusthoff, 1999). The dashed, dotted and dot-dashed curves correspond to leading twist transverse $q\bar{q}$, leading twist transverse $q\bar{q}g$ and higher twist longitudinal $q\bar{q}$ contributions, respectively.



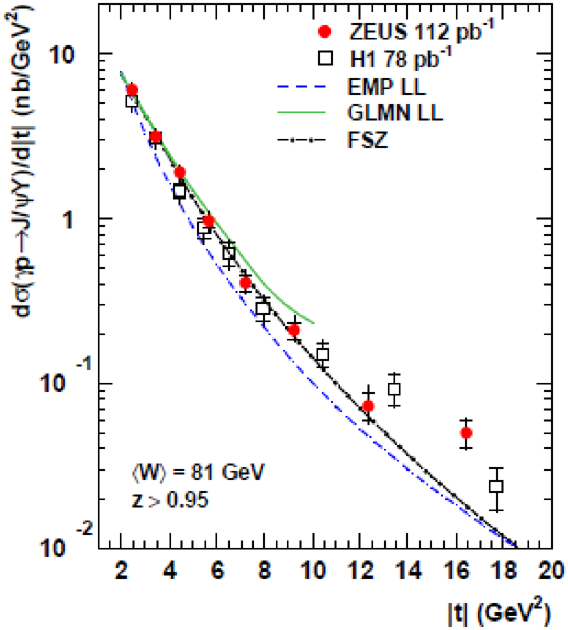
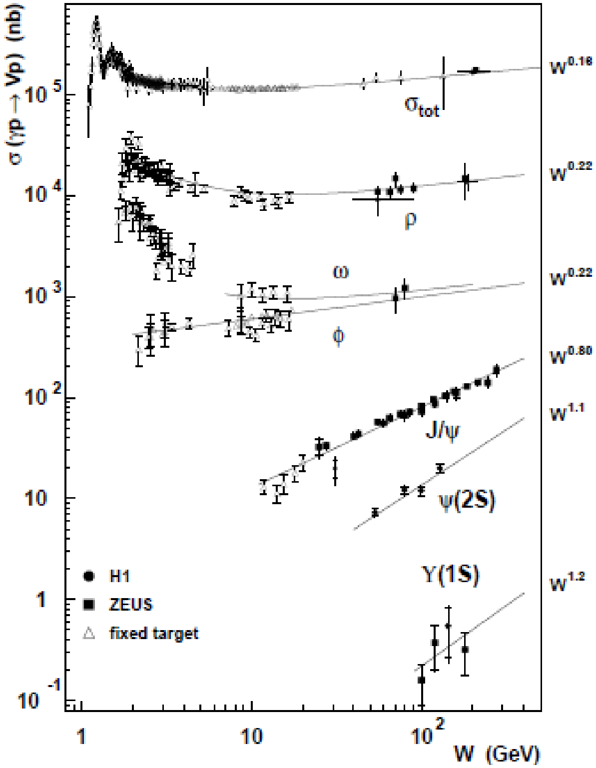
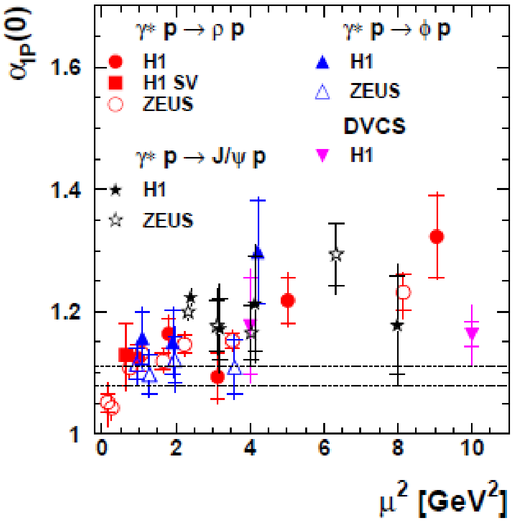
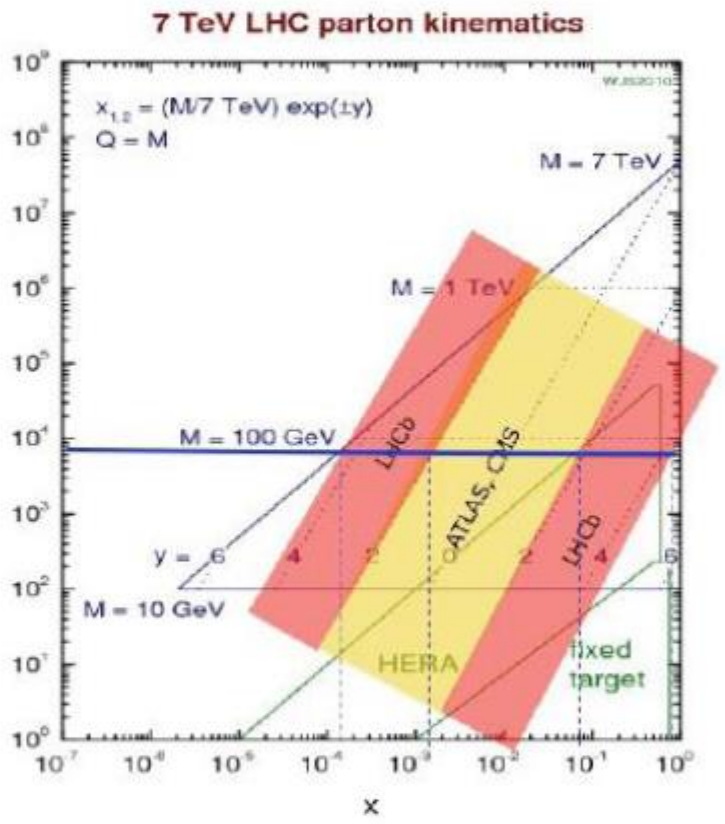


