

Need for solid mathematical foundation for Risk-based Verification, Validation & Accreditation (VV&A)

A close-up, low-angle photograph of the front of an F/A-18 Hornet fighter jet. The aircraft is white with blue and grey camouflage markings. The number '521' is visible on the nose, and a stylized blue and white logo is on the side. The cockpit canopy is open. The background is a dark, out-of-focus blue.

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A QUICK VISIT TO BASIC PRINCIPLE

Risk-based VV&A stems from the statistical principle of hypothesis testing

- VV&A has become the standard way to answer the question “ Is this M&S credible for the intended use?”
- In statistical language this problem would be stated as testing the “Null Hypothesis” that asserts

H_0 : The M&S is credible for the intended use
With Alternate Hypothesis”

H_a : The M&S is not credible for the intended use

CONDUCT THE EXPERIMENT:

- Build a simulation, collect data, and test the above hypotheses for acceptance or rejection

A QUICK VISIT TO BASIC PRINCIPLE

CONDUCT THE EXPERIMENT:

- Build a simulation, collect data, and test the above hypotheses for acceptance or rejection
- Since this type of testing is based on sampled data, there is always the possibility of committing errors; and the two categories of possible errors that result, are namely:

“Type I” and “Type II” errors:

Type I error denoted by α ; occurs if one rejects H_0 when it is true (**rejecting simulation results when it is valid**); and

Type II error denoted by β ; occurs if one accepts H_0 when it is false (**accepting simulation result when it is false**).

PROBLEM:

- α is selected as the p-value or level of significance
- Is there a formal way for the experimenter to control β ?

RISK-BASED VV&A: Current State Of The Art

- There are two widely circulating approaches for “*calculating*” the risk associated with M&S use:
 - The NAVAIR Risk-based VV&A
 - The Johns Hopkins’ APL MURM (MURM = M&S Use Risk Methodology)

Let's Start with NAVAIR process .

NAVAIR REQUIREMENTS

Method Must Be Flexible, & Usable In Real Practical Applications

- ***OTHER REQUIRED CHARACTERISTIC***
 - Must Reflect industry standards and best practice
 - Must build on structures and practices already in place in DoD acquisition program (program's existing risk management approaches, working group/IPT structure, delegation agreements, etc.)
 - Be consistent and based on experiences with M&S accreditation efforts, supporting major acquisition programs

THE NAVAIR RISK-BASED PROCESS

Risks associated with the M&S use are related/evaluated based on the following 4 criteria:

- Likelihood of the M&S being inaccurate and/or inappropriate for the application.
- Consequences of the M&S being inaccurate and/or inappropriate for the application.
- Importance of the (acquisition) decision supported by the M&S.
- Level of Reliance of the (acquisition) decision on the M&S.

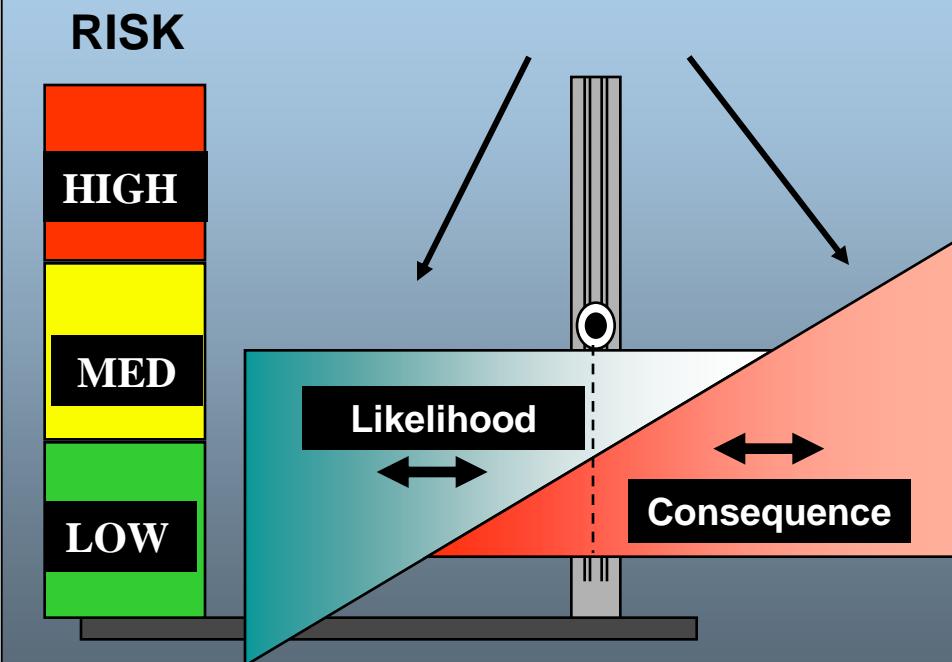
THREE LEVELS OF RISK: Low, Medium and High

What is Risk?

RISK = LIKELIHOOD X CONSEQUENCE

- *In the risk management community, risk is generally defined as the likelihood that something (usually bad) will happen times the consequences if it does*
 - Sometimes in casual speech people use the word “risk” to mean likelihood of occurrence
- *To reduce risk, either reduce the likelihood that something will occur or reduce severity of the consequence or both*
 - Risk literature also discusses the idea of exposure, which we'll come back to shortly

