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Mathematics of the Electric Grid

March 13: ***Recording posted***
Probability for People and Places

April 10: ***Recording posted***
Social and Biological Networks

May 8: ***Recording posted***
Mathematics of Redistricting

June 12:
Number Theory: The Riemann Hypothesis

July 10: *Topology*

August 14:
Algorithms for Threat Detection

September 11:
Mathematical Analysis

October 9: *Combinatorics*

November 13:
Why Machine Learning Works

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Mathematics of Epidemics

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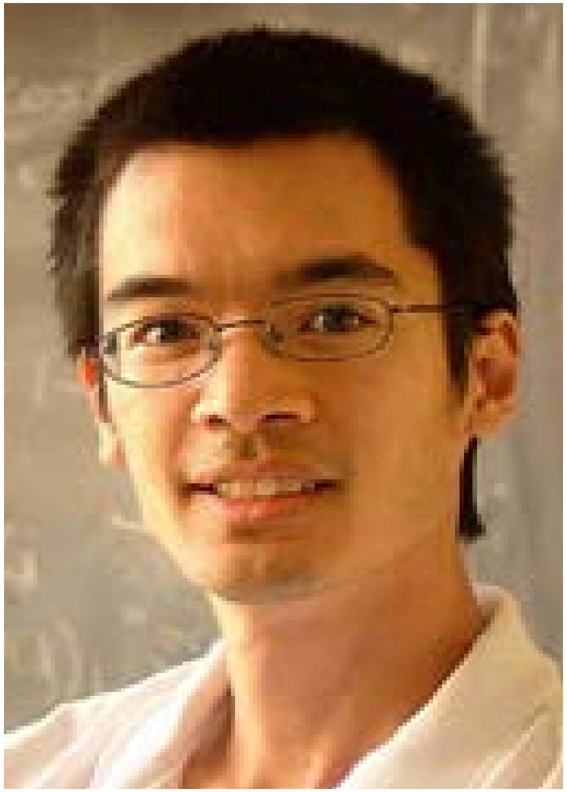
MATHEMATICAL FRONTIERS
Number Theory: The Riemann Hypothesis

Ken Ono,
Emory University



Terence Tao,
UCLA

David Chu,
Institute for Defense Analyses

MATHEMATICAL FRONTIERS
Number Theory: The Riemann Hypothesis

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Emory University

Asa Griggs Candler Professor of Mathematics, Department of Mathematics and Computer Science

History and Motivation

HILBERT AND THE RIEMANN HYPOTHESIS

David Hilbert (1862 – 1943)

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“If I were to awaken after having slept for a thousand years, my first question would be: Has the Riemann Hypothesis been proven?”

RIEMANN HYPOTHESIS (1859)

Bernhard Riemann
(1826-1866)

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Conjecture (Riemann)
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Question. What does this mean? Why does it matter?

PRIMES

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2, 3, 5, 7, 11, 13, 17, 19, 23,

29, 31, 37, 41, 43, 47, 53, 59,

61, 67, 71, 73, 79, 83, 89, 97

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"Primes grow like weeds… seeming to obey no other law than that of chance… nobody can predict where the next one will sprout…

…Primes are even more astounding, for they exhibit stunning regularity. There are laws governing their behavior, and they obey these laws with almost military precision."

SIEVE OF ERASTOTHENES (~200 BC)

Algorithm for listing the primes up to a given bound.

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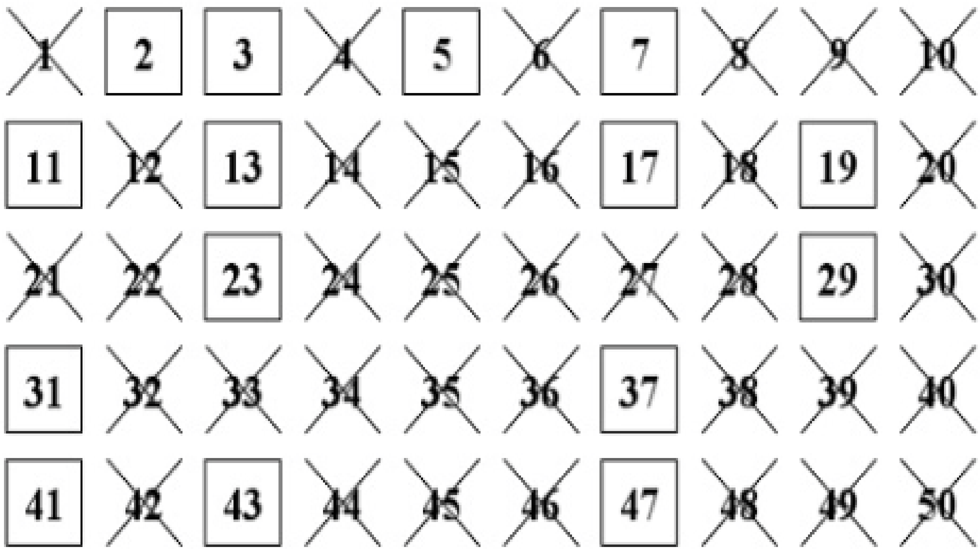

