ATTRACTING AND RETAINING STUDENTS TO COMPLETE TWO- AND FOUR-YEAR UNDERGRADUATE DEGREES IN STEM: THE ROLE OF UNDERGRADUATE MATHEMATICS EDUCATION

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INTRODUCTION

The mathematical sciences are unique among the disciplines of Science, Technology, Engineering, and Mathematics (STEM) in that they serve both as important and evolving fields of advanced study and as a source of foundational knowledge required of every STEM major. While much could be said about the challenges and opportunities of introducing students to the potential of a major in the mathematical sciences, this report will only address the issues facing Mathematics as gateway to other STEM disciplines

Many STEM departments teach introductory courses required of students pursuing other majors, but none provide as extensive a role as gatekeeper as does Mathematics. Moreover, while few students engage in a serious study of engineering or the physical or biological sciences before high school, all have been subjected to mathematics instruction from their earliest schooling. They arrive at college or university having accumulated many years of assessment of their mathematical abilities and with deeply ingrained attitudes toward mathematics.

For many students heading into STEM careers, their mathematical performance in high school has been very high. For others, it has not. Each of these two groups faces its own particular set of challenges and obstacles. The bulk of this report is devoted to identifying what these are and how, if at all, they are being addressed. Perhaps the most striking finding about high performing students is the tremendous loss of confidence that hits them in the first term of university-level mathematics. This affects women particularly hard. They are twice as likely as men to abandon the calculus sequence after the first semester.

Lower performing students face the obstacle of College Algebra/Precalculus as well as the potential for needing to take precollege level mathematics (*i.e.* non-college credit bearing courses). All of these courses are notoriously ineffective at advancing students to the level needed for success in calculus. We will describe some efforts that show promise in modestly improving these courses. We also will explore several initiatives that completely transform them.

There also are issues affecting all students. These include class size, the need for curricular coherence, and the roles of instructors, student support services, and departmental efforts at continuous improvement.

This report concludes with a brief summary of the ways in which the mathematical community has responded to the challenges in the *Engage to Excel* report from the President's Council of Advisers on Science and Technology (PCAST 2012).

ISSUES FACING HIGH PERFORMERS

The Role of High School Performance: Nothing predicts success in a STEM field as effectively as K-12 mathematics performance. The best single predictor of successful completion of a STEM degree is whether or not the student studied calculus in high school (Chen 2009). Of those who declare a STEM major, 60% of those who have also studied calculus in high school will complete their STEM degree. This beats entrance examination scores (59% of those who were in top quartile complete the degree), high school GPA (52% of those with a GPA at or above B complete the degree), or parental education (51% of those with a parent who completed a bachelor's degree complete their STEM degree). Combined with the fact that one of the most accurate predictors of success in an Advanced Placement (AP) course is a student's 8th grade academic preparation (Dougherty and Mellow 2010), we see the importance of a student's entire mathematical trajectory.

Achieving success requires more than just doing well in mathematics. Tyson *et al.* (2007), using the longitudinal data available for the state of Florida, have found that the more rigorous the mathematics courses taken in high school, the more likely it is that a student will chose to major in a STEM field and will succeed in completing that major. Among students for whom the highest mathematics taken in high school was at the level of Algebra II or lower, there were no significant differences in completion of a STEM degree. But there was a significant difference, at the level of $p < 0.001$, between students who stopped after Algebra II and those who went on to complete a course that encompassed trigonometry. Each additional course in the progression toward calculus produced a highly significant increase in the percentage of students who went on to declare and complete a STEM degree until it peaked at 35% among high school students who had taken a course of calculus. Notably, the significance of this effect was still present when there was control for prior student performance (Madigan 1997). It is not just that better students take more rigorous courses. The challenge and rigor of these courses appears to be adding something to student ability to persevere through a STEM major. Madigan also documented that the benefit from rigorous courses is greatest for the students who have already demonstrated high levels of performance. As we shall see later, the key is not simply to take a rigorous course, but that the student is well prepared for and succeeds in that course.

These findings are also supported by those of Sadler and Tai (2007) who found that, controlling for student performance as measured by college entrance exams, AP Biology, Chemistry, and Physics give students an advantage in the particular discipline—Biology, Chemistry, or Physics—that has been tested but with little or no transfer to other disciplines. On the other hand, having taken AP Calculus had a highly significant positive effect on performance in all other science disciplines.