To reduce risk, reduce likelihood, consequence or both

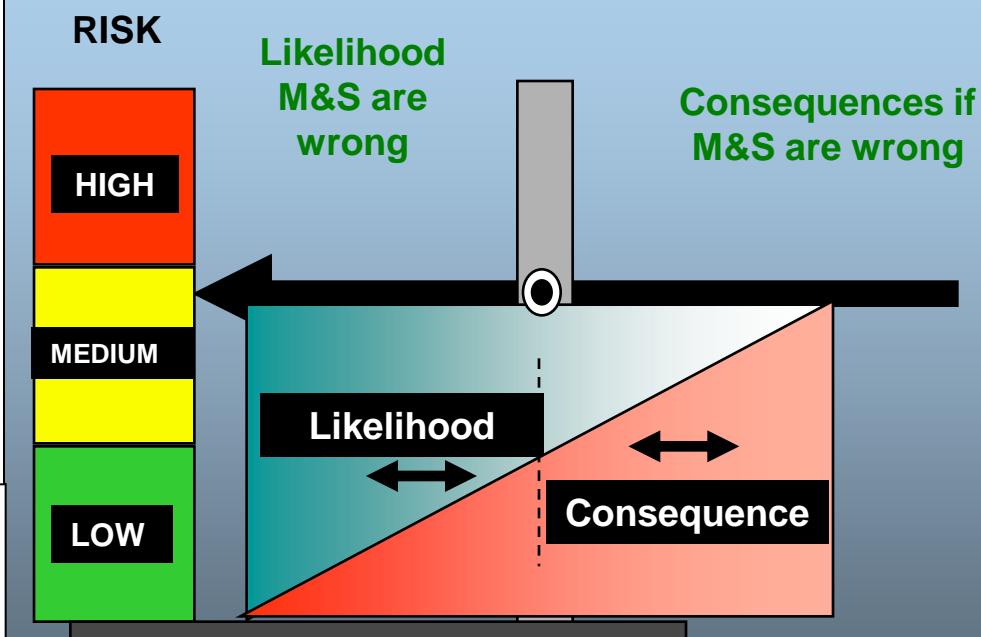


Risk Associated with M&S Use

- **Here, the risk of interest is the risk associated with using M&S**
 - M&S includes the models and simulations as well as the necessary input data
- **Likelihood is the odds that the M&S and/or their input data are incorrect or inappropriate to your intended use**
- **Consequence is the impact if the M&S output is wrong but you believe it and act on it**

Note: The risk associated with model development (will it be done on time and within budget?) is an important but separate issue which we treat as a special case. For this presentation we focus on the practical M&S operational risk.

$$\text{RISK} = \text{LIKELIHOOD} \times \text{CONSEQUENCE}$$



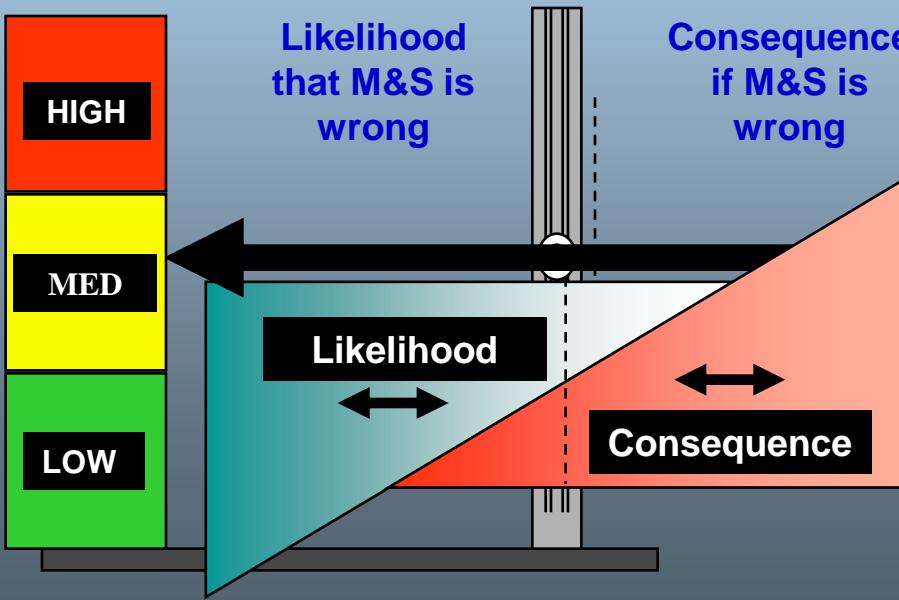
Consequence of a Poor Decision vs. Consequence if Model is Wrong...

- **Consequence if model is wrong depends on:**
 - Role M&S play in the decision-making process
 - Consequence of a poor decision

- Here, the role of M&S in decision making is similar to the concept of exposure in the risk literature

- Reduce risk by limiting exposure
- One way to reduce the risk associated with M&S use is by limiting the role of M&S in the decision process

RISK



Consequence if model is wrong =
f (role of M&S in decision
and
consequence of poor decision)

- *Risk associated with use of M&S is driven by likelihood M&S is wrong and consequence of M&S being wrong and user (knowingly or unknowingly) applies the M&S*
- *VV&A addresses likelihood of M&S error (and thus confidence in model results)*
 - Level of risk you can accept and consequences if model is wrong drive the amount of effort required to establish an acceptable level of confidence
 - Also, likelihood M&S is wrong and consequence if the model is wrong drive risk you accept if you use M&S
- *If you had a practical method of apply these principles, you could determine how much effort to put into VV&A*
 - What kind and how much evidence is required to establish confidence and reach accreditation decision for particular uses
 - Extent of appropriate review process
 - Level of independence in V&V and review
 - Appropriate level of accreditation authority

Drive
Resources

THE PROBLEMS

- *You can't always (or even often) come up with actual numbers for either consequence (cost, lives lost, etc.) or likelihood.*
- *How can you multiply what you can't quantify numerically?*
- *How does one obtain a mathematically objective value of the likelihood (a probabilistic value)?*
 - *In Practice:*
 - Usually resort to using estimates within defined bands or levels or bins: High, Medium, Low, etc.
 - Adopt a scheme for combining levels to arrive at a single value (combine likelihood value and consequence value to get risk value)
 - Borrow some practical ideas from System Safety community.
 - Actual assessment are subjective rather than objective estimates
 - We will show in a practical example later
- **NOW LET'S REVIEW JHU APL MURM NEXT**

“M&S Use Risk Mathematical Form”

- The M&S Use Risk is defined by the following statement:
- **“The probability that [(inappropriate application of M&S Results for the intended use will produce unacceptable consequences to the decision-maker) AND (that inappropriate application of M&S Results for the intended use occur) AND (unacceptable consequences to the decision-maker occur)].”**
- In symbols:
 - Cause = $C :=$ inappropriate application of M&S Results for the intended use
 - Effects = $E :=$ unacceptable consequences to the decision-maker
- Which results in the equation:

$$\text{M&S Use Risk} = P[(C \wedge E) \wedge (C \Rightarrow E)]$$

M&S Use Risk = $p[(C_1 \wedge C_2) \wedge (C_3 \rightarrow E)]$

- Where

$$P(\text{Causes}) = P(C_1 \cup C_2 \cup C_3)$$

- With

C_1 := Lack of clarity of intended use leading to misuse (i.e., Clarity)

C_2 := Adverse impact on decision if capability is not achieved (i.e., Importance), and

C_3 := Incorrect recommendation to employ or not to employ M&S Results relative to that capability (i.e., Confidence).

- And where

P(Effects) is the probability of the effects of unacceptable consequences to the decision-maker.

