Physics of gluon saturation

"U.S.-Based Electron Ion Collider Science Assessment Committee"

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Outline

Partons and linear evolution equations

Gluon saturation, and the saturation momentum

Propagation of a color dipole in matter

MV model, saturation in the MV model

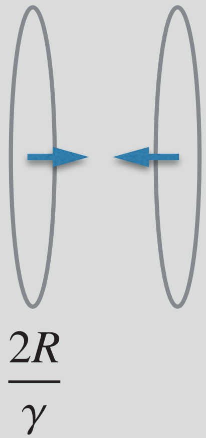
Non linear evolution equations

Phenomenology (heavy ions)

Elementary introduction to the main concepts. For details and references see Rep. Prog. Phys. 80 (2017) 032301 [arXiv:1607.04448]

High gluon density in high energy hadronic collisions

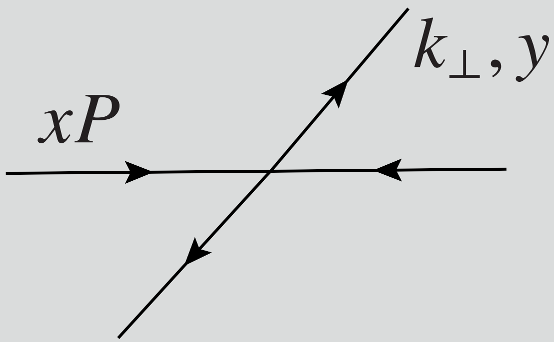
Consider nucleus-nucleus collision at high energy



RHIC $\gamma \approx 10^2$

LHC $\gamma \approx 10^3 - 10^4$

Particle production from collisions between "constituents" of the nuclei



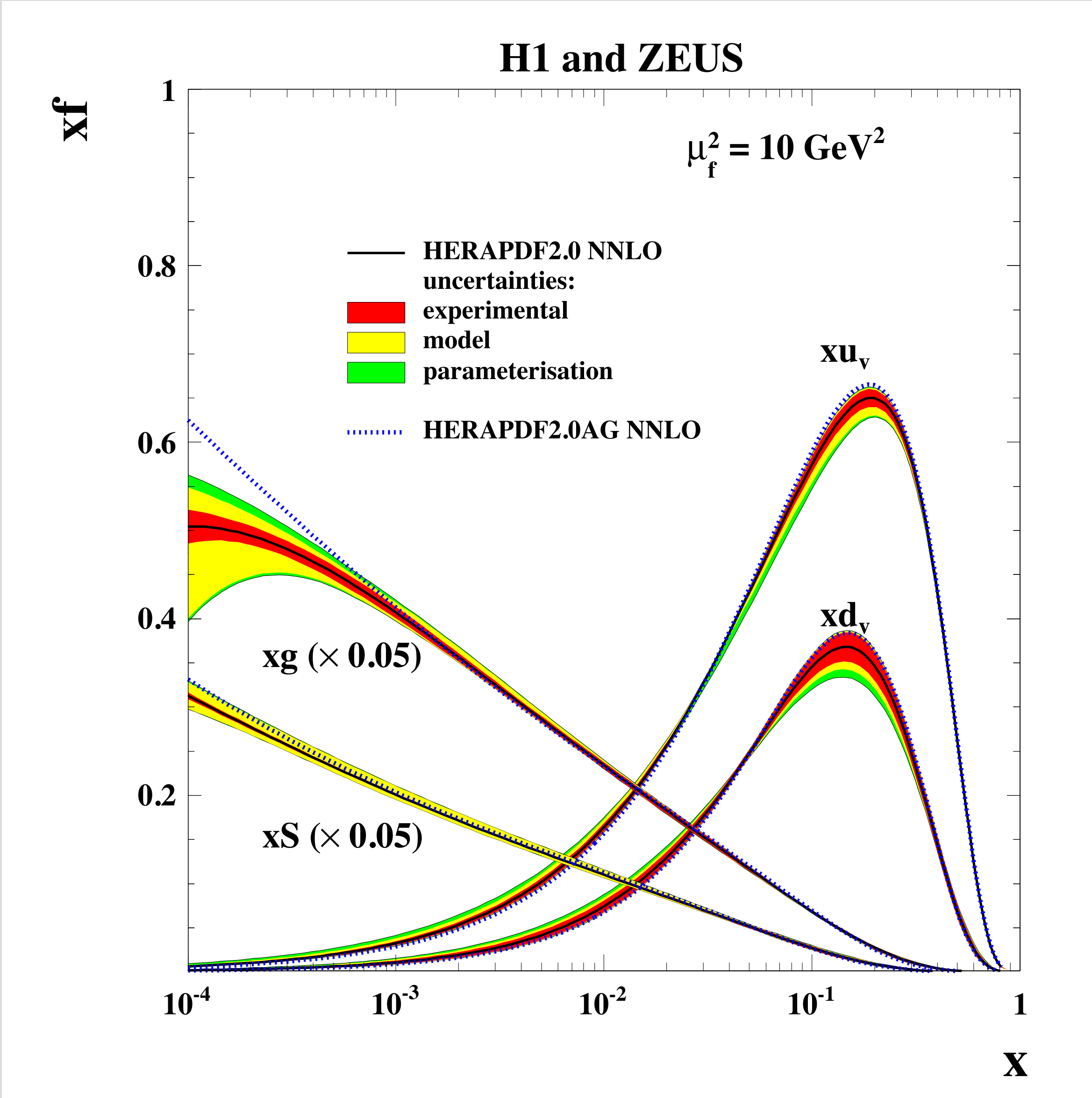
$$x \simeq \frac{k_\perp}{\sqrt{s}} e^{\pm y}$$

Most produced particles have $k_\perp \approx 1\ \mathrm{GeV}$ and small x values