Calculus I is the mathematical entry point at the collegiate level for most STEM majors. Most STEM majors require at least this course. Many, in particular Engineering and the Physical Sciences, view Calculus I as the first of three, four, or more mathematics courses that build sequentially, each requiring a degree of mastery of the previous course.

It therefore should come as no surprise that the national study of Calculus I instruction undertaken by the Mathematical Association of America (MAA), *Characteristics of Successful Programs in College Calculus* (CSPCC), found that students enrolled in Calculus I had been high performers in mathematics in high school. Most had a GPA of 3.5 or higher in their high school mathematics course, almost all were in the top quartile of the college entrance exam, and over half had taken calculus in high school (Bressoud *et al* 2013).

Calculus in High School: If taking rigorous mathematics classes, especially calculus, in high school is the best single predictor of successful completion of a STEM degree, then one approach to improving success must be to increase access to such programs. There are real differences in access. Among Calculus I students at research universities, 88% attended a high school where calculus instruction was available, either as AP, IB, Dual Enrollment, or some other program. In contrast, only 73% of students at two-year colleges came from high schools where such instruction was available (Bressoud *et al* 2013). Schools with 0–25% of their students eligible for free/reduced-price lunch offered an average of 1.4 courses in AP Mathematics or Statistics.

Those with 75–100% of their students eligible offered an average of 0.7 such courses (NSB 2014). From the College Board (2014), while Black students made up 14.5% of the graduating class of 2013, they accounted for only 9.2% of the students taking AP exams and a tiny 4.6% of the students who earned a 3 or higher on an AP exam.

The College Board, working with programs such as the National Math and Science Initiative (NMSI), has been actively promoting the spread of AP to schools that serve significant numbers of African-American and Hispanic students, preparing new teachers to work in these schools and providing professional development for the teachers already there. NMSI reports significant and accelerating progress in enabling students to successfully complete AP Calculus[2].

Another approach to rigorous mathematics courses relies on dual enrollment or early college programs. Early College High School[3] was started with funding from the Gates Foundation. Dual enrollment programs, enabling high schools to partner with local colleges, usually two-year colleges, for the purposes of introducing college-level classes into the high schools, are now common across the country.

The result of all of this activity has been an enormous expansion in the number of students who now study calculus while in high school. Each year, almost 400,000 students take the AP Calculus exam and roughly 700,000 high school students begin the study of some form of calculus (Bressoud 2014).

Like any large-scale movement, this push toward calculus in high school has both benefits and unintended negative consequences. For college-bound students at privileged schools, the expectation is that they will study calculus on or before their senior year, creating an even greater disadvantage to those students who have not had access to such a course when they get to college and must compete against students who have at lest some familiarity with this material. Worse, there is now considerable pressure from parents, school boards, and administrators to get as many students into calculus as possible (Farkas and Duffett 2009). Not all of these students are adequately prepared for this course. Dougherty and Mellow (2010) have shown that students who take but do not pass an AP course have no better chance of success in college than those who did not take such a course.

In addition, the push to enroll as many students as possible in calculus can result in hollowed-out courses that do little to prepare students for the mathematics needed for STEM. The Longitudinal Studies of the National Center for Education Statistics have revealed that 31% of those who have studied calculus in high school take Precalculus in college (NELS:88). Even more discouraging is that 13.5% of those who study calculus in high school take remedial mathematics—precollege level mathematics—when they get to college (NCES 2013).

The lack of accountability for the quality of high school calculus is particularly problematic in dual enrollment programs that suffer from very uneven professional development for and oversight of the faculty who teach these courses (Bressoud 2007). The National Alliance of Concurrent Enrollment Programs[4] provides accreditation for such programs and holds them to

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[2] See www.nms.org/Programs/ResearchResults.aspx

[3] See www.jff.org/sites/default/files/publications/materials/Early-College-Expansion_031714.pdf

[4] www.nacep.org

rigorous standards, but only 89 of the concurrent or dual enrollment programs across the United States are accredited.