P(Effects) is based on M&S Impact and M&S Reliance

“JHU APL M&S Use Risk



Mathematical Form” (Cont.)

- Where

$$P(\text{Causes}) = P(C_1 \cup C_2 \cup C_3)$$

C_1 := Lack of clarity of intended use leading to misuse (i.e., Clarity)

C_2 := Adverse impact on decision if capability is not achieved (i.e., Importance), and

C_3 := Incorrect recommendation to employ or not to employ M&S Results relative to that capability (i.e., Confidence).

ISSUES

- How does one determine C_1 C_2 and C_3 objectively (standardized)?
- How does one compute $P(\text{Causes})$ solid probabilistic foundation?

JHU proposes an approach, but is it based on a sound mathematical/probabilistic or statistics foundation?

- In “Mathematical Logic”

M&S Use Risk = $P[(\text{Causes} \wedge \text{Effects}) \wedge (\text{Causes} \Rightarrow \text{Effects})]$

- “Algebraic Form for Probability Calculations”

M&S Use Risk = $P(\text{Causes}) * P(\text{Effects}) * [1 - P(\text{Causes}) + P(\text{Causes}) * P(\text{Effects})]$

- Defines Risk as:

The probability of Cause C times probability of Event E times probability that the cause C implies Event E".

- **Critical definition Issues with the logical computation of probabilities:**

- If C implies E, and you have C, you don't need to include "E" in the premises.
- If you have C, and C implies E, then you have E as well (by *Modus ponens*).
- This Double counts the probability of E.

- MURM Defines Risk as:

"The probability of Cause C times probability of Event E times probability that the cause C implies Event E".

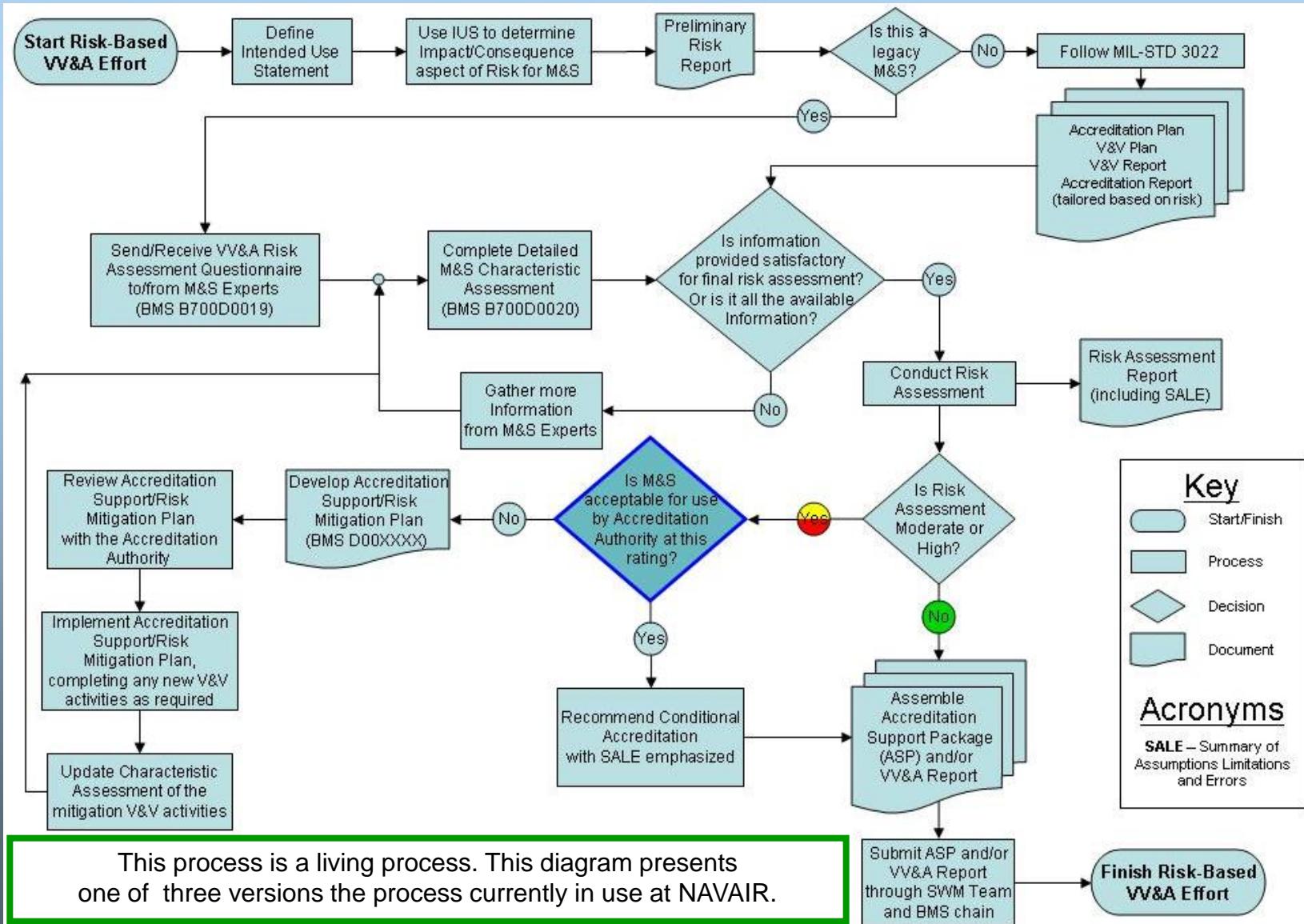
- **Definition is not consistent with Official DoD Definition of Risk**
- **MURM defines Risk as a probability, but Risk is not a probability**
- **Most everybody defines Risk as an “Expected Bad Outcome”**

CONSIDER THE FOLLOWING EXAMPLE:

- If I invest \$2M (\$1M each) in two separate investments, with the first investment having historical likelihood (probability) of a 20% loss, while the other investment has a historical likelihood of 15% loss, is the risk in this investment 20% or is it 15%? The answer is **NONE OF THE ABOVE!** .
- **THE CORRECT RISK IS THE EXPECTED LOSS:**
- $\text{Risk} = (0.2)(\$1M) + (0.15)(\$1M) = \$200,000.00 + \$150,000.00 = \$350,000.00$

TOOLS OF THE TRADE

- You'll need scales and rules
 - Scale and selection criteria for
 - Levels of risk associated with M&S use
 - Levels of likelihood of error (and an inverse scale for the level of confidence in M&S results)
 - Levels of consequence if model is wrong
 - Levels for role of M&S in decision making
 - Levels of consequence if decision is poor
 - Level combining rules
 - Combine (role of M&S in decision making) & (level of consequence of a poor decision) to get (Level of consequence if model is wrong)
 - Combine (likelihood of model error) & (level of consequence if model is wrong) to get (risk level)