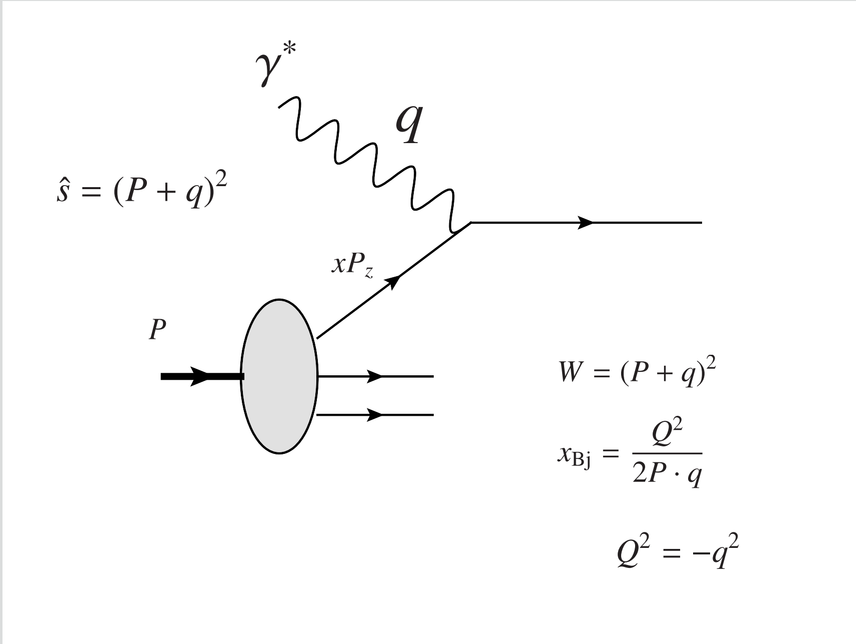
RHIC $x \approx 10^{-2}$ LHC $x \approx 10^{-3} - 10^{-4}$

NB. Small x constituents completely overlap longitudinally.

Gluons dominate the "wave function" of hadrons at small x



DIS and the parton picture



DIS → $Q^2 \gg \Lambda^2_{QCD}$

Interaction takes place on a short time scale compared to the scale of internal dynamics ('naive parton model')

$\hat{s} \gg Q^2$ → $x \approx \frac{Q^2}{\hat{s}} \ll 1$

Gribov-Regge limit (high energy limit and limited Q^2)

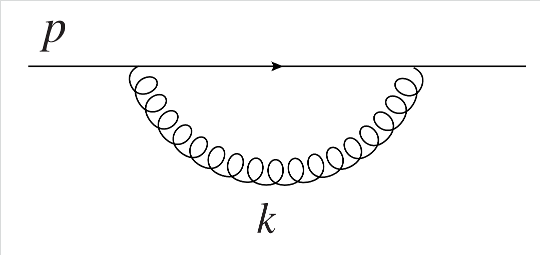
Light cone perturbation theory

(leading order)

Fock states

$$|q\rangle = a|q\rangle_0 + b|qg\rangle_0 + \cdots$$

Lifetime of fluctuations $(k_\perp \ll k_z,\ k_z \ll p_z,\ x \ll 1)$



$$\Delta t \simeq 2xp_z/k_\perp^2$$

Probability of radiation

$$d\mathcal{P} \simeq \frac{\alpha_s C_R}{\pi^2} \frac{\mathrm{d}^2\boldsymbol{k}}{k_\perp^2} \frac{dx}{x}$$

Integrated and **unintegrated** gluon distribution

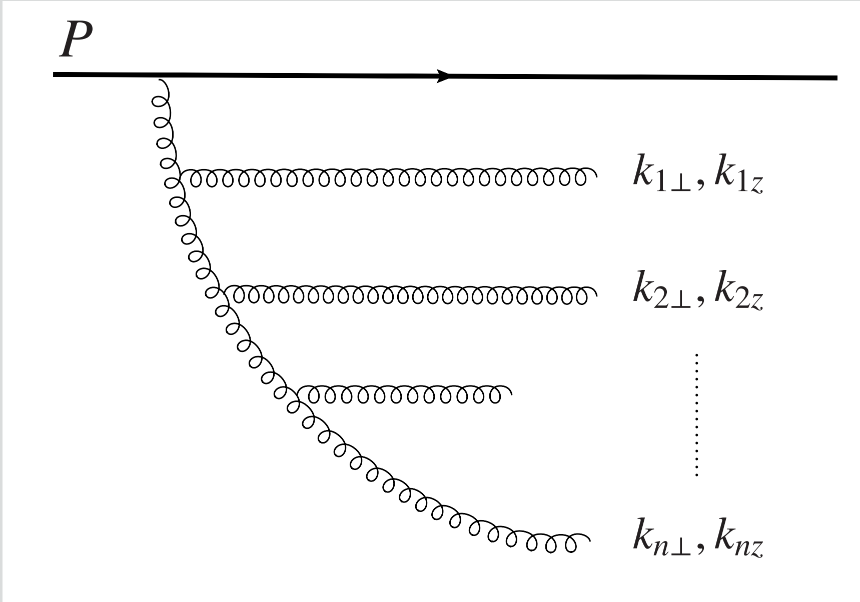
$$xG(x,Q^2) = \frac{\alpha_s C_F}{\pi} \ln\left(\frac{Q^2}{\Lambda_{QCD}^2}\right) = \int_{\Lambda_{QCD}^2}^{Q^2} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \varphi(x,k_\perp)$$

Gluon 'cascades'

LO not sufficient when

$$\alpha_s \ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right) \approx 1$$

Coupling 'Density'



Same when $\alpha_s \ln(1/x) \approx 1$

Large logarithms are contained in simple diagrams

Specific ordering of successive momenta

DGLAP $k_{1\perp} < k_{2\perp} < \cdots < k_{n\perp}$

BFKL $k_{1z} > k_{2z} > \cdots > k_{nz}$

Lifetime of fluctuation decreases along the cascade

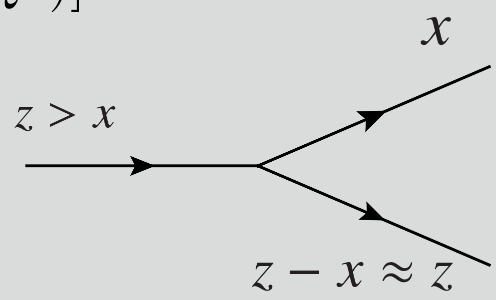
$$\Delta t_i = 2k_{iz}/k_{i\perp}^2$$

Linear evolution equations

DGLAP

At small x, for gluons, evolution of the integrated pdf

$$Q^2 \frac{\partial}{\partial Q^2}[xG(x,Q^2)] = \frac{\alpha_s(Q^2)C_A}{\pi}\int_x^1 \frac{dz}{z}[zG(z,Q^2)]$$