FIG. 65 Dependence on $|t|$ of proton dissociative J/ψ production integrated over proton dissociation masses satisfying $M_Y^2/W^2 < 0.05$. The data are compared with a prediction ('GLMN LL' (Gotsman *et al.*, 2002)) which is based on DGLAP evolution and is expected to be valid for $|t| < m_\psi^2$. They are also compared with a BFKL-based prediction ('EMP LL' (Enberg *et al.*, 2002b)) and a more phenomenological approach ('FSZ' (Frankfurt *et al.*, 2008). From (Chekanov *et al.*, 2010f).





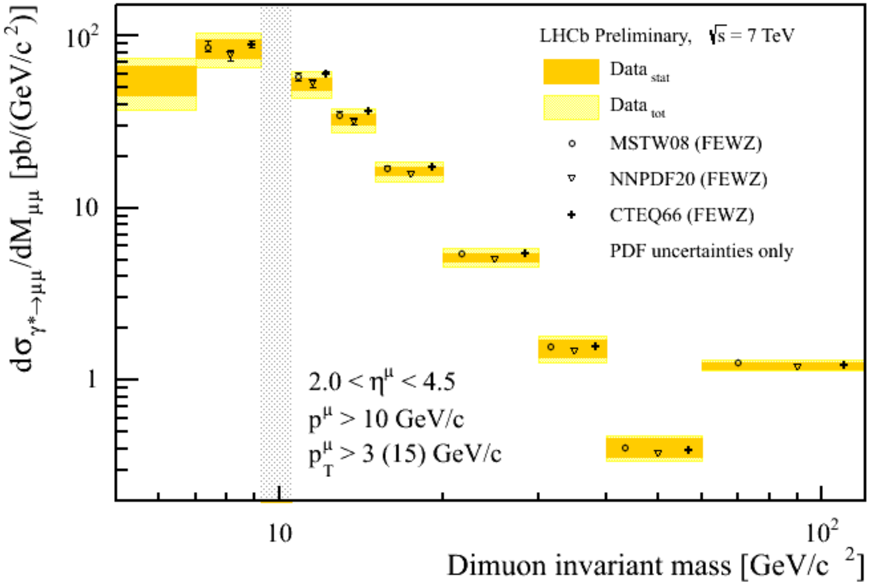


Does the inclusion of low x, Q2 data in the PDFs bias the predictions at the LHC?

