Mathematics Education and Young Dual Language Learners

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Demographics on Dual Language Learners in the U.S.

In this report, I use young dual language learners (DLLs) to refer to "...children learning two or more languages at the same time, as well as those learning a second language while maintaining the first (or home) language" (Administration for Children and Families and U.S. Department of Health and Human Services, 2013, p. 3). According to the National Academies of Sciences, Engineering and Medicine (2017), the term dual language learners (DLLs) is used to include children from birth to age 5, and English Learner (EL) is used for students enrolled in grades Pre-K-12. There are discrepancies in the number of young DLLs reported from birth to age 5 because there are different surveys that have been used to capture language skills. A report by O'Hare (2016) also indicates that there are discrepancies in counting young learners from birth to under age 5 because they may be living in difficult-to-count places, such as multi-unit buildings or a high number of renters, there is high mobility, or households may have multigenerational families or complex family relationships. Overall, however, the U.S. Department of Education (2016) has reported 4.8 million ELs in the U.S. during the 2014-2015 school year, which represents about 10% of the student population. In addition, there are over 400 languages spoken in the U.S., with Spanish being the most commonly spoken language by over 75% of ELs. The next most commonly used languages include Arabic, Chinese, and Vietnamese, each spoken by about 2% of ELs.

There are three main sections that are included in this report. I begin by providing a brief background on research that has contributed to our understanding on young learners and mathematics education, as well as language development. The second section focuses on reviewing the literature on promising teaching practices that can advance the mathematics education of young DLLs in particular. The third section includes the role of the second language teacher and professional developers in supporting the mathematics learning of young DLLs and professional learning opportunities for teachers. Based on the review of the literature, I conclude with a summary of recommendations for teaching mathematics to young DLLs.

Young Learners, Mathematics Education, and Language Development

Before young learners experience formal schooling, they engage in exploring mathematics concepts and processes, as well as on developing their language through interactions with others. They count, sort, and classify objects; they notice patterns and shapes through daily experiences and play (Ginsburg, 2006; Wager & Parks, 2014). Similarly, young learners develop their home and second languages through sociodramatic play (Salinas-Gonzalez, Arreguín-Anderson, & Alanís, in press). Thus, teachers must listen to what children communicate in order to understand topics that are of interest / relevance in their everyday lives and connect these to their mathematics teaching practices. Having a strong foundation in early mathematics education for later learning has been well documented (Duncan et al., 2007; NAEYC & NCTM 2002/2010; Watts et al., 2014). Furthermore, Carpenter et al. (1999) have shown that when young children enter kindergarten and primary grades they can engage in

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solving complex problems using different operations, including multiplication and division. These findings include young Latinx students when they are given an opportunity to engage in similar problem solving activities (Turner & Celedón-Pattichis, 2011).

To provide a strong foundation in early mathematics education, the National Research Council (2009) recommended three areas that should be considered. One recommendation involves a focus on developing two mathematical strands: number concepts, which includes whole numbers, operations, and relations; and geometry, spatial relations, and measurement, with more time devoted to developing number concepts. A second recommendation includes "all early childhood programs...provide high-quality mathematics curricula and instruction" (NRC, 2009, p. 3). A final recommendation to promote mathematics learning in young learners includes forming collaborations between schools and families/communities.

Cognitively Guided Instruction (CGI) studies have been instrumental in understanding young children's mathematical thinking and the strategies they use to solve a range of problems (Carpenter, Fennema, Franke, Levi, and Empson, 1999, 2014). More recently, Math Thinking Conversations have been used to assess children's mathematical thinking and observe how they respond to specific tasks (Freeman, Ginsburg, Bautista, & Uscianowski, 2017). At the center of this work is the need for teachers to understand strategies students use to problem solve using basic operations, rather than tell children how to solve problems, and use that information to clarify, support, and extend students' mathematical thinking. Furthermore, listening to children's mathematical thinking is important for all students but particularly essential for DLLs. A critical finding of these studies points to the underestimated capacity of young children to engage in challenging mathematical problem solving.