Probabilistic interpretation

Only evolution calculable

Universal character of pdf's- Collinear factorisation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp^2} = \int \mathrm{d}x_1 \int \mathrm{d}x_2\, x_1 G(x_1,\mu^2)\, x_2 G(x_2,\mu^2)\, \frac{\mathrm{d}\hat{\sigma}_{gg\to gg}(\hat{s},\mu^2)}{\mathrm{d}p_\perp^2}$$

pdf

Monte Carlo (HIJING, etc), hard probes, nuclear pdf

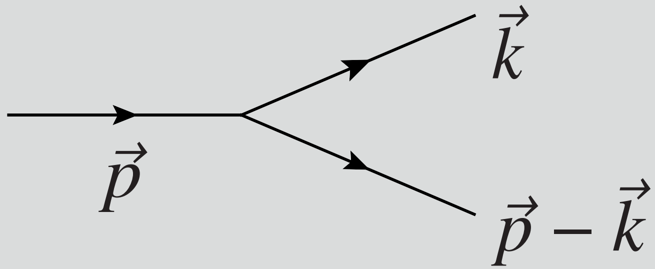
Linear evolution equations

BFKL

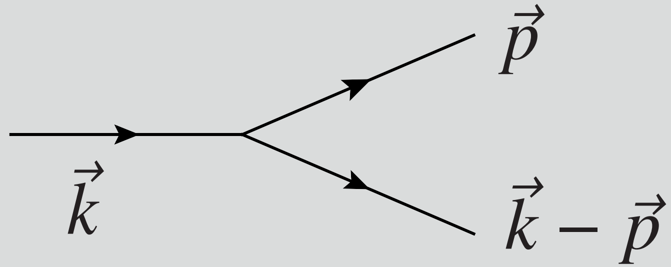
Evolution of the unintegrated pdf

$$\frac{\partial \varphi(y,\boldsymbol{k})}{\partial y} = \frac{\bar{\alpha}_s}{2\pi}\int_{\boldsymbol{p}} \frac{\boldsymbol{k}^2}{\boldsymbol{p}^2(\boldsymbol{k}-\boldsymbol{p})^2}\left[2\varphi(y,\boldsymbol{p}) - \varphi(y,\boldsymbol{k})\right] \qquad y = \ln(1/x) \qquad \bar{\alpha}_s \equiv \alpha_s C_A/\pi$$

Probabilistic interpretation



(gain)

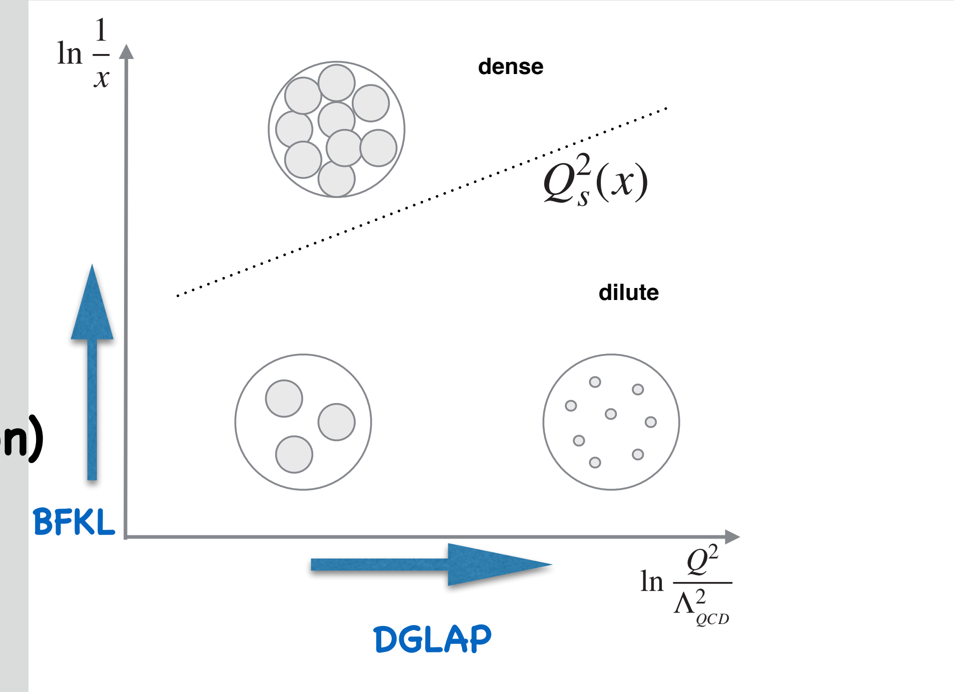


(loss)

Exponential growth

$$\varphi(y,\boldsymbol{k}) \sim \mathrm{e}^{\omega\bar{\alpha}_s y} \qquad (\omega = 4\ln 2)$$

Saturation



Growth of gluon density must be tamed
(bound on cross section)

Early approach (correction to the linear evolution)

$$\frac{\partial^2 \, xG(x,Q^2)}{\partial \ln(1/x)\, \partial \ln Q^2} = \bar{\alpha}_s \, xG(x, Q^2) - \# \, \bar{\alpha}_s^2 \, \frac{[xG(x,Q^2)]^2}{R^2 Q^2}$$

A scale emerges: the saturation momentum

$$Q_s^2 \sim \alpha_s(Q_s^2) \frac{xG(x, Q_s^2)}{\pi R^2}$$

fixes the strength of the coupling

gluon density per unit area
in a nucleus $\sim A^{1/3}$

phase space occupation $\quad \frac{xG(x, Q_s^2)}{\pi R^2} \frac{1}{Q^2} \sim \frac{1}{\alpha_s}$ $\quad$ suggest classical fields

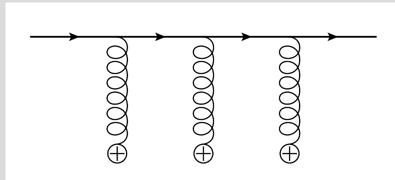
NB. Saturation is not a sharp transition