Example Scheme for “Quantifying” Likelihood

An Example Scheme for “Quantifying” Likelihood

**The number of items should be specified

Likelihood Description	Likelihood of Occurrence over Lifetime of an Item	Likelihood of Occurrence Per Number of Items**
Frequent	Likely to Occur Frequently	Widely Experienced
Probable	Will Occur Several Times in Life of Item	Will Occur Frequently
Occasional	Likely to Occur Some Time in Life of Item	Will Occur Several Times
Remote	Unlikely but Possible to Occur in Life of Item	Unlikely but can Reasonably be Expected to Occur
Improbable	So Unlikely, it can be Assumed Occurrence May Not Be Experienced	Unlikely to Occur but Possible

Example Scheme For Levels Of Confidence/Likelihood of M&S Error

- Here's one suggestion based upon our experience and guidelines in VV&A RPG

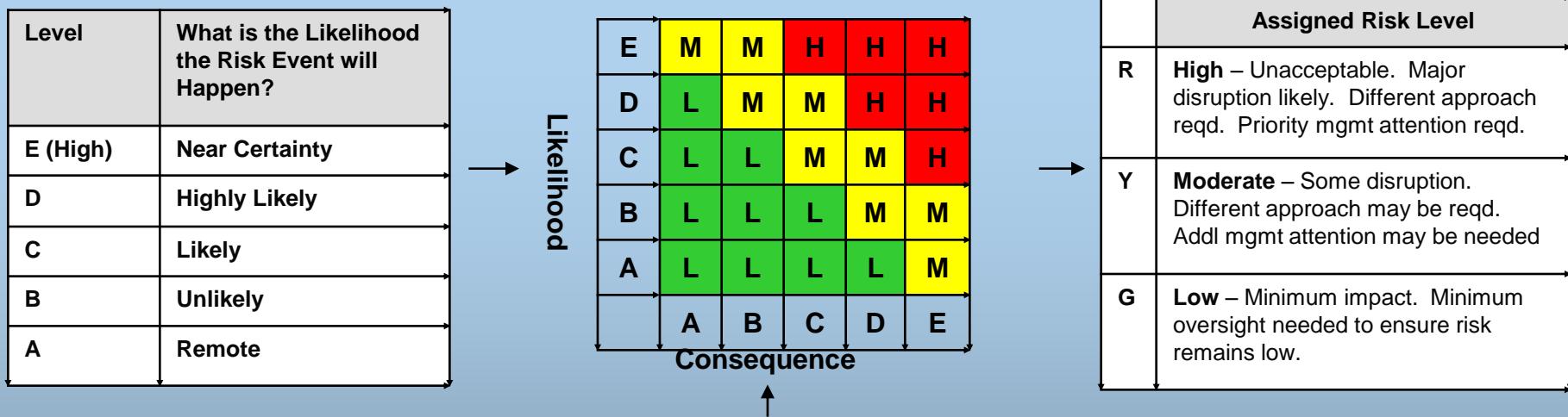


Include one level for either low or unknown level of confidence so that your approach has a minimal effort option to cover emergency or low consequence situations

Likelihood of Error	Confidence Level	Description
1	4	Very high confidence based upon extensive documented V&V relevant to intended use
2	3	High confidence based on face validation by SMEs
3	2	Moderate confidence based upon previous usage history
4 (High)	1	Low or unknown level of confidence. M&S appears to have the functionality required but credibility is unknown.

Conduct Risk Assessment

Example From Management Guide for DoD Acquisition



Level	Technical Performance	And/or	Schedule	And/or	Cost	And/or	Impact on Other Teams
A	Minimal or no impact		Minimal or no impact		Minimal or no impact		None
B	Acceptable, some reduction in margin		Additional resources reqd; able to meet need dates		<5%		Some impact
C	Acceptable; significant reduction in margin		Minor slip in key milestones; not able to meet need date		5 – 7%		Moderate impact
D	Acceptable; no remaining margin		Major slip in key milestones or critical path impacted		7-10%		Major impact
E (High)	Unacceptable		Can't achieve key team or major program milestones		>10%		Unacceptable

Scales, Rules and Tables

- Examples
- Some Tips and Advice

Levels of Risk

- Here's an example of a risk scale with three levels
 - Many programs use a three level high/medium/low risk scale
 - Very conducive to the use of stoplight charts



Give strong consideration to starting with the risk level structure already in use on your program and adapting it for use in your VV&A approach

Risk Level	Definition
High	Unacceptable. Major disruption likely. Different approach required. Priority management attention required
Moderate	Some disruption may occur. Different approach may be required. Additional management attention may be needed
Low	Minimum impact. Minimum oversight needed to ensure risk remains low.

Levels of Confidence / Likelihood of M&S Error

- Here's one suggestion based upon JASA's experience and guidelines in DMSO VV&A RPG



Include one level for either low or unknown level of confidence so that your approach has a minimal effort option to cover emergency or low consequence situations

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Levels of Consequence

- Here's an extremely simple example of consequence levels with four broadly defined levels



Whatever scheme you choose, you should make provisions to consider consequences of varying natures including cost, schedule, personnel safety, political, operational

- Also be sure you take into consideration all of the ways the model output could be wrong (e.g. M&S could erroneous over- or under-estimate performance of a military system, and the consequences might be different for each case)

Consequence Level	Definition
High	Major disruption to program. Different approach required. Priority management attention and resource allocation required immediately.
Moderately High	Significant disruption to program. Different approach required. Priority management attention required.
Moderate	Noticeable disruption. Different approach may be required. Additional management attention may be needed.
Low	Minimum impact. Minimum oversight needed to ensure risk remains low.