Although there are several research studies that document young learners' experiences in mathematics classrooms, there is a paucity of studies that are particular to young children who are DLLs and who are learning mathematics. The following section presents teaching practices that have been shown to be promising in advancing the mathematics education of young DLLs.

Promising Approaches Used in Teaching Mathematical Concepts to Young Dual Language Learners

Research conducted by several scholars through the Center for the Mathematics Education of Latinos/as (CEMELA)'s Kindergarten Study drew from CGI as a framework to understand children's mathematical thinking and from Funds of Knowledge to integrate language and culture into mathematics teaching and learning (Turner, Celedón-Pattichis, & Marshall, 2008). This study included 45 dual language kindergartners in post assessments conducted similarly to Carpenter et al.'s (1999) study. These researchers also studied teaching practices that teachers drew from to support young DLLs in learning mathematics.

Turner et al. (2008) found that the majority of Latinx kindergarteners could solve the most basic addition and subtraction problems (80% and 73% respectively), and join change unknown, multiplication, and partitive division problems were each solved by approximately half of the students (56%, 49% and 43%, respectively). The most difficult problem types for students were those involving comparison and division with a remainder, and even so approximately one-fourth of the students solved each problem correctly. What is also interesting is that although one might expect that multi-step problems would be too difficult for young learners, 44% of students solved this problem type correctly. Also, it is interesting to note that Latina/o kindergarteners' performance on the addition and subtraction problem types was comparable to that of the bilingual, Latina/o first graders who solved similar problems in Secada's (1991) study. Kindergarteners were also more successful on a comparable join change unknown problem (55% solved, as compared to 29% of first graders). [An example of a join change unknown problem type is as follows: Mario wants to buy a toy plane that costs $11. Right now he has $7. How many more dollars does he need to buy the toy?] Furthermore, kindergarteners solved a much wider range of problem types than what the results of a national assessment of 22,000 kindergarten students might predict (NCES, 2000). Whereas the NCES study found that only 18% of kindergarteners could solve simple addition and subtraction problems, and that only 2% could solve multiplication and division problems, the kindergarteners could solve similar problem types at much higher rates. The studies from CGI, Ginsburg, and the CEMELA Kindergarten study point to promising teaching practices that can advance the mathematics education of young DLLs. The next sections discuss these teaching practices.

Practice #1: Use Asset-based Teaching Approaches

The use of sociocultural theory has been instrumental in understanding children's mathematical thinking from an asset-based approach (Civil, 2007, 2016; Gonzalez, Moll, & Amanti, 2005; Gutiérrez, 2013; Moschkovich, 1999, 2002, 2010). An asset-based approach capitalizes on students', family's, and community's ways of knowing, including their language and culture, as intellectual resources that can be drawn upon to make meaning in and out of the mathematics classroom (Civil, 2007, 2016, 2017). Additionally, a basic tenet of asset-based approaches is to consciously move away from perspectives that have deficit views of students, families, and communities in relation to school readiness (Coleman et al., 2016).

Using this theoretical lens in research has contributed much to our understanding of how young DLLs use their native language (Celedón-Pattichis & Turner, 2012; Espada, 2012; Turner & Celedón-Pattichis, 2011), code-switching or translanguaging (Celedón-Pattichis, 2008; Khisty, 1995; Moschkovich, 1999, 2007), gestures (Domínguez, 2005; Fernandes & McLeman, 2012; Moschkovich, 1999), and other tools to make mathematical meaning. For example, researchers have documented how DLLs in upper elementary grades used keystrokes on a calculator as a tool to communicate mathematical thinking on a word problem that involved finding the area of a cone, covering geometry and measurement concepts and standards (Razfar, Chval, & Khisty, 2011). This particular approach is especially critical for students who are at beginning stages of developing English as a second language to communicate their mathematical thinking since they can draw on keystrokes on a calculator as symbols to represent their solutions.