Propagation of a fast parton in matter

$$x^{+} = \frac{t+z}{\sqrt{2}}$$

Eikonal approximation

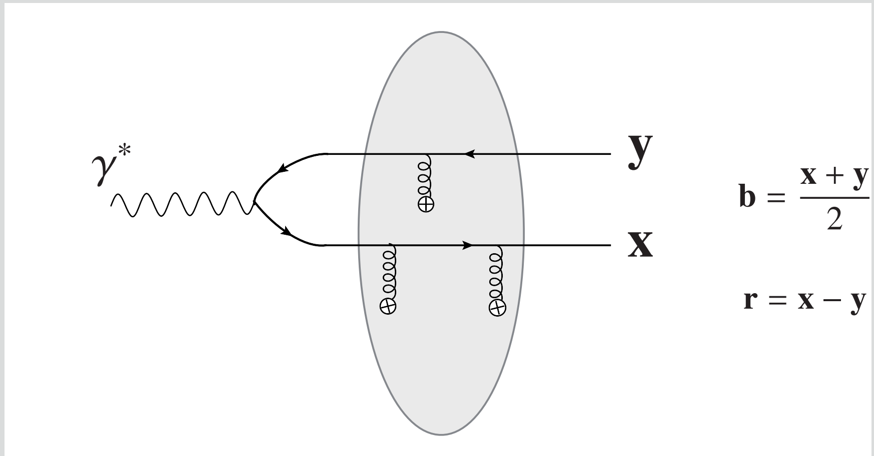
$$U(x) \equiv \mathrm{T}\exp\left(ig\int_{-\infty}^{x^{+}} dz^{+} A_{a}^{-}(z^{+}, \boldsymbol{x})t^{a}\right)$$



DIS in 'dipole frame'

$$\sigma_{\gamma^{*}p}(x, Q^{2}) = \int_{0}^{1} dz \int_{\boldsymbol{r}} \left|\psi(Q^{2}, z, \boldsymbol{r})\right|^{2} \sigma_{\mathrm{dip}}(\boldsymbol{r})$$

$$\sigma_{\mathrm{dip}}(\boldsymbol{r}) = 2\int \mathrm{d}^{2}\boldsymbol{b}\,(1 - \mathrm{Re}\,S(\boldsymbol{b}, \boldsymbol{r}))$$



Dipole nucleon/nucleus S-matrix

$$S(\boldsymbol{b}, \boldsymbol{r}) \equiv \frac{1}{N_{c}}\langle \mathrm{tr}\,(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^{\dagger})\rangle$$

Average over the field fluctuations of the target

Saturation as a result of multiple scattering

Dipole-nucleon, at leading order (2 gluon exchange)

$$\sigma_{\rm dip}(r_\perp) = \frac{\pi^2\alpha}{N_c} r_\perp^2 xG_N(x,Q^2) \qquad (Q^2 = 1/r^2)$$

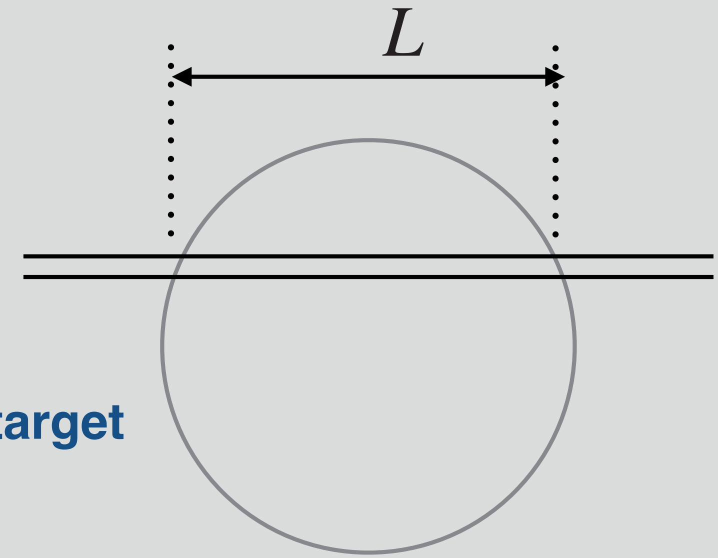
Color transparency for "small" dipoles

Dipole-nucleus, multiple scattering

Survival probability $S^2(\boldsymbol{b},\boldsymbol{r}) = \mathrm{e}^{-L/\lambda}$ $\qquad \frac{1}{\lambda(r_\perp)} = \rho\,\sigma_{\rm dip}(r_\perp)$