Not at high-x, Q^2 where predictions from Q^2 cut fits Q^2>10 GeV^2
agree with Q^2 cut fits Q^2>3.5 GeV^2

But it could matter at low-x (x < 0.0001) and moderate Q^2, Q^2 ~25-100 GeV^2.
This is the LHCb region

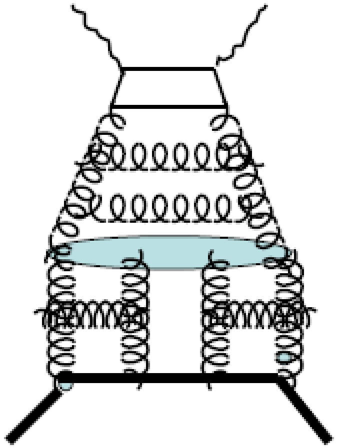
LHCb low-mass Drell-Yan data show no sign of deviation from DGLAP predictions– but this is a log plot, uncertainties are large

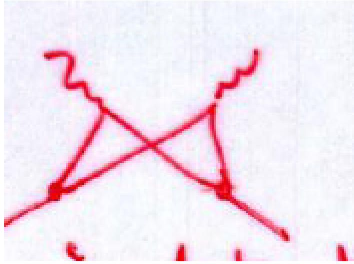


The use of non-linear evolution equations also *improves* the shape of the gluon at low x, Q^2

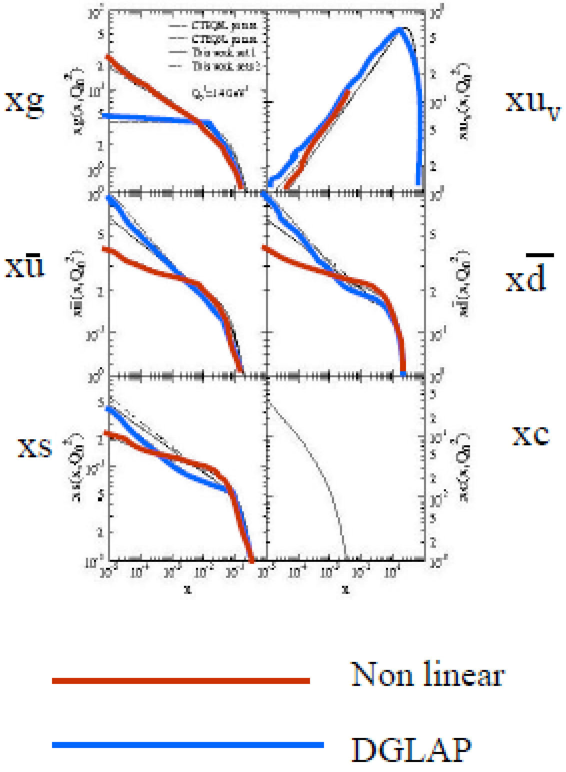
The gluon becomes steeper (high density) and the sea quarks less steep

Non-linear effects $gg \rightarrow g$ involve the summation of FAN diagrams – higher twist