Research that draws on language and culture as intellectual resources with young DLLs has shown the need to know the students' communities to pose word problems using meaningful contexts for kindergarten and second grade DLLs (Turner & Bustillos, 2017). Furthermore, Monette and Parks (2017) highlight ways to engage families with mathematical activities without requiring attendance at after-school events. They illustrate a two-way process where the teacher sends materials to promote mathematical conversations in the home, and caregivers send back notes or digital photos so that the teacher can use these artifacts as launch points for mathematical discussions in the classroom. These take-home activities can allow teachers to learn more about rich mathematical practices in students' home lives that might otherwise go unnoticed. The next section turns to how teachers can create a classroom community that talks mathematics.

Practice #2: Create a Mathematics Discourse Community

Discourse in the mathematics classroom matters (Khisty & Chval, 2002). The pedagogical practices that teachers use to create a community where DLLs support one another to develop not only everyday language but also mathematics discourse are important (Khisty & Chval, 2002). Drawing from Gee's work on discourse (2004) and Wenger's (1998) community of practice, Willey (2010) conceptualized a Mathematics Discourse Community (MDC) to involve the ways of being, doing, thinking, and speaking as these manifest in teachers' and students' interactions in the mathematics classroom. The role of the teacher in creating this environment is critical for young DLLs. In a Kindergarten Study conducted through the Center for the Mathematics Education of Latinos/as (CEMELA), researchers found several components that are important in creating a MDC. These include the use of storytelling, multimodal representations to communicate mathematical thinking, addressing academic language demands in learning a second language while learning mathematics, and positioning young DLLs as competent problem solvers.

Use storytelling. Storytelling has been an effective teaching practice that helps to frame problem solving among young DLLs in mathematics classrooms (Lo Cicero, Fuson, & Allexsaht-Snider, 1999; Lo Cicero, De la Cruz, & Fuson, 1999; Turner, Celedón-Pattichis, Marshall, and Tennison, 2009). Lo Cicero, Fuson, & Allexsaht-Snider (1999) illustrated how bilingual students' stories can be mathematized in the classroom. For example, students sharing a story of how many blocks they walked to get to school in the morning can be mathematized to create problem solving lessons using basic operations. Studying three primarily Latina/o kindergarten classrooms, one in which mathematics was taught in Spanish, one bilingual, and one in English as a second language, Turner and Celedón-Pattichis (2011) found that, although there was growth across all three classrooms in problem solving, students showed the most growth in solving word problems in the classroom where the teacher used storytelling twice as often; used the home language, Spanish, more often; and spent more time solving a wide range of problem types. What is important to note is that all teachers drew from familiar ways of talking and negotiating meaning within students' cultural contexts (Delgado-Gaitan, 1987; Villenas and Moreno, 2001), telling and sharing authentic, storytelling conversations, and inviting young DLLs to co-construct these stories when engaged in mathematical problem solving (Turner, Celedón-Pattichis, Marshall, and Tennison, 2009). Teachers used endearing terms as part of storytelling: "Fíjense amorcitos, pues les voy a contar una historia" (Listen my little loved ones, I am going to share a story").

For example, kindergarten and elementary school teachers leveraged students' familiar contexts of going to the mercado (the supermarket), to the flea market, to the park, and to the corn tortilla factory as a way to co-construct these stories. Teachers and students engaged in telling stories in the following way: "Yo fui al mercado y compré cuatro __________ y los metí en una bolsa." (I went to the supermarket and bought four __________ and I put them in a bag." At this point, students were invited to include items they liked, in which case children chose four toy horses. The teacher continued with the story to expand on it and create the following multiplication type problem: "I went to the supermarket and bought four toy horses. I put them all in a bag. How many legs are in the bag?" Students engaged in solving the problem using their small white boards, and the teacher called a student, who was considered to be in special education, to communicate his solution. The student began explaining his solution by drawing four toy horses and counting each leg verbally and by pointing to each leg as he counted, and then writing an equation (4 X 4 = 16) that connected to his pictorial representation (see Figure 1). At this point, I should note that this was the first time that this particular student was making sense or appropriating the multiplication symbol. Because not all students were using the multiplication symbol yet, the teacher used the details of the story as well as the student's explanation to scaffold for the rest of the class what the multiplication symbol meant, (i.e., 4 + 4 + 4 +4 = 16), an equation that most students were familiar with in this mathematics classroom. She pointed to the student's pictorial representation and connected each detail to the representation using repeated addition.