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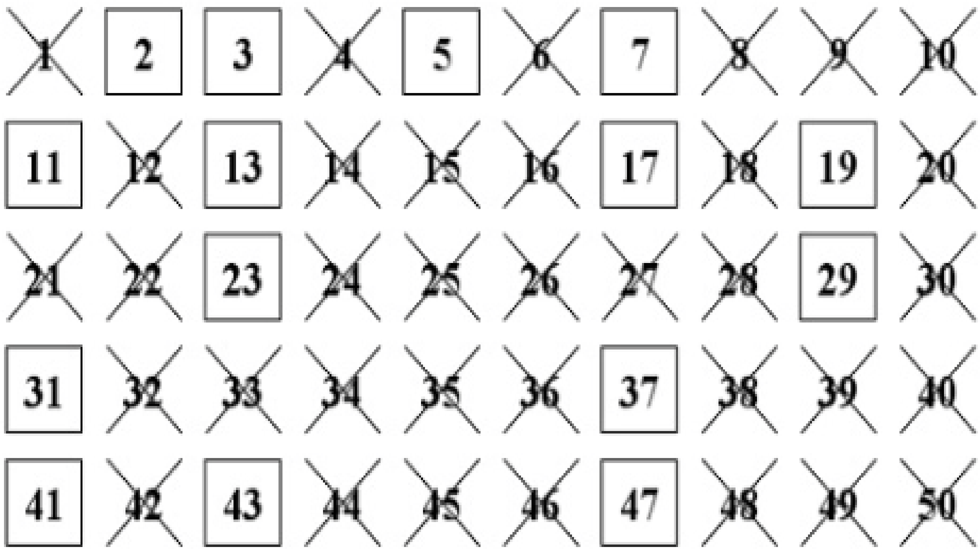

Algorithm for listing the primes up to a given bound.

Problem. This does not reveal much about the primes.

EUCLID (323-283 BC)

Theorem (Euclid)
There are infinitely many primes.

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There are infinitely many primes.

Proof: Suppose that $p_1=2 < p_2 = 3 < \ldots < p_r$ are all of the primes.

Let $P = p_1 p_2 \ldots p_r + 1$ and let p be a prime dividing P.

Then p can not be any of $p_1, p_2, \ldots, p_r$, because otherwise p would divide the difference $P - p_1 p_2 \ldots p_r = 1$, which is impossible.

EULER (1707-1783)

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Geometric Series. If $|r| < 1$, then

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Examples. Strange infinite series expressions

$$2 = \frac{1}{1-\frac{1}{2}} = \sum \frac{1}{2^{a_1}}$$

$$3 = \frac{1}{1-\frac{1}{2}} \cdot \frac{1}{1-\frac{1}{3}} = \sum \frac{1}{2^{a_1}3^{a_2}}$$

$$\frac{15}{4} = \frac{1}{1-\frac{1}{2}} \cdot \frac{1}{1-\frac{1}{3}} \cdot \frac{1}{1-\frac{1}{5}} = \sum \frac{1}{2^{a_1}3^{a_2}5^{a_3}}$$

$$\frac{35}{8} = \frac{1}{1-\frac{1}{2}} \cdot \frac{1}{1-\frac{1}{3}} \cdot \frac{1}{1-\frac{1}{5}} \cdot \frac{1}{1-\frac{1}{7}} = \sum \frac{1}{2^{a_1}3^{a_2}5^{a_3}7^{a_4}}$$