It was in response to this push to calculus and the recognition that prior preparation is the key to success that MAA and the National Council of Teachers of Mathematics (NCTM) issued their joint statement on the importance of adequate preparation before beginning the study of calculus (Bressoud 2012).

The Crisis of Confidence: There is no better proxy for high performance in high school mathematics than enrollment in a STEM major at a research university[5]. These are also the institutions that produce the largest share of STEM majors. Here, Calculus I students were almost uniformly full-time students in their first semester at college, 17 to 19 years old, and with an average GPA over 3.8 in high school mathematics. Most were in the top decile of the mathematics portion of the college entrance exam, and over 70% had studied calculus in high school (Bressoud *et al* 2013).

Despite these advantages, 25% of the students who take Calculus I at a research university receive a D or F or withdraw from the course (DFW), and another 23% receive a C (Bressoud *et al* 2013), a grade that is widely perceived as a signal that one is not adequately prepared to succeed in Calculus II (Tyson 2011).

Perhaps the most striking finding from CSPCC was that even among the students who successfully completed this course, there was a dramatic drop, half a standard deviation, in student confidence in their mathematical abilities (Sadler *et al* 2014). Confidence is important for all students, but it is particularly important for women. Among women of equal ability, those recognized as being a "mathematics person" were more than eight times as likely to choose or persist in an engineering major as those who were not (Cass 2011).

Rasmussen and Ellis (2013) have found that women who complete Calculus I with a grade of A or B are less likely than men to continue on to Calculus II. The reasons may be complex. Women in Calculus I are much more likely to be seeking a degree in the biological sciences than are men, and the biological sciences often require only a single semester. Nevertheless, of those who entered Calculus I with the intention of continuing through Calculus II and who earned an A or B in Calculus I, at the end of the term 20% of the women decided not to continue to Calculus II, while only 10% of the men made this decision.

It is worth noting that among high performing students of the same gender, neither Tyson (2011) nor Rasmussen and Ellis (2013) observed any differences by race or ethnicity. They also found the disparity between men and women to be consistent across all race and ethnic categories.

ISSUES FACING LOWER PERFORMERS

Racial and Ethnic Disparities: If research universities provide a useful proxy for high performing students, two-year colleges are a proxy for lower performing students. Lower but not necessarily low performing: Students who make it into Calculus I at a two-year college may not

[5] For the purposes of the CSPCC study, "research university" was defined as an institution that offers a PhD in Mathematics.

have been as sterling in high school as those at the research universities, but they were still good students: average high school math GPA of 3.4, most in the top quartile on the college entrance examinations, 24% studied calculus in high school. In contrast to the research universities, only 25% of Calculus I students in two-year colleges are in their first year of college. Most have had to fill in some gaps in their mathematical preparation (Bressoud *et al* 2013), and, as the next subsection shows, even just needing Precalculus can place a significant stumbling block in the path of these students.

Two-year colleges are also where we find the largest concentrations of Black or Hispanic students. At research universities, these categories account for 14% of Calculus I enrollment. At two-year colleges, they make up 28%. While race and ethnicity do not appear to play a role for high performing students, they are reflected in discrepancies in rates for seeking and completing STEM degrees. For students of Asian descent, 47% declare a STEM major and 40% of them complete it. For White, Black, or Hispanic students, the percentage declaring a STEM major is comparable, 20–23%, but completion rates are much higher for White students, 44%, than for Black or Hispanic students, 32–33% (Chen 2009). There may be cultural factors explaining the high rates for Asian-American students, but clearly access to quality K-12 education is a major explanatory variable.

College Algebra and Precalculus: College Algebra and Precalculus are titles of courses that usually carry college credit and that are usually seen as preparation for calculus and other mathematics courses required for a STEM major. Some post-secondary institutions offer both, often as a sequence; others offer either one or the other as the last mathematics class before calculus. Thus, College Algebra could be anything from what is essentially a repetition of high school Algebra II to the final preparation for calculus. When referring to the first college credit-bearing course in the progression toward calculus, we shall call it College Algebra. When designating the last class before calculus, we shall call it Precalculus. At many colleges and universities, these are the same course.