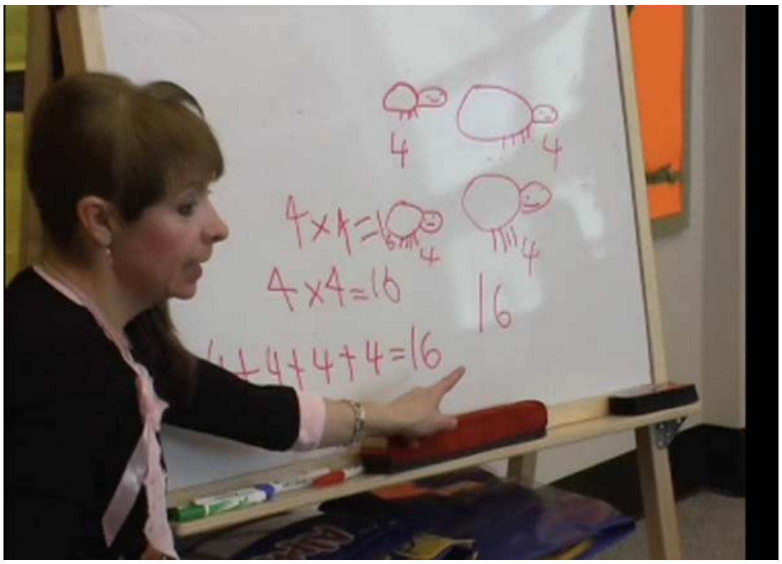

Figure 1: Scaffolding to make connections from oral counting to pictorial to symbolic representations

Use multimodal representations. Figure 2 below illustrates how kindergarten and elementary teachers supported young DLLs to transition from concrete to abstract concepts in mathematics (Musanti & Celedón-Pattichis, 2013, p. 52). As students developed their everyday and mathematical discourse, teachers encouraged students to use pictorial representations to communicate their solutions, especially for young learners who were in beginning stages of developing their home language and English as a second language. Teachers used storytelling in the mathematics classrooms to provide multiple entry points for young DLLs to retell details of the story as well as quantities expressed in the context of these stories, thus supporting young DLLs to develop oral explanations of their solutions. The three teachers also pushed for details by asking students to represent their solutions using symbols such as written numbers and equations that connected to their oral and pictorial representations. As young DLLs developed writing skills in their home language, they were afforded opportunities to create their own word problems using their journals, which occurred during the beginning of spring semester of kindergarten year, and students used these word problems for other students to solve (see figure 3; Celedón-Pattichis & Musanti, 2014). What is important to note is that young DLLs developed not only oracy skills but also writing skills in their home language and transferred these mathematical concepts to their second language, English (Musanti & Celedón-Pattichis, 2013).