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The Fund. Thm of Arithmetic and geometric series give

$$\sum_n \frac{1}{n^s} = \prod_p \frac{1}{1 - \frac{1}{p^s}}$$

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Letting *s=2* (or **any positive even**) Euler obtained formulas such as

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots=\frac{\pi^2}{6}$$

INFINITUDE OF PRIMES APRÉS EULER

Theorem. If $\pi(n)$ is the number of primes < n, then

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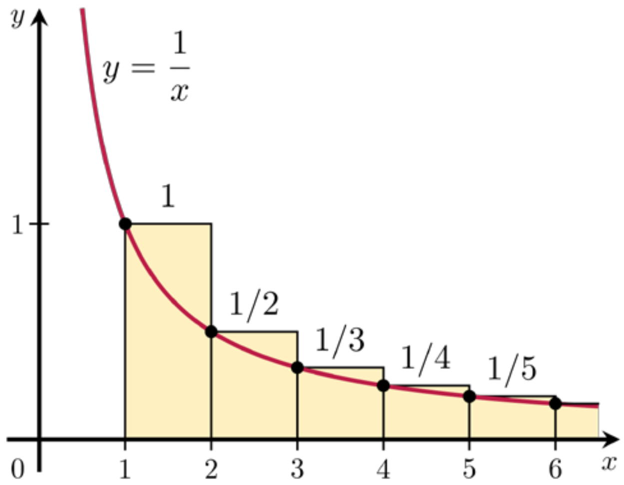

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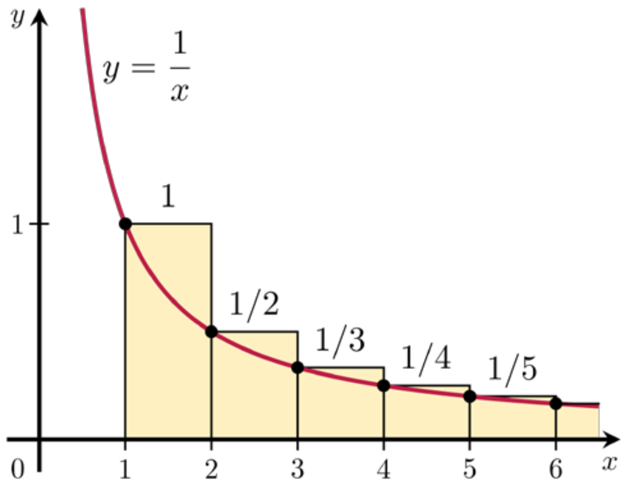

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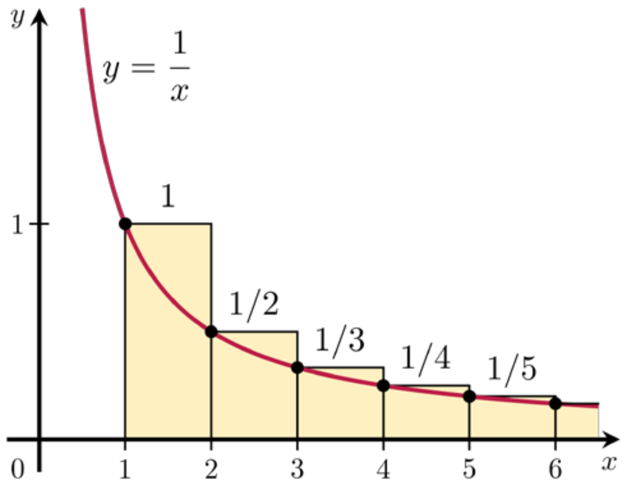
- If $\pi(n) = k$, then Euler's product gives
  $$\ln(n) < \prod_{j=1}^{k} \frac{1}{1-\frac{1}{p_j}}.$$
- By telescoping we get
  $$\ln(n) < \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \frac{k+1}{k} = k + 1 = \pi(n) + 1.$$



GAUSS (1777-1855)

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x	$\pi(x)$	$\frac{x}{ln(x)}$
10^2	25	22
10^3	168	145
10^4	1229	1086
10^5	9592	8686
10^6	78498	72382
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GAUSS (1777-1855)

Carl Friedrich Gauss

Conjecture (Gauss).
If we let $\mathrm{Li}(X) := \int_2^X \frac{dt}{\log t}$, *then we have*

$$\pi(X) \sim \mathrm{Li}(X) \sim \frac{X}{\log X}.$$

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- Strategy to prove Gauss' Conjecture

RIEMANN'S ZETA-FUNCTION

Theorem (Riemann, 1859)

(1) $\zeta(s) := \sum_{n=1}^{\infty} \frac{1}{n^s}$ *is defined for* $\mathrm{Re}(s) > 1$.