College Algebra often serves the role of default option for college students who must satisfy a distribution requirement in mathematics. As Herriott and Dunbar (2009) have discovered, only 10–20% of the students in College Algebra intend to pursue a STEM major that requires a yearlong sequence of mainstream calculus. While there is a strong movement to direct those students who see this as their last mathematics course toward quantitative reasoning or statistics instead, College Algebra still suffers from the wide disparity in the goals of the students who take it.

Even for those who intend to take calculus, Precalculus is problematic. First of all, good placement procedures are rare (Carlson *et al* 2010). Under the auspices of MAA and with support from NSF, Carlson, Madison, and West (2010) have developed a Calculus Concept Readiness assessment that is based on the best research in undergraduate mathematics education on the conceptual misunderstandings as well as technical difficulties that are most likely to hinder students from successful completion of Calculus. The CSPCC study has revealed that most colleges and universities still rely on homegrown assessments.

Second, as Thompson *et al* (2007) discovered at Arizona State, the fact that a student has successfully passed Precalculus is no guarantee that she or he will enroll in calculus, even when it is required for the intended major. Among those who earned a C or higher in Precalculus, 38% of those with a declared engineering major never enrolled in calculus. The percentages were higher in other STEM fields: 62% in Life Sciences, 56% in Mathematical Sciences, and 55% in Physical Sciences. There are similar patterns at the University of Nebraska-Lincoln (UNL) and both public and private post-secondary institutions in Illinois. Only half of the STEM-intending students who took Precalculus at UNL also enrolled in Calculus I. Only 40% of them made it as far as enrolling in Calculus II (Herriott & Dunbar 2009).

Finally, there is evidence that those who take Precalculus and then enter Calculus I have received little benefit from this extra course. Sonnert and Sadler (2014) have applied discontinuity regression to investigate the effect of placement in Precalculus on performance in Calculus I, examining a range of possible cut-offs measured in terms of high school performance: a weighted combination of SAT/ACT Math scores, information on whether precalculus and/or calculus was studied in high school, and grades in these courses. They found that for students between 0.7 standard deviations (SD) below the mean and 0.1 SD above, taking Precalculus in college raised their Calculus I grades by 1 to 2 percentage points, an increase that was not statistically significant. For students below 1 SD below the mean, taking Precalculus in college actually *lowered* their Calculus I grade. Once a student reached 0.3 SD above the mean, taking Precalculus in college lowered the Calculus I grade by a statistically significant 5 or more percentage points. Thus, there is a narrow range of students who may benefit from placement in Precalculus, but, when they exist, these benefits are small. The findings of Sonnert and Sadler are also supported by an unpublished dissertation by Ruddock (1996) (see also Hsu *et al* 2008) that showed that even when controlling for scores on SAT Math, students who went directly into Calculus I were more likely to earn an A or B than those who first took Precalculus in college.

One of the problems with Precalculus is that for most students this is a review of what they did not learn in high school, but taught at a faster pace with higher expectations. Many remedies have been suggested for the problems associated with the general ineffectiveness of Precalculus, many of which can be found in *A Fresh Start for Collegiate Mathematics* (Baxter-Hastings 2006). There is evidence of improvement from modeling-based courses (Ellington 2005), for the use of clickers (Brusi *et al.* 2013), for Inquiry-Based Learning (Cooper *et al.* 2012), and for using technology to move most of the traditional instruction out of the classroom and replace class time with active hands-on learning, individual assistance, and small group projects (Thompson & McCann 2010).

A particular version of the use of technology in Precalculus is ALEKS[6]. It is an adaptive testing platform that can be used to evaluate student knowledge of fine-grained topics up to and including Precalculus. Building a Precalculus course around ALEKS has proven to be very successful at several universities including the University of Illinois, Urbana-Champaign (Ahlgren and Harper 2011, see also Hagerty *et al.* 2005, 2010). ALEKS has several advantages: Exams are individualized and offered online so that students can take them where and when they so wish, and adaptive testing means that there is an opportunity to drill into particularly competencies to assess exactly what students can and cannot do.