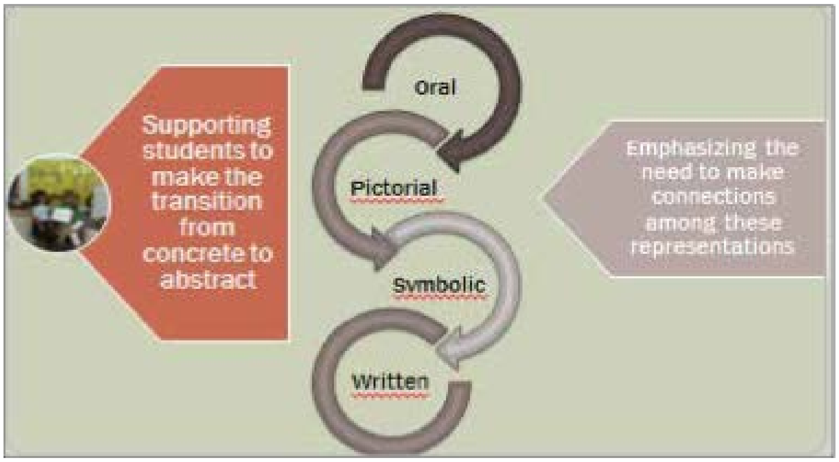

Figure 2: Supporting students to transition from concrete to abstract mathematical concepts (Taken from Figure 1 in Musanti & Celedón-Pattichis, 2013, p. 52)

Figure 3: Students create their own word problems to pose to others.

Address academic language demands of learning English as a second language while learning mathematics. Contrary to popular belief, mathematics is not a universal language. Several scholars have demonstrated that going beyond the teaching of vocabulary and addressing the academic language demands embedded in mathematics are critical (Celedón-Pattichis & Musanti, 2013; Civil & Turner, 2014; Khisty, 1995; Moschkovich, 1999, 2007). Symbols and procedures of immigrant students also differ from what is typically taught in the U.S. (Perkins and Flores, 2002). Teachers should address academic language demands in learning English as a second language while learning mathematics as well as nuances in the meaning of everyday

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language and mathematics. For example, words such as rational have different meanings--a rational personal versus a rational number; table in everyday use versus a table to organize data in mathematics, among others.

In working with Ms. López, a first grade teacher through the CEMELA Primary Study who was implementing a standards-based curriculum, she reported that "how many more" was a difficult concept for young learners to understand because this is a concept that involves a comparison of two or more quantities (Carpenter et al., 1999) and students do not have a starting point to directly model with manipulatives the amounts specified as they do with a word problem that involves an action (i.e., Luisa has 10 marbles. Rafael has 3 marbles. How many more marbles does Luisa have than Rafael? vs. Luisa has 10 marbles. Rafael give her 3 more marbles. How many marbles does Luisa have in total?). Additionally, as described in a later section on professional development, Ms. López shared this concern with her colleagues in a professional learning community and raise two issues. One was that the use of "more" is counterintuitive for students, and that they would want to add rather than subtract the quantities. This is especially true for teachers who tend to use keywords of "more" indicating addition. Another issue was that students did not want to have more than their peers. Ms. López indicated that students wanted to have the same amount of objects.

Drawing from students' understanding that they wanted to have the same amount of objects as their peers, Ms. López adapted highly cognitively demanding language in comparison type problems. She began by scaffolding the comparison type problems by asking students who had more, who had less, and how many more Rafael needed to have the same as Luisa. Most of the students knew Luisa had more marbles. Based on students' understanding, Ms. López posed the questions as follows: "¿Cuántos faltan?" (How many are missing?) to reinforce the concept. Progressively, she was able to ask: "How many more marbles does Luisa have than Rafael?"

Position young DLLs as competent problem solvers. Sociocultural perspectives on learning draw attention to the relationship between participation and identity, in particular, how the ways that students participate can impact how they view themselves as learners (Nasir & Hand, 2006). When students are afforded repeated opportunities to participate in ways that honor their experiences, their ideas, and their ways of knowing, the learning can be transformative (Wenger, 1998). When teachers position students as knowledgeable and competent problem solvers who have something important to contribute to the Mathematics Discourse Community, they not only support students' understanding, but they also place them on a trajectory toward greater competence and participation (Empson, 2003; Turner et al., 2009). Positioning young DLLs as competent problem solvers meant that teachers believed in students' capabilities to show their peers how they solved word problems. Teachers used strategic open and close ended questioning coupled with positioning to support students' participation in a Mathematics Discourse Community. Teachers invited students to share their mathematical thinking whereby even the shyest students at beginning stages of English language development in an ESL classroom showed a learning trajectory that led them to make a shift from problem solvers to problem posers from beginning to end of the kindergarten year (Turner et al., 2009). In the CEMELA Kindergarten Study, we observed teachers positioning students as competent when they took the following actions: They asked student to "Fold your hands and let's listen to Amalia" and calling on students to "Show us" on the board. Teacher actions were coupled with support of students as they explained their mathematical thinking to others in the class. Another way they positioned students as problem posers occurred by having students write their own word problems so that they could ask other students how to solve them.