$$S(b,r_\perp) = \mathrm{e}^{-Q_s^2 r_\perp^2/4}$$

$$Q_s^2 = \frac{2\pi^2\alpha}{N_c}\frac{AxG_N(x,1/r_\perp^2)}{\pi R^2}$$



"small" or "large" depends on the gluon density of the target

"small" $(r_\perp Q_s \ll 1)$ **color transparency**

"large" $(r_\perp Q_s \gg 1)$ **black disk limit**

Geometrical scaling

The Golec-Biernat Wüsthoff model

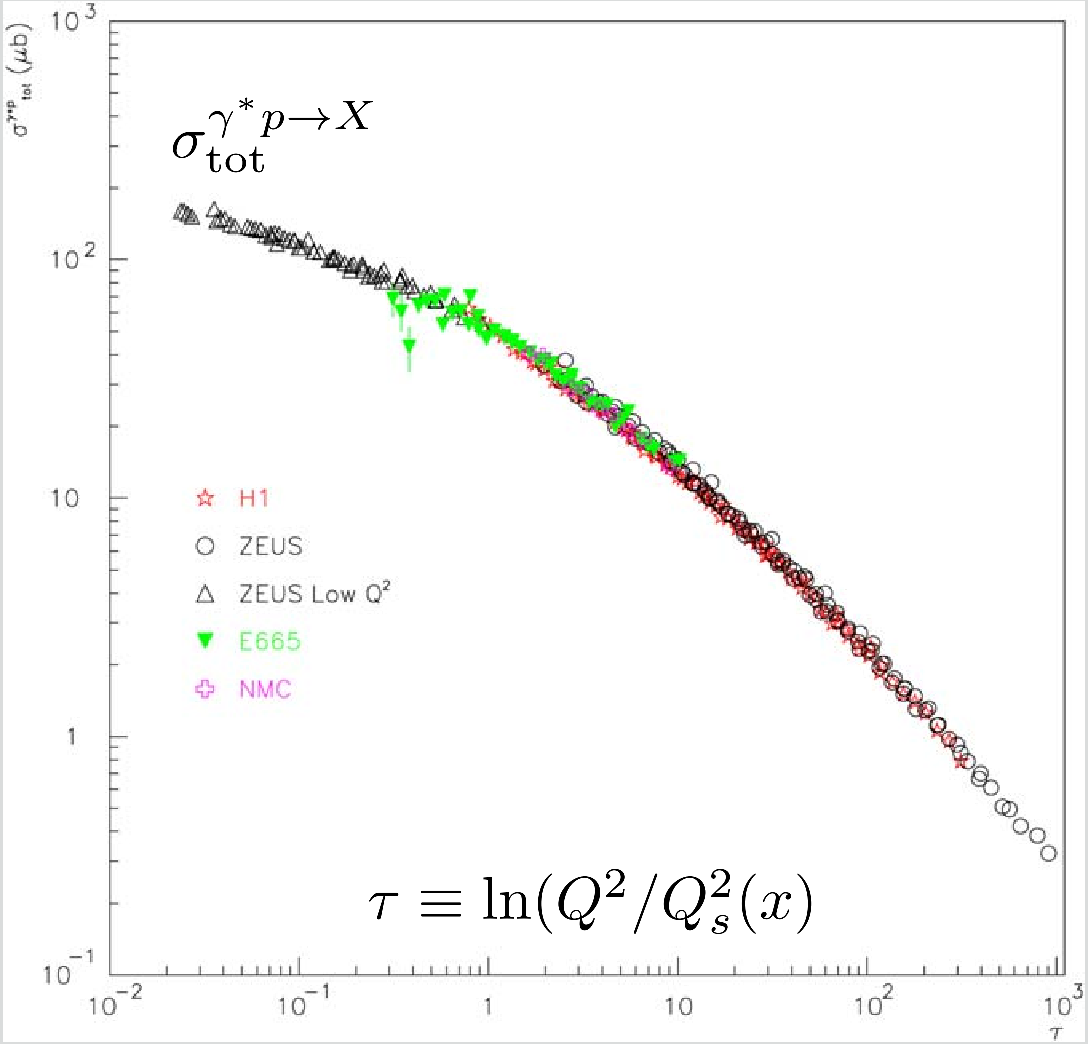
$$\sigma_{\rm GBW}(x, r_\perp) = \sigma_0 \left[1 - e^{-\frac{1}{4}Q_s^2(x) r_\perp^2}\right]$$

$$Q_s^2(x) \equiv Q_0^2 \, (x_0/x)^\lambda$$

$$\lambda \simeq 0.288$$

$$Q_0 = 1 \text{ Gev}$$

$$x_0 \approx 3 \times 10^{-4}$$



Geometrical scaling is the best experimental evidence for the existence of a saturation momentum

The McLerran-Venugopalan (MV) model

Dipole-nucleus interaction

$$S(\boldsymbol{b},\boldsymbol{r}) \equiv \frac{1}{N_c}\langle \mathrm{tr}\,(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle$$

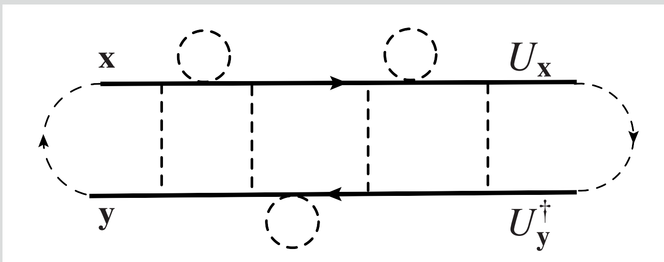
Average over the field fluctuations of the target

Assume small x gluons dominate, and are represented by a classical (random) field

Average over the (random) color field of the target

Assume Gaussian distribution W[A] for the random field

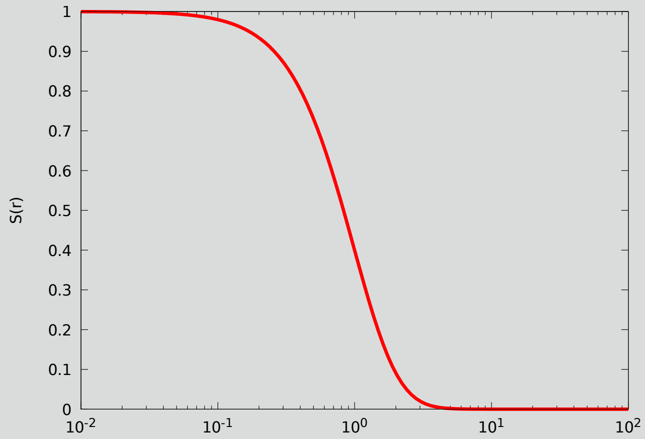
This allows for simple and explicit (non perturbative) calculations of the gluon distribution