(2) Analytic continuation to $\mathbb{C}$ *(simple pole at* $s = 1$*).*

(3) It satisfies $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \cdot \zeta(1-s)$.

(4) It has **trivial zeros** *at* $s = -2, -4, -6, -8, \ldots$.

$1+2+3+4+5+ \ldots = -1/12$

Srinivasa Ramanujan
(1887-1920)

"Under my theory

$1+2+3+4+\ldots = -1/12.$

If I tell you this you will at once point out to me the lunatic asylum,,,"

1+2+3+4+5+ . . . = -1/12

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Proof.

(Euler) $\zeta(2) = \frac{\pi^2}{6}$

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Proof.

$$\text{(Euler)} \qquad \zeta(2) = \frac{\pi^2}{6}$$

$$\text{(Riemann)} \qquad \zeta(-1) \text{ ``}=\text{''} \ 1 + 2 + 3 + 4 + \dots$$

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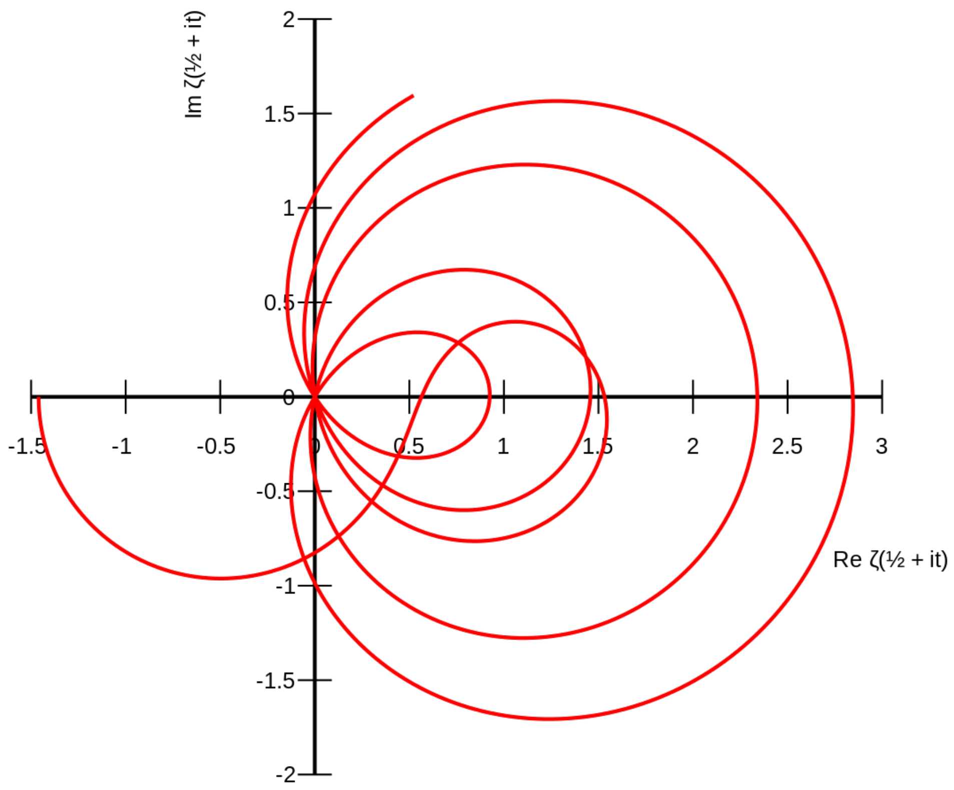
(Riemann) $\zeta(-1) = \frac{1}{2} \cdot \frac{1}{\pi^2} \cdot \sin(-\pi/2)\Gamma(2)\zeta(2) = -\frac{1}{12}.$ □