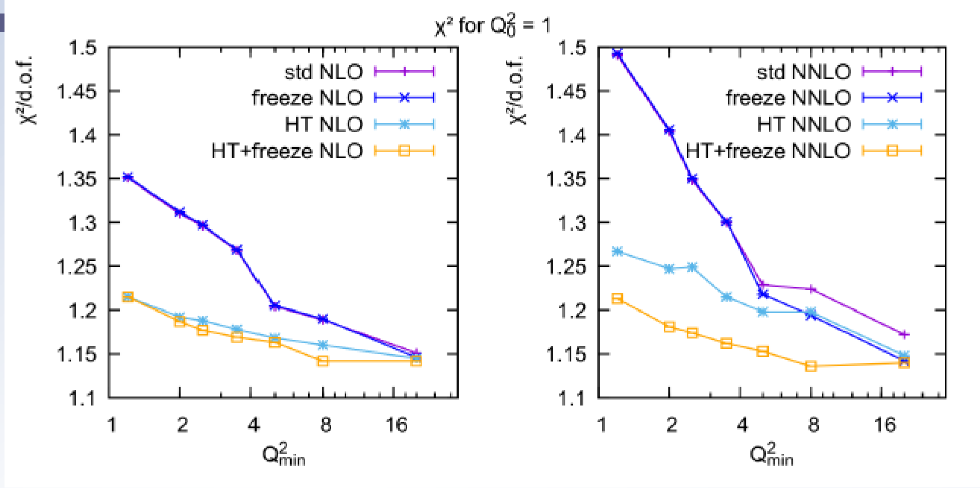


$Q^2 = 1.4\ \mathrm{GeV}^2$



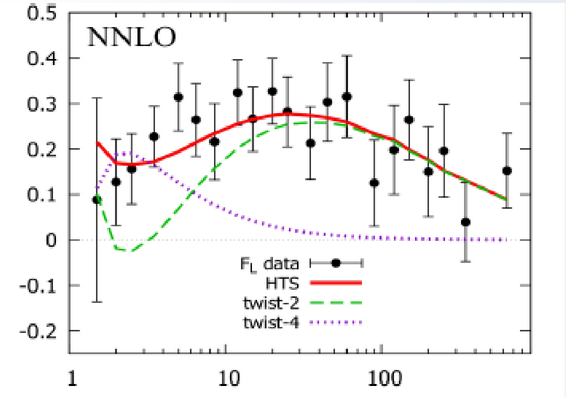
Another example of higher-twist effects in low-x HERA data from Motyka et al

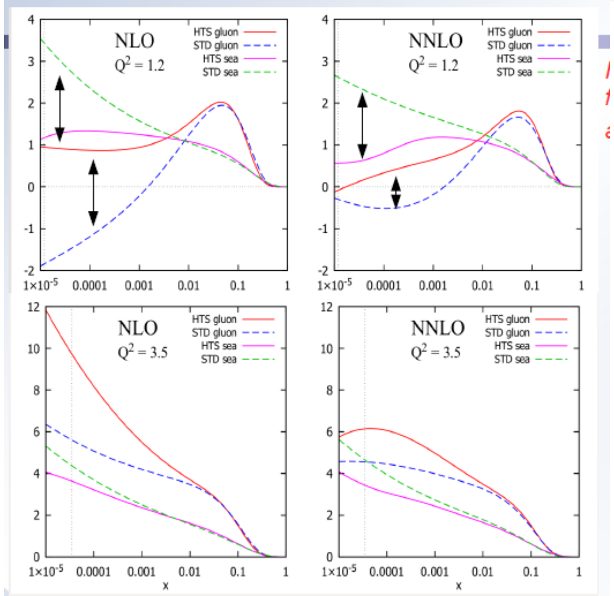
Fit results with and without higher twist corrections:



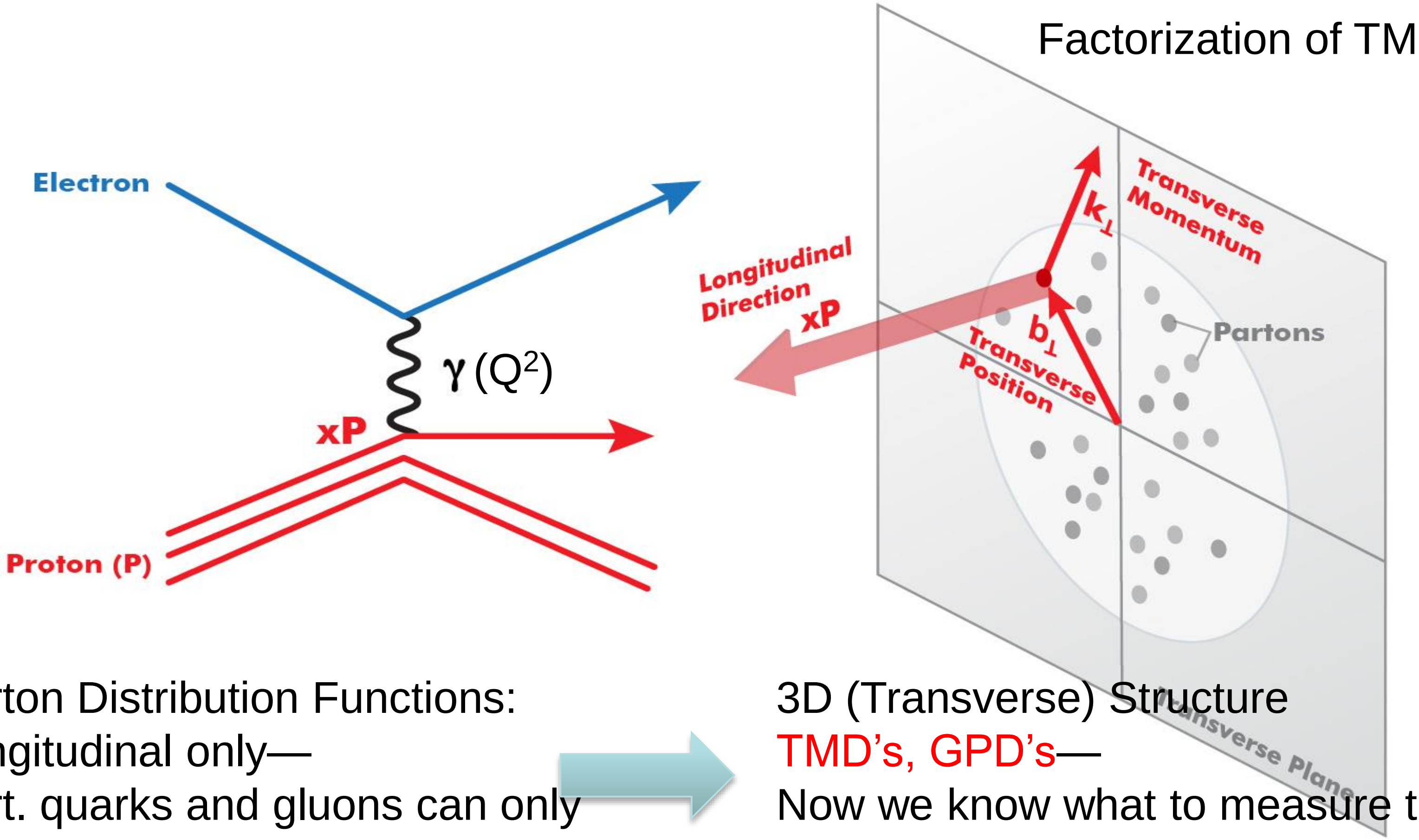
- Clear improvement of fit quality by inclusion higher twist corrections
- Effect of improvement much stronger for NNLO DGLAP fits
- Saturation input damping (freezing) of the quark sea improves fits significantly for NNLO fit with higher twists, but has little effect in NLO HT fit

- Good description of F_L obtained down to lowest Q^2
- NLO DGLAP + HT fit: big contribution of HT below 5 GeV^2
- 10% HT contribution at ~10 GeV^2





Factorization of TMD, GPD



Parton Distribution Functions:
Longitudinal only—
Pert. quarks and gluons can only
be thought of longitudinally making up p.