At Illinois, students must pass the ALEKS assessment before being admitted to calculus. Over the three-month period before Calculus I begins, students may retake the sections of the assessment in which they have not demonstrated proficiency as many times as they wish, giving them a chance to come up to speed and separating the students who have learned the material but need a refresher from those who never mastered it. Illinois has discovered that students who fail

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[6] **A**ssessment and **LE**arning in **K**nowledge **S**paces. www.aleks.com/

to qualify over the three-month period generally acknowledge that this assessment has uncovered deficiencies that need to be addressed. As a result, they are more willing to enroll in Precalculus. Using ALEKS as the core of the Precalculus class allows students to move through these assessments at their own pace, focusing on the topics where they need the most work.

A totally different approach to the problems of Precalculus that is now employed by many post-secondary institutions is a stretched-out version of Calculus I. Such a course, spread over two terms, imbeds review of precalculus topics on a just-in-time basis. This combines new and challenging mathematics with the opportunity to backfill areas of weakness. Materials for such a course were first developed at Moravian College in the 1990s (Sevilla and Somers 1993). At some universities such as Worcester Polytechnic Institute, it has been a great success. At others, such as Rutgers University, the program was perceived early on as remedial and so rejected by the students who might have benefited[7].

Programs in Support of At-Risk Students: As Uri Triesman discovered 40 years ago when he began to study the obstacles facing Black and Hispanic students in calculus, many students who have the necessary background to succeed in calculus still fail this course. He discovered a considerable social component contributing to these failures. Comparing African-American and Chinese-American students at Berkeley, he discovered that while African-American students kept their educational and social lives separate and almost always worked in isolation on their mathematics, Chinese-American students built an active social life around their study groups. Out of his efforts to build self-supporting intellectual communities among at-risk students came the Emerging Scholars Program (ESP) (Hsu *et al* 2008).

One of basic tenets of ESP is that it is not remedial. Students identified as promising but at-risk are invited to participate in a program of extra enrichment where they are challenged to work in community to solve difficult and unfamiliar problems. When it works, the results are remarkable. Asera (2001) and Hsu *et al* (2008) have provided overviews of both successes and failures of ESPs with suggestions for what it takes to make these programs succeed.

One of the hallmarks of ESP is the reliance on the students to wrestle with the key concepts of the mathematics and construct their own understandings. This also is the guiding principle behind Inquiry Based Learning (IBL). IBL, as practiced in mathematics, is a broad umbrella that covers a wide variety of active learning strategies, but always active learning with intention. IBL incorporates individual sense-making, building toward personal construction of key ideas. In mathematics, this effort is supported by the Academy of IBL[8]. This organization provides curricular materials, pairs new practitioners with seasoned mentors, provides small start-up grants for those who wish to try implementing IBL, and runs a large annual national conference. IBL centers have been established at University of Michigan, University of Chicago, University of Texas-Austin, and University of California-Santa Barbara.

Sandra Laursen of the *Ethnography & Evaluation Research* group at the University of Colorado, Boulder has been studying the effectiveness of IBL at the four IBL centers and has found that this approach significantly improves not just student success in the IBL course but also student persistence through and success in the courses following those in which IBL was experienced. The benefits are particularly strong for students from at-risk groups (Kogan and Laursen 2013).

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[7] Private communication from a Mathematics faculty member at Rutgers.

[8] www.inquirybasedlearning.org/

Alternate Pathways for Underprepared Students: Most of the students who were low performing in high school and who seek a STEM career enter a two-year college to make up this deficiency[9]. In fact, many public universities have abandoned trying to meet the needs of these students, directing them to local two-year colleges. These students face a long and difficult succession of courses that must be negotiated before arriving at calculus. It is typical for the pass rate in these courses to be around 50%. When combined with an average persistence rate of 50%, only a quarter of the students make it from one course to the next. If we then compound this with the fact that at two-year colleges preparation for calculus is often a two-, three-, or even four-course sequence, we see that the likelihood of completing a STEM degree becomes vanishingly small.

Several organizations, most notably the Carnegie Foundation for the Advancement of Teaching and the Dana Center at the University of Texas have developed curricular programs designed to move these students through to the successful completion of the mathematics they actually need for their intended careers. The Carnegie programs, *Community College Pathways*[10], and the Dana Center's *New Mathways Project*[11] have demonstrated greatly improved success in getting underprepared students through to completion of an Associate's degree or to transfer to a four-year institution, but they have only begun to tackle the problem of preparing these students to succeed in calculus.