Role of M&S in Decision Making

- Here's an example scheme

Role Level	Definition
4	M&S will be the <u>only method</u> employed to make a decision
3	M&S will be the <u>primary method</u> , employed with other non-M&S methods
2	M&S will be a <u>secondary method</u> , employed with other non-M&S methods, and will <u>provide significant data unavailable through other means</u>
1	M&S will be a <u>supplemental method</u> , employed with other non-M&S methods, and will <u>provide supplemental data already available through other means</u>

Combination Schemes

Sample Method of Generating
Consequence / Evidence
Required to Support Accreditation

Evidence Required to Support Likelihood of Error

- *For each level of likelihood of error and confidence level, the table summarizes the information necessary to support an accreditation assessment*
 - More rigorous verification, validation, configuration management, discipline in model development, and oversight and review are required to drive down likelihood of error
 - As likelihood of error goes down, confidence in model results goes up

Likelihood of Error	Confidence Level	Evidence Required to Support Accreditation Assessment
1	4	Level 3 + extensive body of documented verification and validation + evidence of disciplined M&S development including history of technical and managerial review over time
2	3	Level 2 + SME face validation relevant to current intended use + evidence of effective configuration management
3	2	Level 1 + usage history + known V&V history
4 (High)	1	Comparison of M&S requirement derived from intended use with capabilities and limitations of candidate simulation

This is based on BMVVB/JASA's rules of thumb adopted by the MSCO/DMSO VV&A RPG. See "Role of Accreditation Agent in VV&A of Legacy Simulations" for more details.

Criticality Analysis: Importance of Decisions

- *Descriptions of Level of Importance of Decision*

Level	Description
4	Intended use addresses <u>multiple areas</u> of significant program risk, key program reviews and test events, key system performance analysis, primary test objectives and test article design, system requirements definition, and/or high software criticality, used to make a technical or managerial decision
3	Intended use addresses an <u>area of significant program risk</u> ...
2	Intended use addresses <u>medium or low program risk</u> , other program reviews and test events, secondary test objectives and test article design, other system requirements and system performance analysis, and medium or low S/W criticality used to make technical or managerial decisions
1	1 = Intended use addresses <u>program objectives or analysis that is not a significant factor</u> in the technical or managerial decision making process

Criticality Analysis: Role of M&S

- *Here's an example scheme*

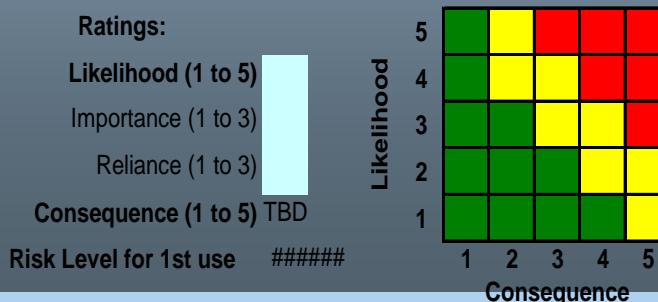
Role Level	Definition
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3	M&S will be the <u>primary method</u> , employed with other non-M&S methods
2	M&S will be a <u>secondary method</u> , employed with other non-M&S methods, and will <u>provide significant data unavailable through other means</u>
1	M&S will be a <u>supplemental method</u> , employed with other non-M&S methods, and will <u>provide supplemental data already available through other means</u>

Actual Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Risk Evaluator(s):	Your name(s)
Group/IPT:	Your group
Model/Simulation:	Short descriptive title
Intended Use(s):	Program requirement(s) or capability(s) being evaluated or supported by the model (in some cases it may be prudent to evaluate the model separately for different uses)
Phase(s):	Program phase(s) in which the model will be used (e.g. design, development, IT/DT, OT, training, production/deployment, FOT&E, etc.)
Milestone(s):	Identify 1st program milestone that the model will support (e.g. PDR, CDR, DRR, GTV Light-off, 1st Flight, OT-B1, MS-C, TECHEVAL, OTRR/IOT&E/OT-C2/OPEVAL, IOC, etc.) Identify all successive program milestones that the model will support
Likelihood Rationale:	Concise sentence or two providing rationale for selection of likelihood assessment
Consequence Rationale:	Concise sentence or two providing rationale for selection of consequence assessment
Safety Considerations:	If applicable, state any safety considerations
Comments:	Additional comments

Instructions for using the worksheet (Enter information in turquoise cells only):

- 1) Determine the "Likelihood of the Model/Simulation Being Wrong" using the Likelihood Guide. Enter Likelihood rating.
- 2) Determine the "Consequence of the Model/Simulation Being Wrong" at the 1st program milestone supported using the Consequence Guide.
- 2a) Determine the "Importance of Technical Decision Supported by the Model/Simulation". Enter Importance rating.
- 2b) Determine the "Level of Reliance on the Model/Simulation". Enter Reliance rating.
- 2c) Consequence rating is automatically calculated from the Importance and Reliance ratings using the Combining Matrix.
Combining Matrix is shown in Table 2 for reference.
- 3) Risk is automatically calculated from the Likelihood and Consequence ratings.



Definitions For Consequence

Importance of Technical Decision Supported by M&S

3 - Same as level 2 but simulation supports a technical decision on multiple ORD thresholds or an ORD Key Performance Parameter (KPP) or Critical Technical Parameter (CTP), or the simulation supports a technical decision involving a catastrophic or critical impact level from Table 1.

2 - Simulation supports a technical decision on the verification of an ORD threshold, key program technical/acquisition reviews and test events, primary test objectives and test article design, and/or high software criticality, or the simulation supports a technical decision involving a marginal impact level from Table 1.

1 - Simulation supports a technical decision on non-required system attributes, minor program reviews and test events, secondary test objectives and test article design, and/or medium or low software criticality, or the simulation supports a technical decision involving a negligible impact level from Table 1.

Greater Importance

Level of Reliance on M&S

3 - the simulation will be the only method used to support a decision or to provide a deliverable, or the simulation provides input to another simulation at this level

2 - the simulation will be the primary method used in addition to other methods to support a decision or to provide a deliverable, or the simulation provides input to another simulation at this level

1 - the simulation will be a secondary method used in addition to other methods to support a decision or to provide a deliverable, or the simulation provides input to another simulation at this level

Greater Reliance

Combining Matrix (Consequence rating shown in green cells)

Importance of Decision	Level of Reliance		
	3	2	1
3	5	4	3
2	4	3	2
1	3	2	1

Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Ratings:

Likelihood (1 to 5)



Importance (1 to 3)

