Bayesian Pseudo Posterior Synthesis for Data Privacy Protection

Based on works of M. Hu and T. D. Savitsky

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Data: CU Income for Consumer Expenditure Survey

- Heavily right-skewed. “Bill Gates” problem
Truncated Dirichlet Process

- Flexible **synthesizer** to preserve data distribution.
- Smooths response values and mixes records.

\[
y_i | x_i, \pi_k, \beta_k^*, \sigma_k^* \overset{\text{ind}}{\sim} \sum_{k=1}^{K} \pi_k N \left( y_i | x_i' \beta_k^*, \sigma_k^* \right)
\]

\[
\pi_1, \ldots, \pi_K \sim D \left( \frac{\gamma}{K}, \ldots, \frac{\gamma}{K} \right)
\]

- Usual practice generate synthetic observations, \((y_1^*, \ldots, y_n^*)\)

\[
y_i^* | y \overset{\text{ind}}{\sim} \int \left[ \sum_{k=1}^{K} \pi_k N \left( y_i^* | x_i' \beta_k^*, \sigma_k^* \right) \right] \times \prod_{k=1}^{K} p \left( (\pi_k, \beta_k^*, \sigma_k^*) | y \right) d \left( (\pi_k, \beta_k^*, \sigma_k^*) \right)
\]
Evaluation of identification disclosure risks

- Fewer synthetic values inside the interval/ball → the intruder has a higher probability of guessing the record of the name they seek.

Scenario 1: \( IR_i = \frac{10}{13} \times 1 = \frac{10}{13} \).
Evaluation of identification disclosure risks

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Pseudo Posterior

- Risk-based record-indexed weights, $\alpha_i \in \{0, 1\}$
  \[ \alpha_i = \min(0, 1 - c_i \times IR_i) \]

- For monotone constants, $(c_i) \in \max(0, [c_{\text{min}} = 1.0, c_{\text{max}} = 1.5])$.

- Used to construct risk-weighted, pseudo posterior:

  \[
  p_\alpha \left( (\pi_k, \beta^*_k, \sigma^*_k)_{k=1,...,K} \mid y, \theta \right) \propto \left[ \prod_{i=1}^{n} p \left( y_i \mid (\pi_k, \beta^*_k, \sigma^*_k)_{k=1}^{K} \right)^{\alpha_i} \right] \times \prod_{k=1}^{K} p \left( (\pi_k, \beta^*_k, \sigma^*_k) \mid \theta \right)
  \]
Application to CE Income: Utility

- Mass of distribution largely unaffected
- See the concentration effect in the tails
- Pulls more isolated records to the modes
CE Income: Compare Risks of Vector vs. Top-coding

- Known pattern: \{gender, age, education, marital status, earner\}.
- Top-coding only protects CUs with extreme incomes (see bulbs).
- Ignores other risky portions of data distribution.
CONTACT INFORMATION

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