In summary, creating a Mathematics Discourse Community affords teachers opportunities to advance the mathematics education of young DLLs. As aforementioned, supporting young DLLs in communicating their mathematical thinking can be accomplished in multiple ways. The next section focuses on integrating mathematics into play activities to develop mathematics concepts and discourse.

Practice #3: Use play to develop mathematical concepts and discourse

Research studies in early childhood education have shown consistently the importance of play in young learners' social, emotional, and cognitive development, including language, literacy, and brain development (Espinosa, 2002; NAEYC, 2009). Vygotsky (1978) also emphasized the importance of play in early childhood as a practice for learning and education. Researchers have also conducted studies on the benefits of play in mathematics education. For example, Seo and Ginsburg (2004) conducted a study with 90 3-5 year old students from five different schools. Students came from a diversity of ethnic and socioeconomic backgrounds. They observed children playing and analyzed 15-minutes video clips of what students could do mathematically through play. They found that preschool and kindergarten children across different income levels could engage more frequently with exploring patterns and shapes, comparing magnitudes (i.e., making side-by-side comparison of objects or making magnitude judgments with or without quantification), and enumerating (i.e., saying number words, counting, subitizing, or reading and writing numbers). They explored dynamic changes, classified, and examined spatial relations less frequently.

In particular to young DLLs, Karabon, Martinez Negrette, Smith, and Wager (2017) provided evidence of how culturally and developmentally responsive mathematics teaching practices can support these students to engage with rich meaningful mathematics that provided agency. Using a dramatic play area, the teacher (Smith) made mathematics apparent when the children were encouraged to write their own recipes or to place prices on the food served in their Mexican restaurant. Within these spaces, Smith created a classroom that talked mathematics (90% of the communication happened in Spanish) through playful activities and communicated a powerful message of agency that positioned Latinx young DLLs and doers of mathematics. These teaching practices showed how mathematics can be integrated into daily activities that involve play with young learners. This study also showed how teachers can provide access to meaningful mathematics learning opportunities, hold high expectations, support students' mathematical identity, and build on students' cultural and linguistic resources.

Similarly, in a second grade classroom with DLLs, Ramirez and Tapetillo (2017) illustrate how students participated in a Mathematics Discourse Community by positioning them as narrators and actors in a theatrical presentation. Using this approach to teaching mathematics showed growth in students' language development and self confidence. DLLs were able to communicate mathematical understanding to their peers, concretely, visually, and in words, both oral and written.

As illustrated, teaching practices that draw from students' language and culture as assets help to inform the work that teachers do in the mathematics classroom. The next section presents professional learning opportunities that teachers can engage with to support young DLLs in learning mathematics.

Professional Learning Opportunities Teachers Need to Support DLLs in Learning Mathematics

Best practices for teacher knowledge and growth have been well documented. Franke et al. (2001) argue that teacher development is an ongoing process that requires the creation of "school cultures where serious discussions of educational issues occur regularly, and where teachers' professional communities become productive places for teacher learning" (p. 654). Darling-Hammond and McLaughlin (1995) pointed out that strong professional development 1) engages teachers in teaching, assessment, observation, and reflection tasks that illustrate learning and development; 2) uses inquiry, reflection, and experimentation that are driven by participants; 3) is collaborative and involves sharing of knowledge within a community of practice of teachers; 4) connects to the work teachers do with their own students in the classroom; 5) is sustained, intensive, and supported with modeling, coaching; and 6) addresses others aspects of school change.