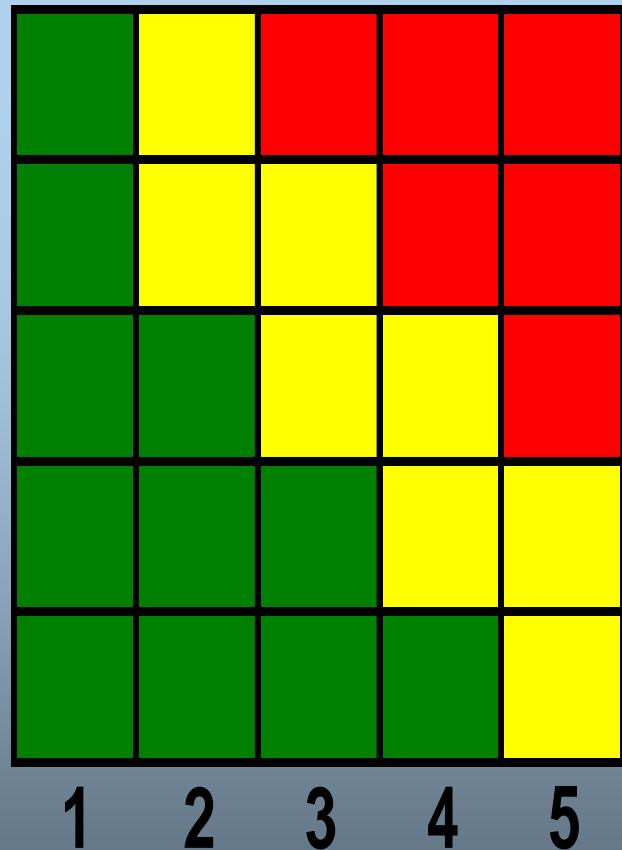
Reliance (1 to 3)

Consequence (1 to 5) TBD

Risk Level for 1st use #####

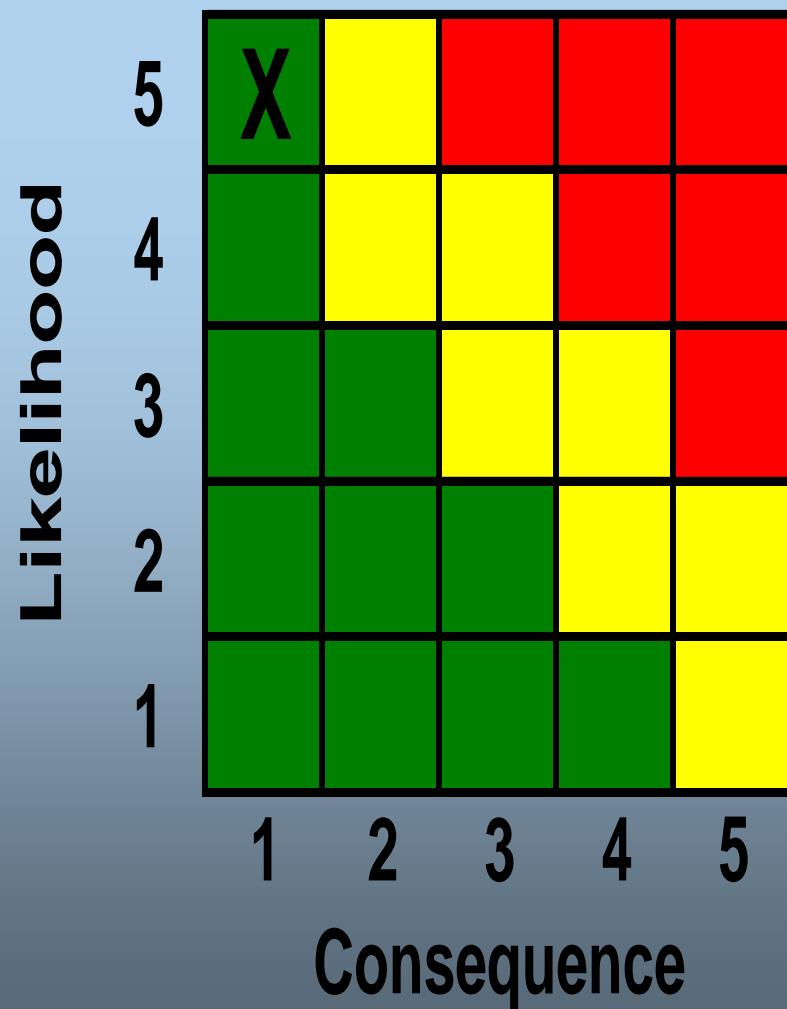
Likelihood

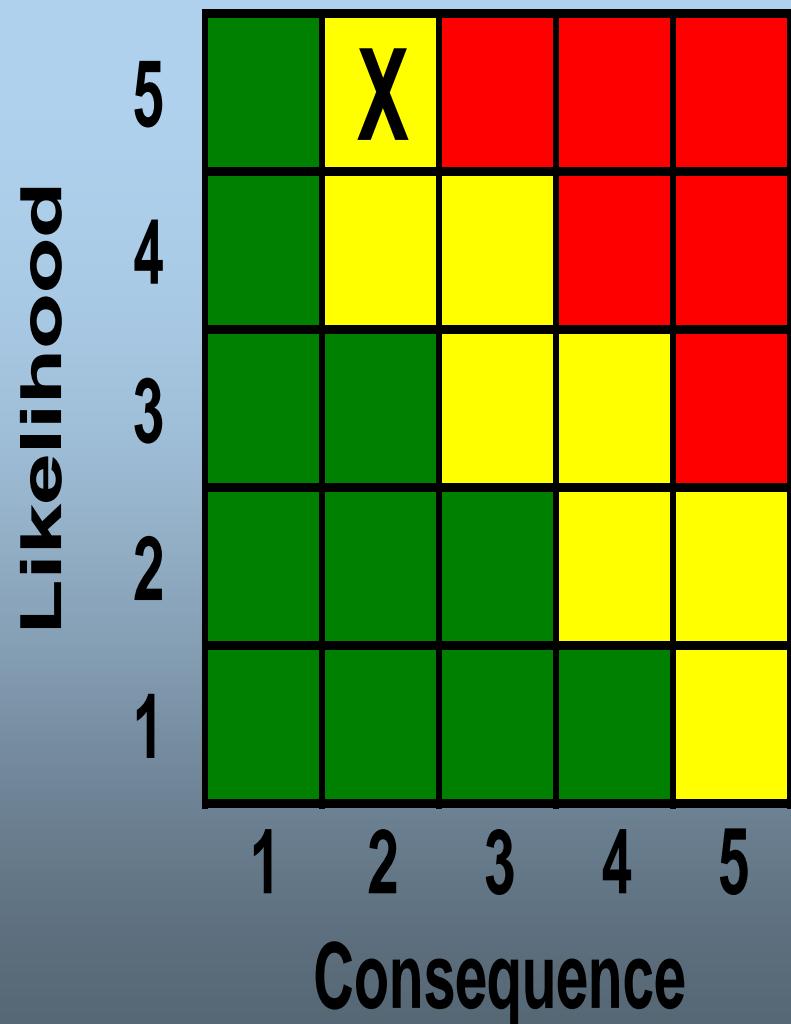
5
4
3
2
1



Consequence

Actual Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Ratings:**Likelihood (1 to 5)** 5**Importance (1 to 3)** 1**Reliance (1 to 3)** 1**Consequence (1 to 5)** 1**Risk Level for 1st use** Low