ADDITIONAL ISSUES FACING ALL STUDENTS

Class Size: Most large research universities find themselves between the Scylla and Charybdis of either using regular faculty with very large sections, where student engagement becomes problematic, or relying heavily on adjunct faculty and graduate students, many of whom are inexperienced in the classroom or poorly motivated. Data collected in the MAA's national study of calculus (CSPCC) show that large classes can work well at top-tier universities with well-prepared and highly motivated students. This is especially true when attention is paid to student engagement in the lecture hall—as with the use of clickers—and to the content and quality of recitation sections. For over 25 years, the University of Michigan has shown how to teach calculus effectively in small sections with large numbers of adjunct faculty and graduate students. It requires a very intentional program of extensive training and close supervision with support from and oversight by departmental faculty (Speyer 2012).

Curricular Coherence: A perennial complaint of the calculus curriculum is that it lacks coherence (Oehrtman *et al* 2008). To most students, calculus is a sequence of obscure and poorly motivated procedures that must be mastered until tested, after which they can be quickly forgotten. In the late 1980s and early '90s, the National Science Foundation poured considerable resources into the Calculus Reform effort in an attempt to correct this (Tucker and Leitzel 1995, Ganter 2001). Some of the fruits of this effort have lasted, including the recognition that competency with algebraic manipulation must be combined with an understanding of how these

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[9] From 2000 to 2010, the number of students enrolled in precollege level mathematics at two-year institutions rose from 763,000 to 1,150,00 while precollege level enrollments at four-year institutions dropped from 218,000 to 209,000 (Blair *et al* 2013).

[10] See www.carnegiefoundation.org/sites/default/files/pathways/CCP_Descriptive_Report_Year_2.pdf

[11] See www.utdanacenter.org/higher-education/new-mathways-project/

relate to the graphical and numerical manifestations of the same ideas. Greater reliance on technology and opportunities to explore challenging problems that mimic real-world situations have also endured.

Some of the accomplishments of Calculus Reform have only survived in pockets, including the attempt to recast this course around the motivating theme of differential equations. This was common in many of the early Calculus Reform efforts, but the difficulty of articulating such an approach with the more common treatment of Calculus—and thus the difficulty this created for transfer students—together with deep suspicion from many math faculty about so radical a change, mean that it has disappeared from all but a few places. The US Military Academy at West Point is the only post-secondary institution that has held onto this approach for the past quarter-century[12]. It works extremely well for them, providing a level entry for most cadets. The first semester looks nothing like what they may have studied as calculus in high school, and it effectively gives meaning and direction to the study of calculus. A few small colleges have also adopted this approach. Macalester College (Flath *et al* 2013) has used it with great effect as an entry to and motivator for the study of calculus.

Role of the Instructor: The MAA's national study of Calculus I (CSPCC) found extremely strong evidence for the importance of the quality of the instructor in maintaining student confidence in their mathematical abilities and student interest in persisting into further mathematics (Sonnert *et al*, 2014). This study found that progressive pedagogies—group work, student reports in class, challenging and unfamiliar problems—could be helpful, but only if built on a foundation of good teaching that includes such basic qualities as encouraging student questions and enlisting their feedback to ensure they have understood, discussing multiple methods of solution and applications, and being accessible outside of class. As James Fairweather (2008) has pointed out, "It may be that the greatest gain in aggregate student learning in STEM is achieved not through the adoption of optimal teaching practices in each classroom but through the elimination of the worst practices."

Role of Student Support Services: A final notable component of the highly successful calculus programs identified by CSPCC was the student support services. The best institutions had very heavily utilized tutoring centers that combine a central location, a conducive atmosphere that encourages students to drop in as a place to do their work, and tight connections between center staff and faculty to identify students who might be struggling and to encourage them to take advantage of the support services.

Role of Continuous Improvement: CSPCC also found that the best and most supportive universities are constantly assessing the effectiveness of their programs and implementing adjustments that are often small but can make a significant difference. For example, the University of Michigan has introduced a Precalculus class that does not begin until after the results from the first Calculus exam are in, so that students who discover they are in over their heads can drop back and only lose a single semester rather a full year.