Bilingual and English as a second language (ESL) teachers of young DLLs are often provided professional development that focuses on language and literacy and rarely addresses content areas such as mathematics (Celedón-Pattichis & Musanti, & Marshall, 2010; Musanti & Celedón-Pattichis, & Marshall, 2009). Conversely, mathematics teachers are provided professional development that focuses on the content area itself and not necessarily the development of English as an additional language. If we are to embrace an asset-based approach to teaching young DLLs mathematics, the professional development opportunities offered to educators should center not only on children's mathematical thinking but also connect to children's language and culture. As young DLLs develop their first (home) language and English as an additional language, it is important educators understand the role that language plays while simultaneously learning mathematics (Ramirez & Celedón-Pattichis, 2012). Additionally, there is a need to prepare teachers to become advocates for young DLLs.

This section highlights projects that have integrated children's mathematical thinking and their funds of knowledge to advance the mathematics education of young DLLs. In particular, I discuss two projects, CEMELA's Kindergarten and Primary Studies and the Teachers Empowered to Advance Change in Mathematics (TEACH MATH) (Turner et al., 2012) as two examples of projects that have undertaken the work of preparing teachers to work with young DLLs. These projects have used Cognitive Guided Instruction as well as Funds of Knowledge as a foundation to understanding children's mathematical thinking and connecting this knowledge base to children's languages and cultures.

Integrate Young DLLs' Mathematical Thinking, Language and Culture

CEMELA's Kindergarten and Primary Study. The work of the Kindergarten and Primary Studies, which included K-5 grade teachers through CEMELA, drew from situated professional development to co-construct the experiences for bilingual and ESL teachers that were meaningful to their own context (Musanti, Marshall, Ceballos, & Celedón-Pattichis, 2011). Teachers wanted to integrate CGI within their reform curriculum because they saw a lack of word problems, and they wanted students to experience a range of context-rich word problems. The researchers formed collaborations with the bilingual and ESL teachers in the one bilingual school to engage in co-designing lessons, debriefing after a lesson was introduced to students, participating in professional development sessions at least three times per semester, and co-presenting at a dual language conference. The professional development sessions focused on reflecting on video clips that the researchers were collecting as data and that illustrated teacher and student interactions while problem solving as well as student work. While watching the video clips, teachers were asked to focus on the following:

1) What is the teacher doing to support, clarify, and extend students' mathematical thinking?
2) How does the teacher draw upon the students' home language and culture to teach mathematics?
3) How are students supporting their own learning?
4) How are students positioned in this classroom (i.e., in what ways are students participating)?

The results of our collaboration with Ms. López, a first-grade teacher who participated in the Primary Study, provide evidence of the importance of creating situated professional development communities "that promote the practice of shared inquiry grounded in teachers' work" (Crockett, 2002, p. 609). Furthermore, reflecting on teachers' practices using student work was an effective strategy to situate professional development within the context of the classroom while keeping in perspective the wider educational context defined by the adopted reform curriculum, the school's bilingual program, and the Latino community in which the work was embedded. Our work with Ms. López illustrated the potential of professional development that validates bilingual teachers' agency in terms of enacting curriculum and language policies (Musanti, Celedón-Pattichis, & Marshall, 2009). Findings from the professional development work with a kindergarten teacher show that her confidence in understanding mathematics herself increased during our collaborative work and that her own students also gained confidence in doing mathematics. This is illustrated in the quote below:

> I think that my main achievement is that they feel more confident. Because I see how much more confident they are, and how secure they feel either in mathematics or anything else. I enjoy that because at the beginning they would lower their heads. Their self-esteem was very low. Not now. (translated from Spanish) (Musanti, Marshall, Ceballos, & Celedón-Pattichis, 2011, p. 228)

A major contribution of the studies conducted through CEMELA and the professional development work is the fact that the researchers and teachers were addressing a high need area of young DLLs learning mathematics in Spanish. Furthermore, the work also included teachers and students who were in classrooms where English was a second language. Professional development tools from TEACH MATH, described below, and Celedón-Pattichis & Ramirez (2012) address the needs of DLLs in mathematics classroom settings where teachers and students do not necessarily share the same language.