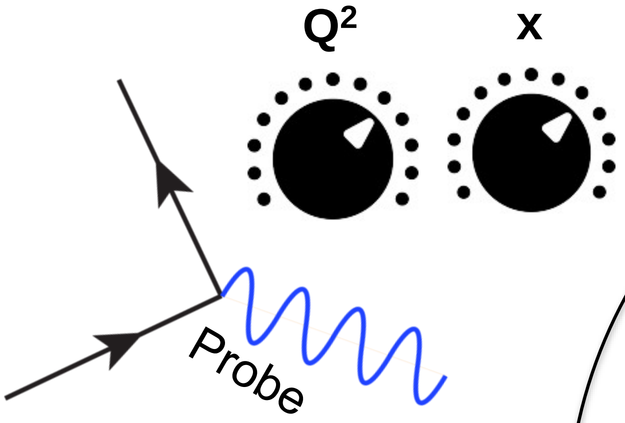
3D (Transverse) Structure
TMD's, GPD's—
Now we know what to measure to
understand the 3D structure of nucleons

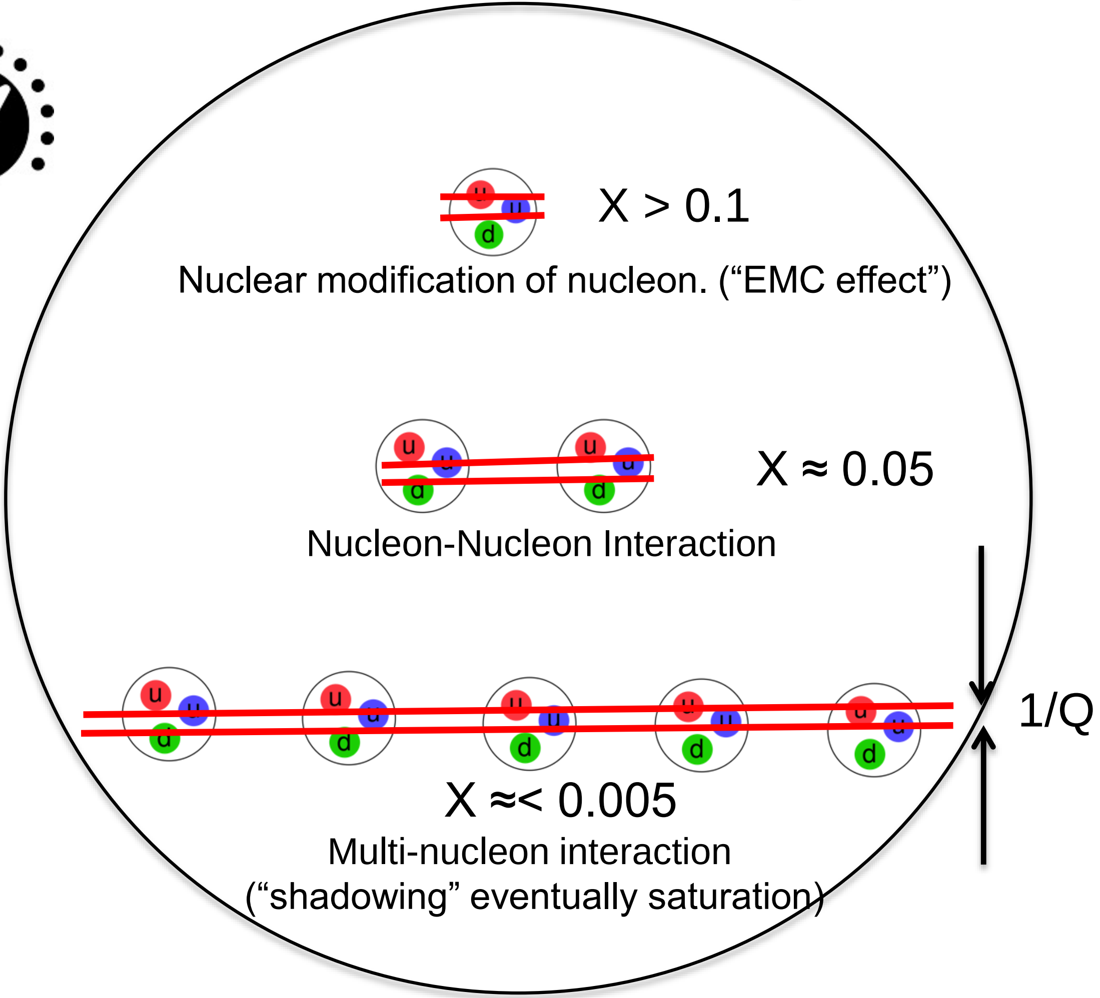
Transverse Momentum Dependent Distributions (TMD): k_t
Generalized Parton Distributions (GPD): b_t