VALUES ON THE CRITICAL LINE



Spiraling $\zeta(\frac{1}{2} + it)$ for $0 \leq t \leq 50$

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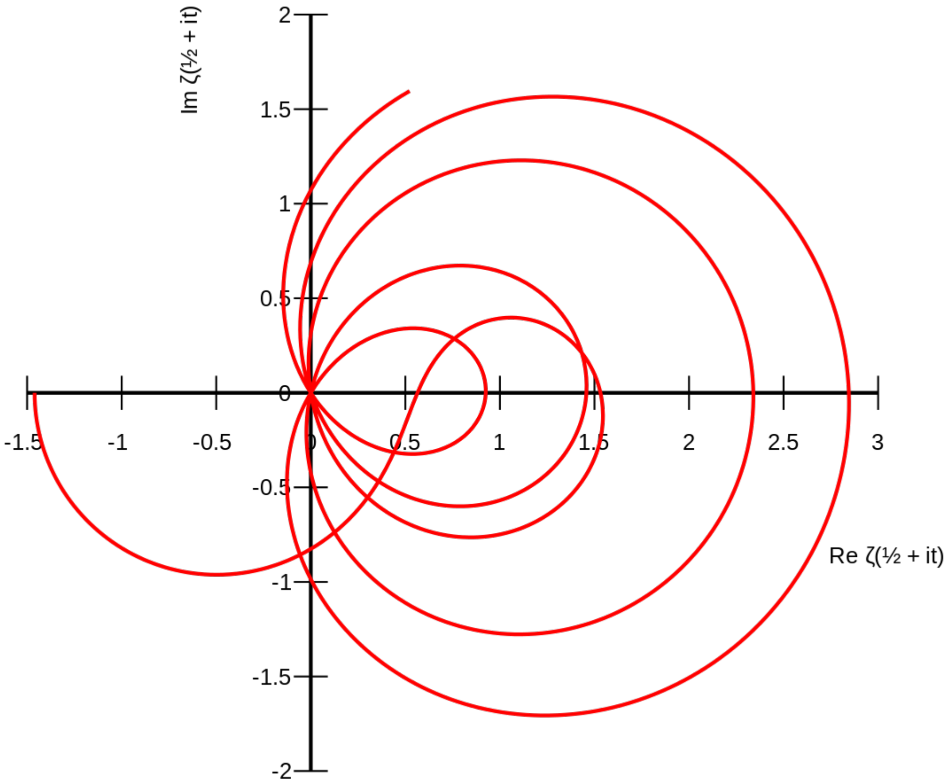
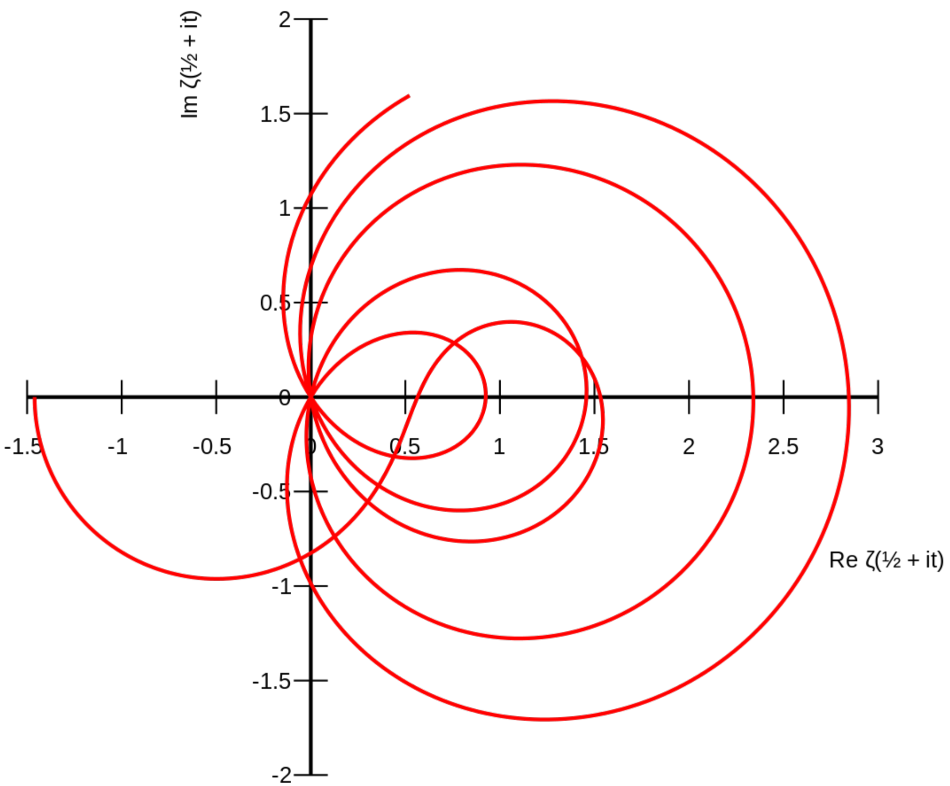