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[12] The first Calculus course at USMA is MA103, *Mathematical Modeling and Introduction to Calculus*. See www.usma.edu/math/SiteAssets/SitePages/Core Math/CMB-AY2013.pdf

EFFORTS OF THE MATHEMATICS COMMUNITY TO ADDRESS THE CONCERNS IN *ENGAGE TO EXCEL*

The following are the four recommendations in *Engage to Excel* (PCAST 2012) that are addressed, at least indirectly, to the mathematical, together with a summary of actions now underway from the Mathematics community.

1. Catalyze widespread adoption of empirically validated teaching practices.

There are several large efforts to catalyze such adoption within the community of mathematical scientists:

A. The Mathematical Association of American (MAA) is completing and will be publishing in 2015 its latest Curriculum Guide, describing best practices and providing examples of worthy programs.

B. Investing in the Next Generation through Innovative and Outstanding Strategies (INGenIOuS)[13] began in 2013 as a joint effort of the American Mathematical Society (AMS), American Statistical Association (ASA), MAA, Society for Industrial and Applied Mathematics (SIAM), and the Institute of Mathematical Statistics, with support from NSF, to find and disseminate innovative solutions to the problem of preparing undergraduate and graduate students for the challenges of the 21st century workplace. It is inspired by the *Partnership for Undergraduate Life Sciences Education* (PULSE)[14].

C. The Carnegie Corporation of New York and the Sloan Foundation are underwriting another effort that began in 2013, *Transforming Post-Secondary Education in Mathematics* (TPSE)[15] to analyze the effectiveness of current curricula to meet the needs of students, to evaluate the potential impact of online education, to change faculty reward structures so that they promote improvement of mathematics education, and to open pathways to STEM success. This effort, run in conjunction with the Institute for Advanced Study, is led by prominent research mathematicians.

D. MAA's program for new faculty, *New Experiences in Teaching* (Project NExT)[16] includes as an important part of its mission the introduction of new faculty to active learning strategies and other empirically validated teaching practices. It also connects them with mentors who can support them as they implement these approaches.

2. Advocate and provide support for replacing standard laboratory courses with discovery-based research courses.

While Mathematics is usually not taught as a laboratory science, it does take advantage of discovery-based methods that fall under the umbrella of Inquiry Based Learning (IBL) that is described under the section on **Programs in Support of At-Risk Students**. While IBL is

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13 www.ingeniousmathstat.org

14 www.pulsecommunity.org

15 www.tpsemath.org

16 www.maa.org/about-maa/support-maa/project-next

particularly effective for these students, it has been adopted and its benefits are currently being assessed at a wide variety of post-secondary institutions.

3. Launch a national experiment in postsecondary mathematics education to address the math preparation gap.

There are at least two major programs designed to address the math preparation gap, the *Community College Pathways* and the *New Mathways Project*, both described under **Alternate Pathways for Underprepared Students**.

4. Encourage partnerships among stakeholders to diversify pathways to STEM careers.

The National Math and Science Initiative, described under **Calculus in High School**, is a good example of business and industry working with non-profits and academic institutions to create opportunities for under-served students.

There also is increasing awareness of the need for cross-disciplinary efforts. This is particularly evident in the AAU *Undergraduate STEM Education Initiative*[17] and the APLU *Science and Math Teacher Imperative*[18]. The MAA's program for *Integration of Strategies that Support Undergraduate Education in STEM*[19] is providing opportunities for the sharing of information and cooperation among the STEM disciplinary societies as they move to improve undergraduate education.

Finally, the community of Research in Undergraduate Mathematics Education[20], which operates under the umbrella of MAA, continues to be an important source of evidence for best teaching practices and insight into how and why they work. MAA regularly disseminates information from these researchers to the entire mathematical community through its publications and meetings. They also are currently gearing up for the first issue of the *International Journal of Research in Undergraduate Mathematics Education*[21].

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[17] www.aau.edu/policy/article.aspx?id=12588

[18] www.aplu.org/smti

[19] serc.carleton.edu/issues

[20] sigmaa.maa.org/rume/Site/About_SRUME.html

[21] www.springer.com/education+%26+language/mathematics+education/journal/40753

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