MV model

How does a dipole sees a nucleus

$S(r_\perp)$



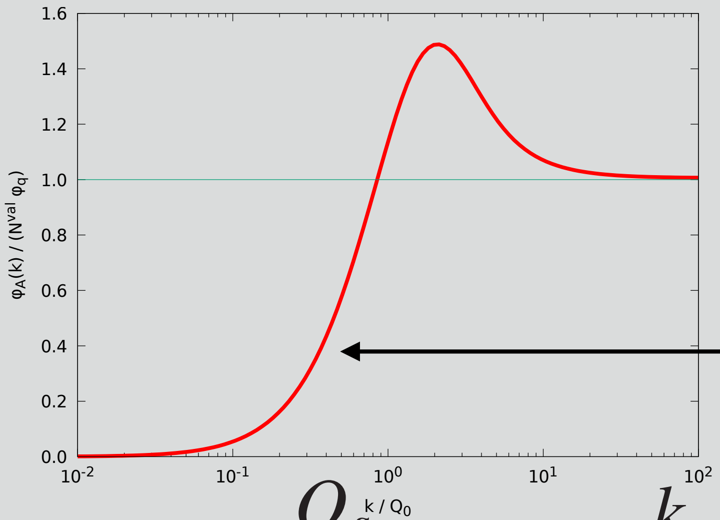
$r_\perp Q_s$ $\quad r_\perp$

Integrated gluon density is additive

$$xG_A(x, Q^2) = A\, xG_N(x, Q^2)$$
$$(Q^2 \gg Q_s^2)$$

Saturation in the MV model

$$\frac{\varphi_A}{N_{\text{val}}\varphi_q}$$



Q_s $\quad k$

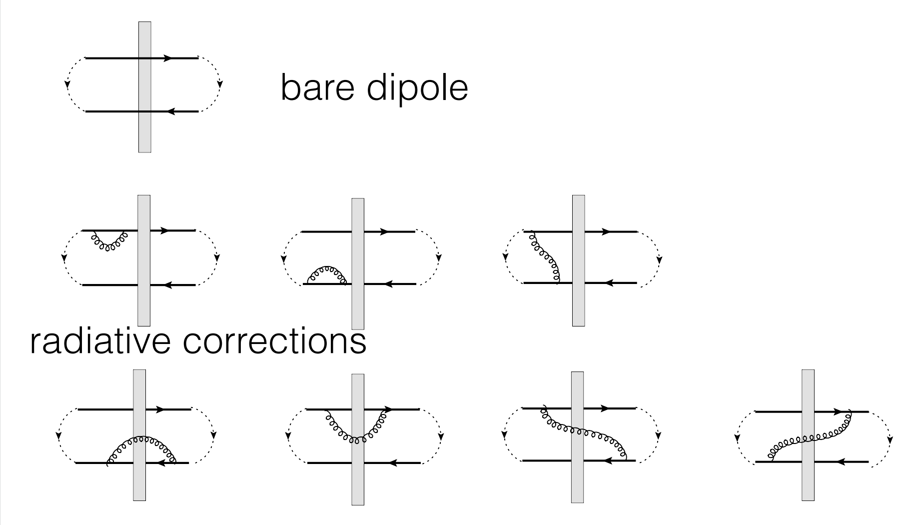
Low momentum modes are suppressed (screening, saturation)

$(k^2 \lesssim Q_s^2)$

Non linear evolution equations

(Leading order correction)

Boosting a "bare" dipole,
keeping the target untouched



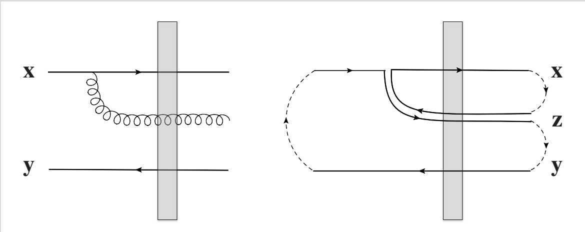
the S-matrix acquires a dependence on the rapidity interval Y

$$S(\boldsymbol{x}, \boldsymbol{y}) \longrightarrow S_Y(\boldsymbol{x}, \boldsymbol{y})$$

and obeys a non-linear evolution equation

$$\partial_Y S_Y(\boldsymbol{x}, \boldsymbol{y}) = -\frac{\alpha_s N_c}{2\pi^2} \int d^2\boldsymbol{z}\, \mathcal{K}_{\boldsymbol{xyz}} \left\{ S_Y(\boldsymbol{x}, \boldsymbol{y}) - S_Y(\boldsymbol{x}, \boldsymbol{z}) S_Y(\boldsymbol{z}, \boldsymbol{y}) \right\}$$

$$\mathcal{K}_{\boldsymbol{xyz}} \equiv \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}$$



Probabilistic interpretation

$$\mathrm{d}P = (\bar{\alpha}/2\pi)\mathcal{K}\,\mathrm{d}Y \qquad S_{Y+\mathrm{d}Y} = (1-\mathrm{d}P)S_Y + \mathrm{d}P\, S_Y^2$$

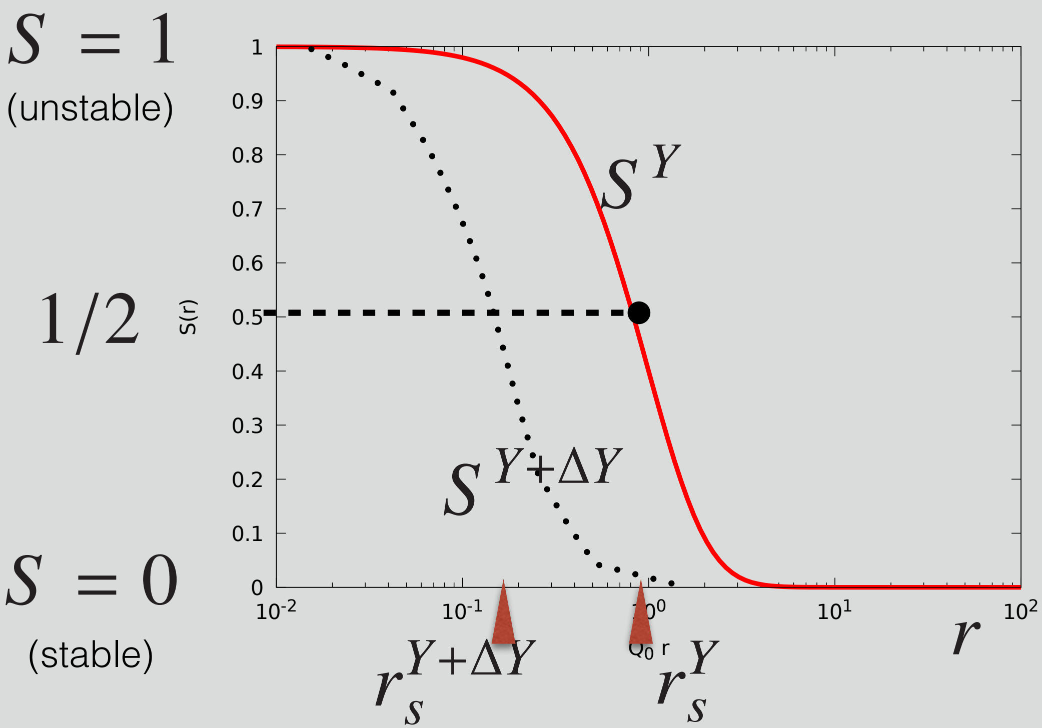
The Balitsky-Kovechegov equation

Take average over target field and assume factorisation

$$\partial_Y \langle S^Y_{\boldsymbol{x},\boldsymbol{y}} \rangle_{Y_0} = -\frac{\bar{\alpha}_s}{2\pi} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{xyz}} \left\{ \langle S^Y_{\boldsymbol{x},\boldsymbol{y}} \rangle_{Y_0} - \langle S^Y_{\boldsymbol{x},\boldsymbol{z}} \rangle_{Y_0} \langle S^Y_{\boldsymbol{z},\boldsymbol{y}} \rangle_{Y_0} \right\}$$

Non linear equation with two fixed points $\partial_Y S = -S(1-S)$

The solution interpolates between the two fixed points, without much change in shape (geometrical scaling)



The dipole knows (almost) nothing about the target. But the result can be interpreted as the scattering of a bare dipole on a target with an increased gluon density:

$$Q_s^{Y+\Delta Y} > Q_s^Y$$

B-JIMWLK, Color Glass Condensate, etc

This observation is central to many developments, where one interpret the radiative correction to the dipole S-matrix as a modification of the target.