Spiraling $\zeta(\frac{1}{2} + it)$ for $0 \leq t \leq 50$

Note.

- $\boldsymbol{\zeta}(\frac{1}{2}) = -1.460354\ldots.$

VALUES ON THE CRITICAL LINE



Spiraling *ζ(½ + it)* for *0 ≤ t ≤ 50*

Note.

- ***ζ*(½)** = *-1.460354….*
- The first few nontrivial zeros are encountered.

RIEMANN'S HYPOTHESIS

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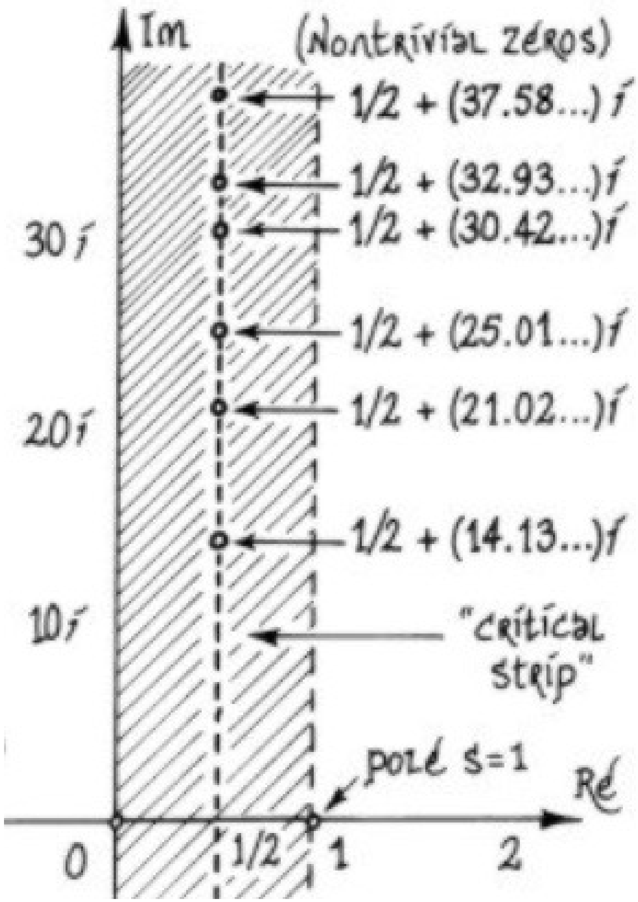

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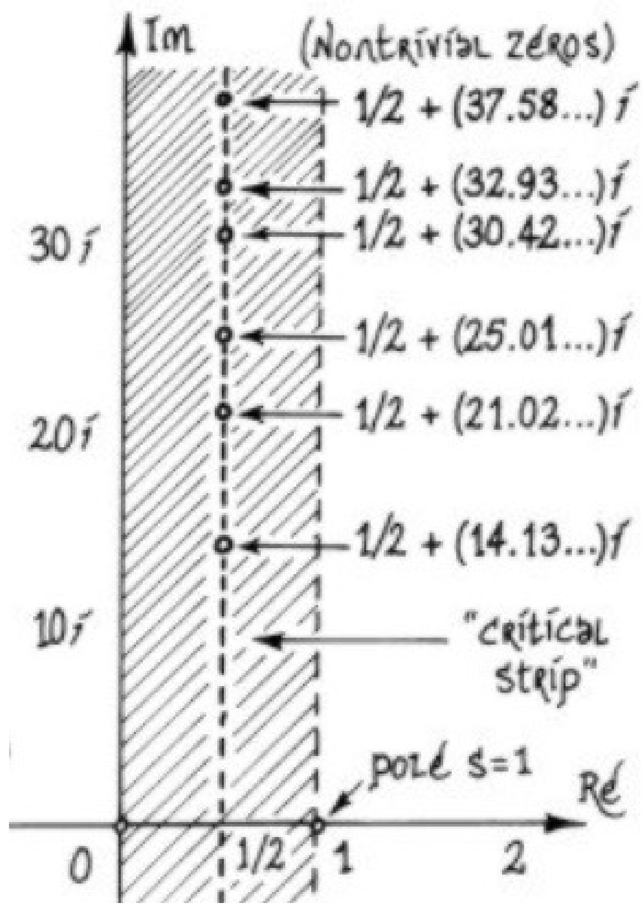
"… it would be desirable to have a rigorous proof of this proposition…"