TEACH MATH. Teachers Empowered to Advance Change in Mathematics (TEACH MATH) was featured in the NCTM Research Commentary as a project that draws from an asset-based approach to research and practice (Celedón-Pattichis et al., in press). The project developed and researched three instructional modules for teacher education and professional development settings to support teachers in connecting to children's mathematical thinking and cultural /linguistic/ community-based funds of knowledge, which the project named a student's *multiple mathematical knowledge base* in instruction (see Bartell et al., 2017; Drake et al., 2015; Turner et al., 2012). Facilitating an asset-based approach to mathematics teaching, the pedagogical tools included in these modules support student mathematical learning and participation.

Getting to know a student who is from a culturally and linguistically different background from the preservice teacher's is the purpose of the *Case Study Module*. By interviewing the student and conducting problem-solving interviews, preservice teachers have an opportunity to get to know the student. The ultimate goal of this module is to support preservice teachers in advancing that child's mathematics learning. Turner et al. (2016) found that 97% of 96 contextualized mathematics tasks developed from the mathematics learning case studies conducted by preservice teachers directly attended to children's mathematical thinking and knowledge of the child's interests. Furthermore, almost half (46%) of the contextualized mathematics tasks aimed to foster the child's mathematical reasoning and connect to specific knowledge about the child's out-of-school experiences to leverage mathematical learning (Turner et al., 2016).

In the *Classroom Practices Module*, there are tools to support teachers in observing and reflecting on their own practice (or that of others) related to mathematical tasks, learning, teaching, and power and participation. For ways to utilize these lenses to analyze mathematics teaching using video cases, see Roth McDuffie et al. (2014b). The researchers found that preservice teachers' use of these lenses to analyze videos of instructional practice deepened their capacities to notice children's multiple mathematical knowledge bases, and instructional moves and interactions that promote mathematics learning. Roth McDuffie et al. (2014a) found that it was more challenging to develop noticing with inservice teachers (Roth McDuffie et al., 2014a).

As part of the *Community Exploration Module*, teachers engage in learning about the mathematical practices in students' families and communities and draw from these to create standards-based mathematics lessons that are meaningful and culturally relevant to the students (see Aguirre et al., 2012; Turner et al., 2014). Aguirre et al. (2013) analyzed 70 Community Mathematics Exploration projects that represented the work of 113 pre-service teachers and found that approximately half of the projects (47%) addressed multiple mathematical knowledge bases in ways that indicate preservice teachers can be supported to design and implement standards-based instruction from an asset-based approach, a challenge that is often cited for inservice teachers to implement into their practice (Aguirre & Zavala, 2013). The theoretical, empirical, and practice-based contributions of the TEACH MATH modules have fostered collaborations with institutions across the country to continue to field-test these modules in mathematics methods courses and to support inservice teachers at different institutions in implementing them (www.teachmath.info).

Conclusions

Based on the research reviewed in this report and the recommendations for early childhood mathematics education, I highlight the following points regarding the mathematics education of young DLLs in particular:

- Young DLLs' capacity to engage in challenging mathematical problem solving is often underestimated. Educators should pose a range of problem types so that students can engage in solving addition, subtraction, multiplication, division, and multi-step type problems, among others (see Carpenter et al., 1999).

- As DLLs make a transition from home to school, creating a mathematics classroom that talks mathematics (Mathematics Discourse Community) is critical in developing not only everyday language but also mathematics discourse. Using the student's home language provides access to mathematics concepts and allows for student participation in a MDC.
- Given that some families may be of immigrant origin, the role of families should be considered since this may be the first time families navigate the schooling system in a new country.
- Pedagogical practices should draw from culturally relevant / responsive teaching that builds on students' language and cultures as strengths.
- Professional development for teachers should integrate children's mathematical thinking, development of a second language, and culture.

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