Worksheet for Evaluating Technical
Risk of Using Modeling & Simulation**Ratings:****Likelihood (1 to 5)** 5**Importance (1 to 3)** 2**Reliance (1 to 3)** 1**Consequence (1 to 5)** 2**Risk Level for 1st use** Med

Worksheet for Evaluating Technical
Risk of Using Modeling & Simulation

Ratings:

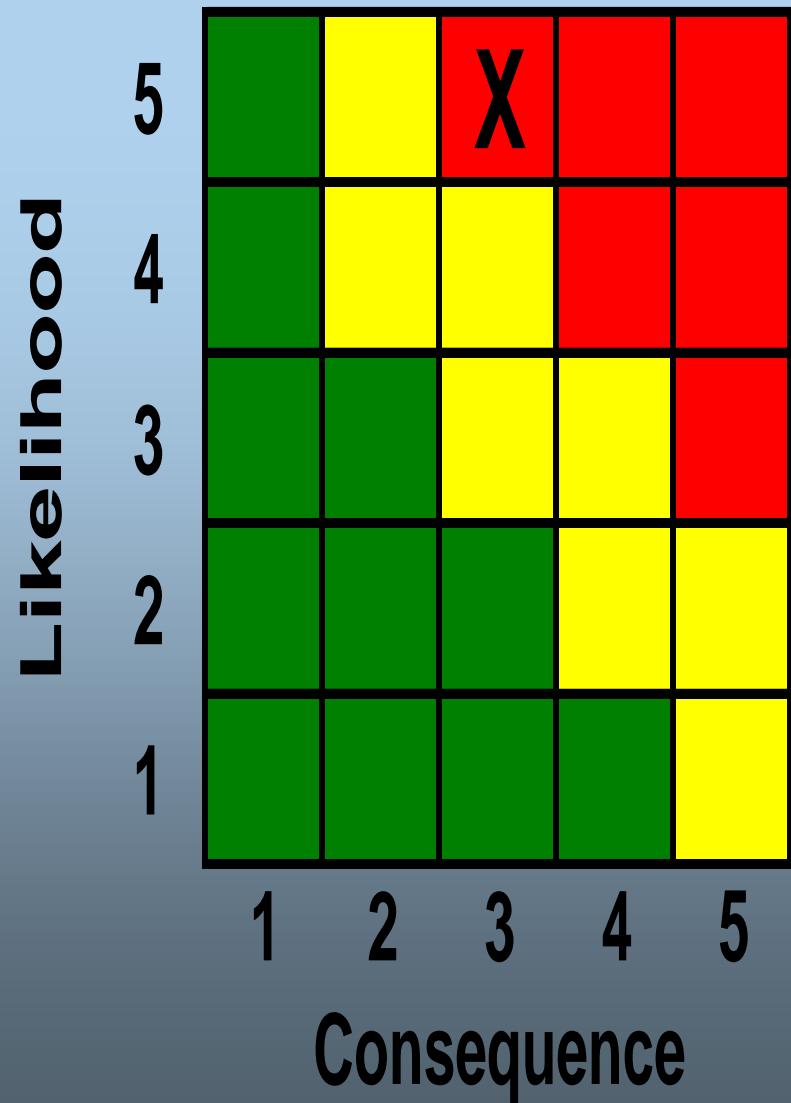
Likelihood (1 to 5) 5

Importance (1 to 3) 2

Reliance (1 to 3) 2

Consequence (1 to 5) 3

Risk Level for 1st use High

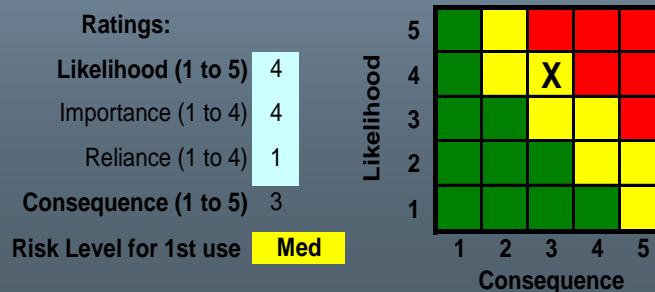


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- 2c) Consequence rating is automatically calculated from the Importance and Reliance ratings using the Combining Matrix.
Combining Matrix is shown in Table 2 for reference.
- 3) Risk is automatically calculated from the Likelihood and Consequence ratings.



A Sample 4X4 5 Consequence Table

Combining Matrix (Consequence rating shown in green cells)

Importance of Decision	Level of Reliance			
	4	3	2	1
4	5	5	4	3
3	5	4	3	3
2	4	3	2	2
1	3	3	2	1

Sample Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Ratings:

Likelihood (1 to 5)

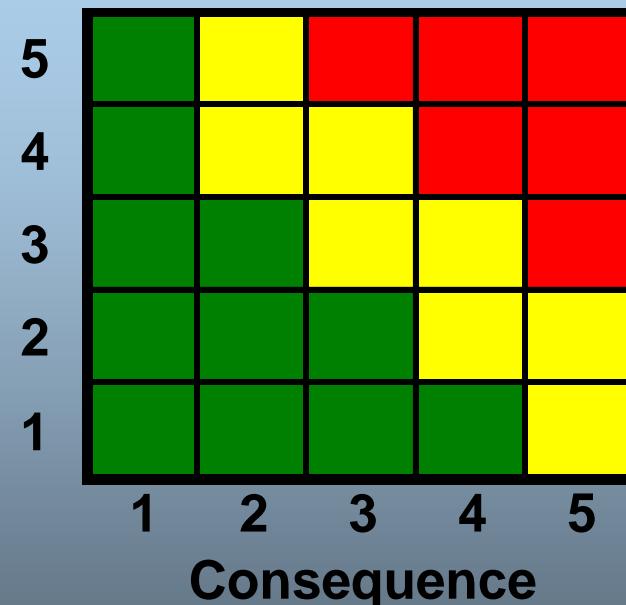
Importance (1 to 4)

Reliance (1 to 4)