The target is characterised by a distribution of "classical" fields, and this distribution evolves with rapidity (or energy):

$$W[A] \to W^Y[A]$$

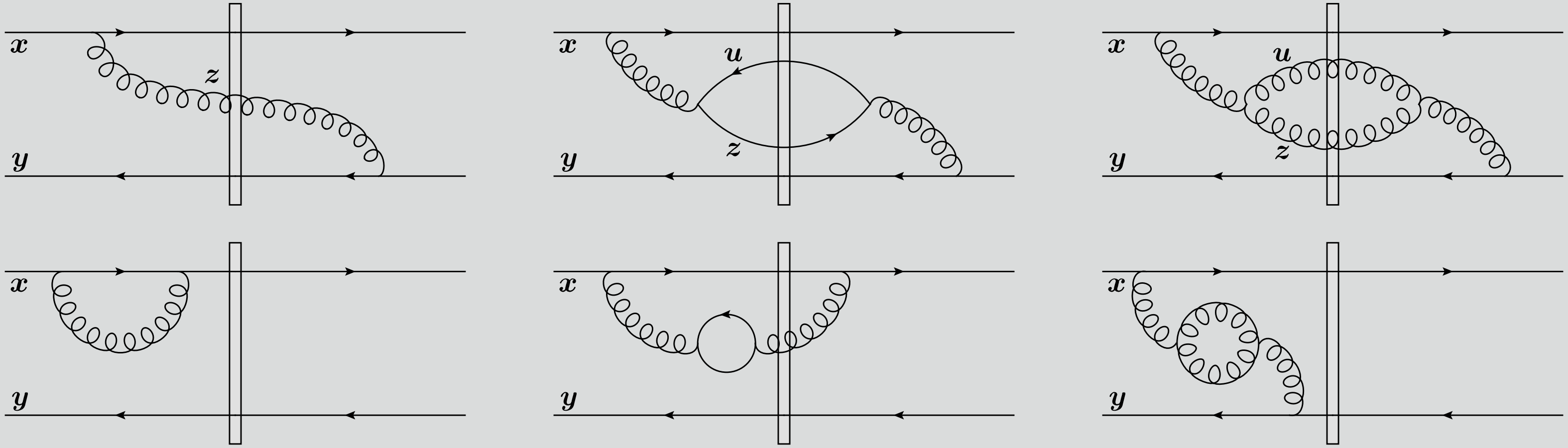
The evolution equation for $W^Y[A]$ is the B-JIMWLK equation (with the structure of a functional Fokker-Planck equation)

$W^Y[A]$ plays a role somewhat similar to that of pdf
(although theoretical status is less solid)

The B-JIMWLK equation summaries an infinite hierarchy of equations that describe the interactions of arbitrary products of Wilson lines with a dense target.

NLO corrections to evolution equations

(present state of the art)



(LO) (NLO)

E. Iancu, J.D. Madrigal, A.H. Mueller, G. Soyez, D.N. Triantafyllopoulos arXiv:1601.06525

Phenomenology of heavy ion collisions

(a few illustrative examples)

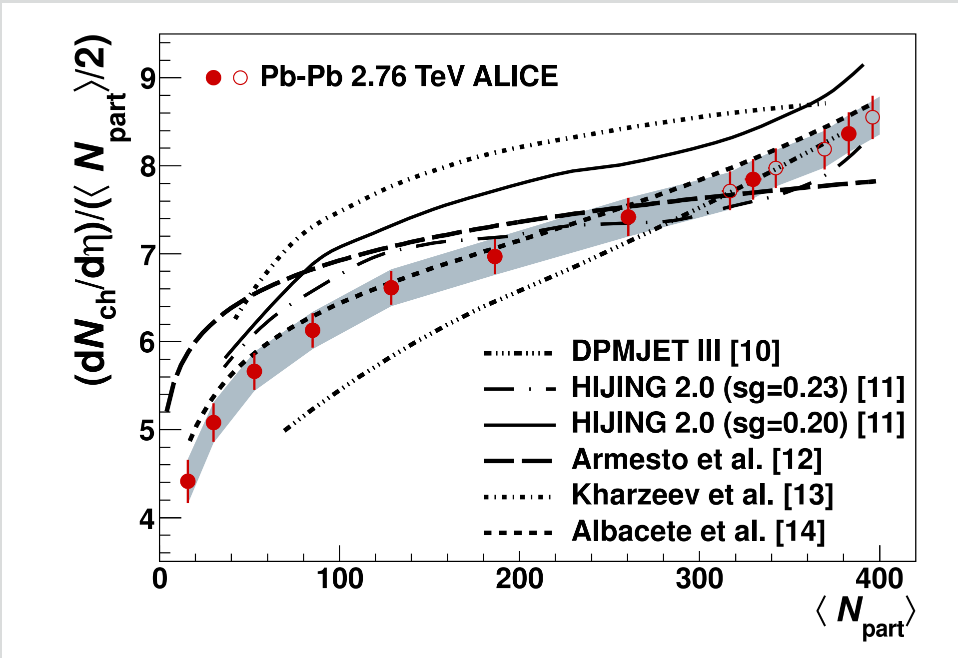
Saturation momentum is the unique momentum scale

Its energy dependence can be controlled from DIS) $Q_s^2 \propto A^{1/3} Q_0^2 (x/x_0)^\lambda$