HERMES,
COMPASS,
JLAB 12

Parameters of the Probe (Nuclei)

[Cosyn, Armesto, Fazio]





$X > 0.1$

Nuclear modification of nucleon. (“EMC effect”)

$X \approx 0.05$

Nucleon-Nucleon Interaction

1/Q

$X \approx< 0.005$

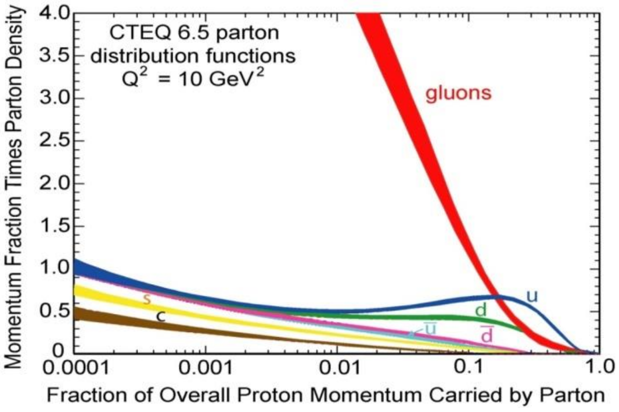
Multi-nucleon interaction
(“shadowing” eventually saturation)

Probing the nucleon interaction in the nuclei
(note this is different from correlation measurements)

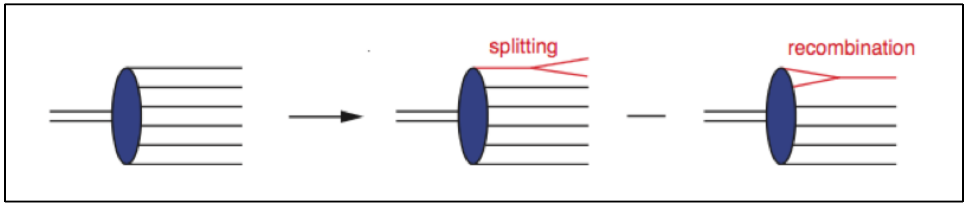
Note: the x range for nuclear exploration is similar to the nucleon exploration

QCD at Extremes: Parton Saturation

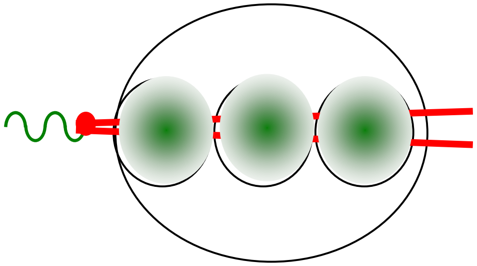
[WG2: Low-x and Diffraction]



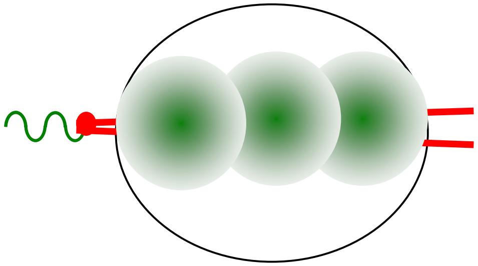
HERA discovered a dramatic rise in the number of gluons carrying a small fractional longitudinal momentum of the proton (i.e. small-x).



This cannot go on forever as x becomes smaller and smaller: parton recombination must balance parton splitting. i.e. Saturation—unobserved at HERA for a proton. (expected at extreme low x)



In nuclei, the interaction probability enhanced by $A^{1/3}$



Will nuclei saturate faster as color leaks out of nucleor