Consequence (1 to 5) TBD

Risk Level for 1st use #####

Likelihood



Sample Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Ratings:

Likelihood (1 to 5) 5

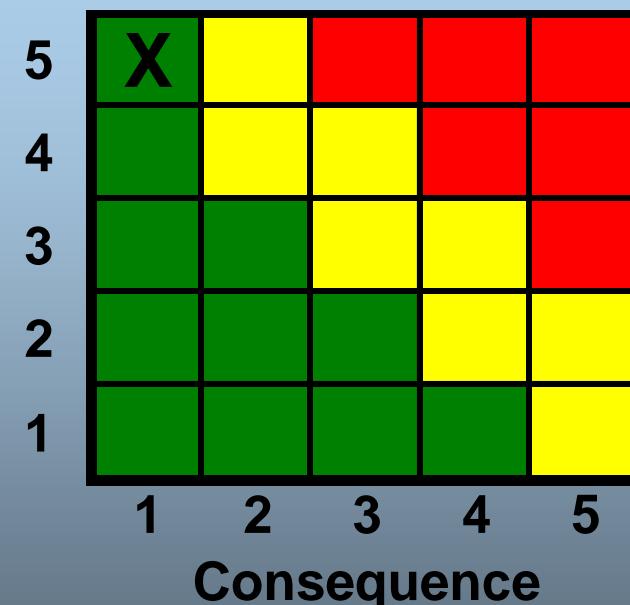
Importance (1 to 4) 1

Reliance (1 to 4) 1

Consequence (1 to 5) 1

Risk Level for 1st use Low

Likelihood



Sample Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Ratings:

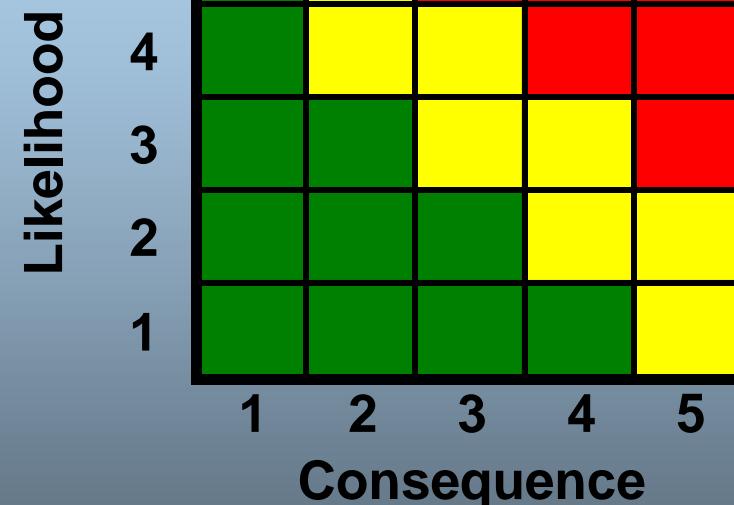
Likelihood (1 to 5) 5

Importance (1 to 4) 3

Reliance (1 to 4) 1

Consequence (1 to 5) 2

Risk Level for 1st use Med



Sample Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Ratings:

Likelihood (1 to 5) 5

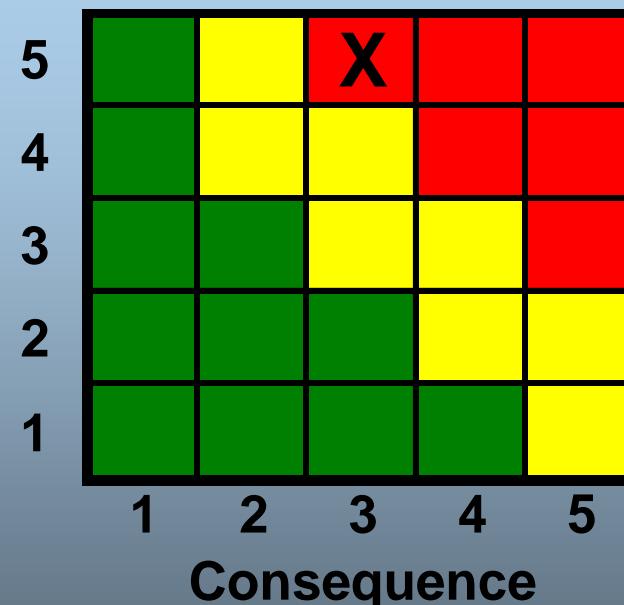
Importance (1 to 4) 3

Reliance (1 to 4) 2

Consequence (1 to 5) 3

Risk Level for 1st use **High**

Likelihood



Sample Worksheet for Evaluating Technical Risk of Using Modeling & Simulation

Ratings:

Likelihood (1 to 5) 5

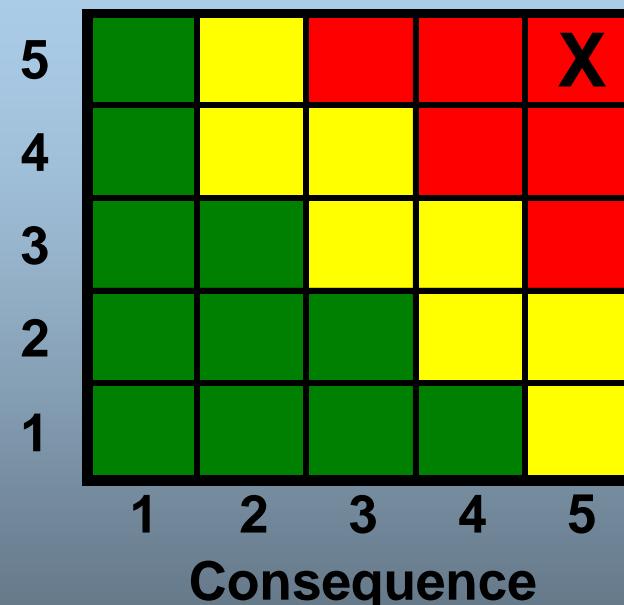
Importance (1 to 4) 3

Reliance (1 to 4) 4

Consequence (1 to 5) 5

Risk Level for 1st use High

Likelihood



CONCLUSION

- In reality VV&A is a risk mitigation process
- Risk-based VV&A provides a way to optimize resources spent on VV&A based on the risk associated with using the M&S to support decision making
- The method is applicable to both Legacy and New M&S
- The method has been used successfully for various military acquisitions program its limitations notwithstanding

BUT:

- Several parameter are subjective estimates
- Several definitions require standardization
- Several likelihood probability values have no mathematical basis or foundations

Questions?