Most gluon taking part in particle production have transverse momenta of order Qs

$$\frac{1}{\pi R^2}\frac{dN}{dy} \sim \frac{Q_s^2}{\alpha_s(Q_s^2)}$$

Charge particle multiplicity

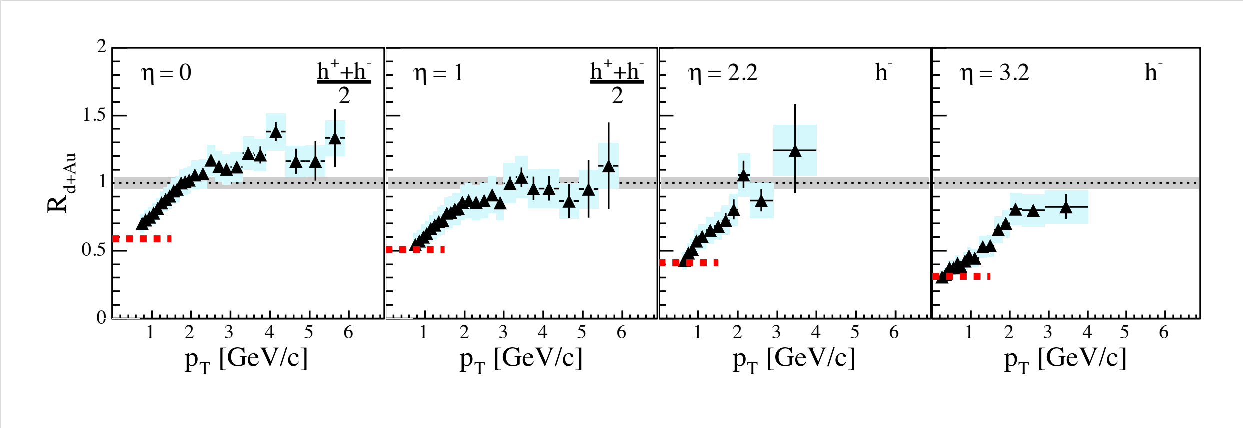


Phenomenology of heavy ion collisions

Forward rapidity (1)

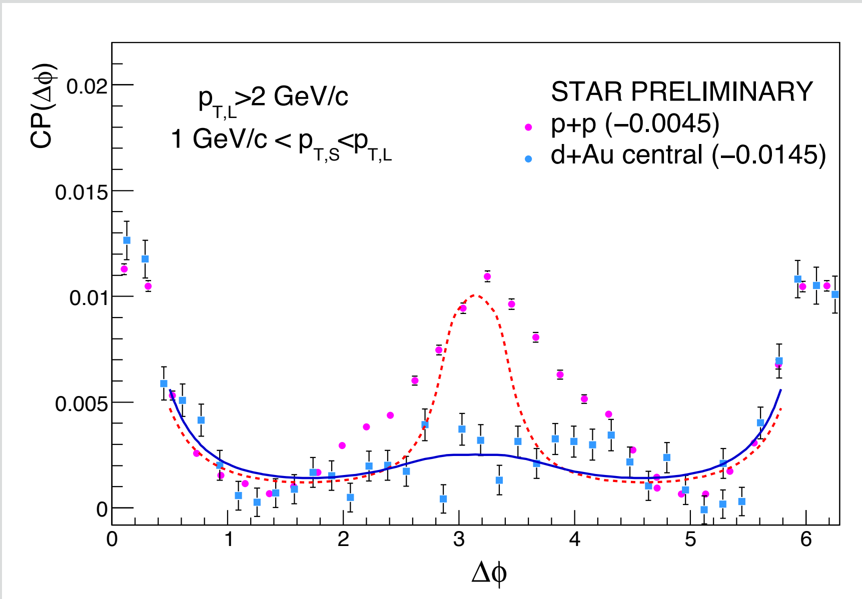
$$R_{\mathrm{dA}} = \frac{1}{N_{\mathrm{coll}}} \frac{dN_{dA}/d^2p_T\,dy}{dN_{pp}/d^2p_T\,dy}$$

Growing suppression (with increasing rapidity) of low momentum particles



Forward rapidity (2)

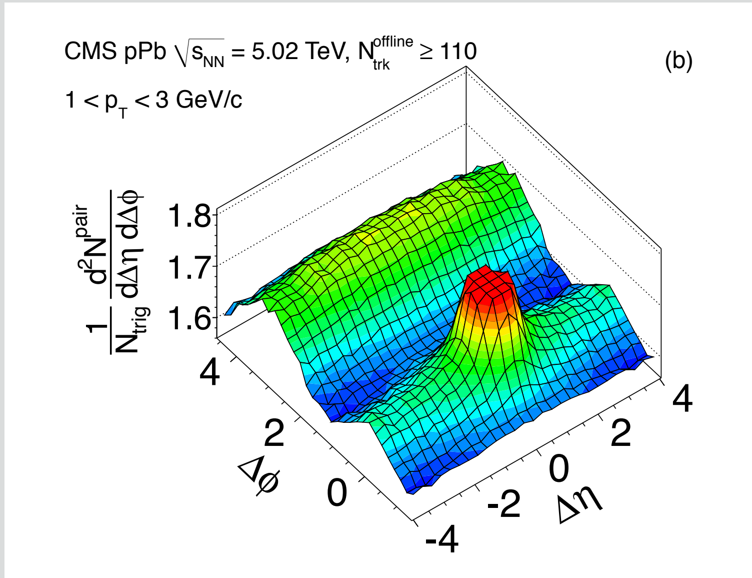
Suppression of back to back correlation



Phenomenology of heavy ion collisions

The "ridge"

Effect of collective flow,
or specific correlations in the "initial state" ?



Conclusions

- Gluon saturation is a subtle phenomenon, with many facets, and not fully understood.
  (tames the growth of gluon density with increasing energy, unitarizes the cross section, induces specific correlations in wave functions, etc)
- Saturation is characterised by one (transverse) momentum scale, the saturation momentum Q_s
  (separates dense and dilute regimes, threshold for breakdown of perturbation theory, momentum broadening, etc)
- High gluon densities potentially play an important role in heavy ion collisions (most produced particles have transverse momentum of order Qs, several phenomena are qualitatively (semi-quantitatively ?) understood in terms of high gluon densities, gluon densities determine the initial stages of collisions)