Bernhard Riemann (1859)



COUNTING PRIMES

Theorem. (Chebyshev, von Mangoldt)

The Prime Number Theorem is equivalent to

$$\lim_{X \to +\infty} \frac{\Psi(X)}{X} = 1,$$

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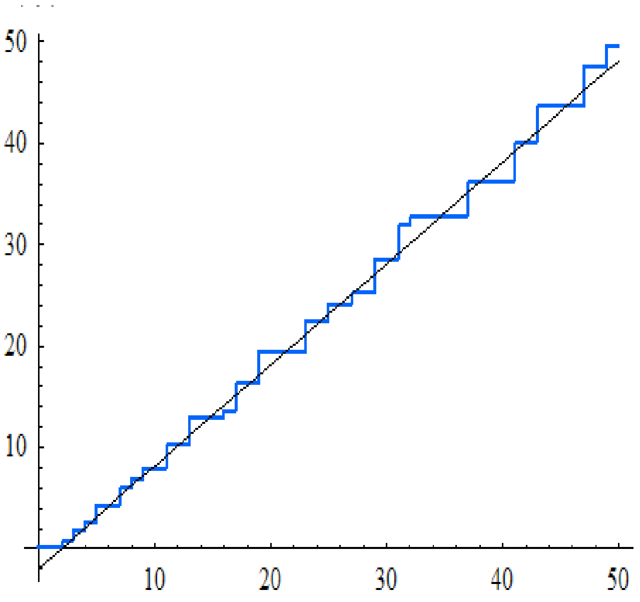

Graph of $Y=\Psi(X)$

WHY DO THE NONTRIVIAL ZEROS MATTER?

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Theorem. (von Mangoldt)

As a sum over the **nontrivial zeros** ρ *of* $\zeta(s)$, *we have*

$$\Psi(X) = X - \log(2\pi) - \frac{1}{2}\log(1 - x^{-2}) - \sum_{\rho} \frac{X^{\rho}}{\rho}.$$

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Gauss' Conjecture is true. We have that

$$\pi(X) \sim \mathrm{Li}(X) \sim \frac{X}{\log X}.$$

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Proof. We always have Re*(ρ)* < *1*. □

Terence Tao,
UCLA

Professor of Mathematics,
Department of Mathematics

Applications and partial progress

APPLICATIONS OF THE RIEMANN HYPOTHESIS

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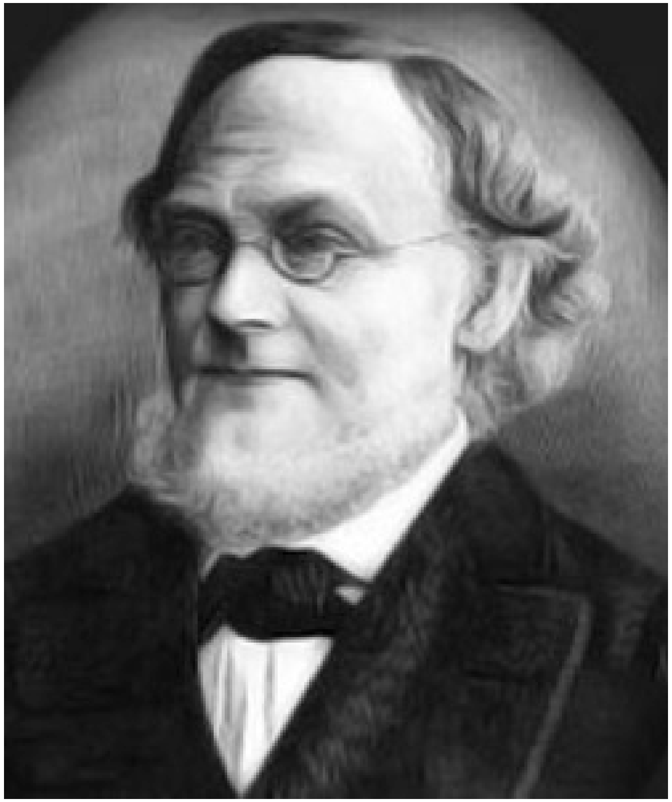
For instance:

APPLICATIONS OF THE RIEMANN HYPOTHESIS



Christian Goldbach (1690-1764)

In 1742, Christian Goldbach posed the following conjecture to Euler:

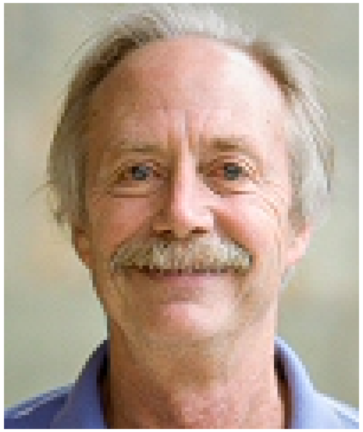
Odd Goldbach conjecture: every odd number greater than 5 is the sum of three primes.

In response, Euler made a stronger conjecture:

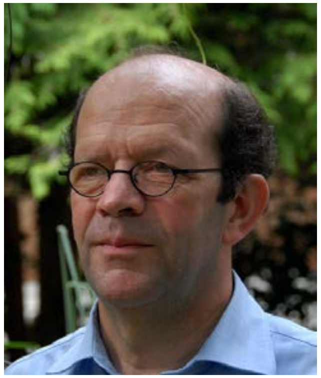
Even Goldbach conjecture: every even number greater than 2 is the sum of two primes.

APPLICATIONS OF THE RIEMANN HYPOTHESIS

Jean-Marc Deshouillers



Gove Effinger



Herman Te Riele

Dmitrii Zinoviev

It was not until 1997 that the **odd Goldbach conjecture** was conditionally resolved by Deshouillers, Effinger, Te Riele, and Zinoviev, assuming (a generalization of) the Riemann hypothesis.

The proof is six pages long and requires a certain amount of computer assistance.

APPLICATIONS OF THE RIEMANN HYPOTHESIS

Harold Helfgott

David Platt

What if one does not assume the Riemann hypothesis?

An unconditional proof of the **odd Goldbach conjecture** was eventually obtained by Helfgott in 2012.

The proof is 317 pages long and requires a substantial amount of computer assistance (including extensive numerical verification of the Riemann Hypothesis, carried out by Platt).

The **even Goldbach conjecture** remains unproven, even under the assumption of the Riemann Hypothesis and its generalisations.

APPLICATIONS OF THE RIEMANN HYPOTHESIS

Srinivasa Ramanujan (1887-1920)

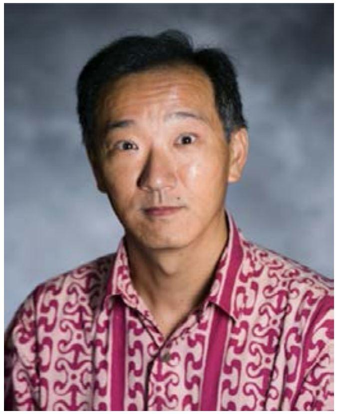
Ramanujan considered the question of what odd numbers cannot be expressed in the form $a^2+b^2+10c^2$ for some natural numbers a, b, c.

Conjecture: there are only eighteen odd numbers that cannot be expressed in the above form, namely

3, 7, 21, 31, 33, 43, 67, 79, 87, 133, 217, 219, 223, 253, 307, 391, 679, 2719.

(There are infinitely many even numbers that are not expressible in the above form, for instance any number of the form 16n+6.)

APPLICATIONS OF THE RIEMANN HYPOTHESIS



Ken Ono

Kannan Soundararajan

In 1997, Ono and Soundararajan established this conjecture assuming (a generalisation of) the Riemann Hypothesis.

At the present time, no unconditional proof of this conjecture is known.

PROGRESS ON THE RIEMANN HYPOTHESIS

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Despite strenuous efforts, we do not appear to be anywhere close to a proof of the Riemann hypothesis.

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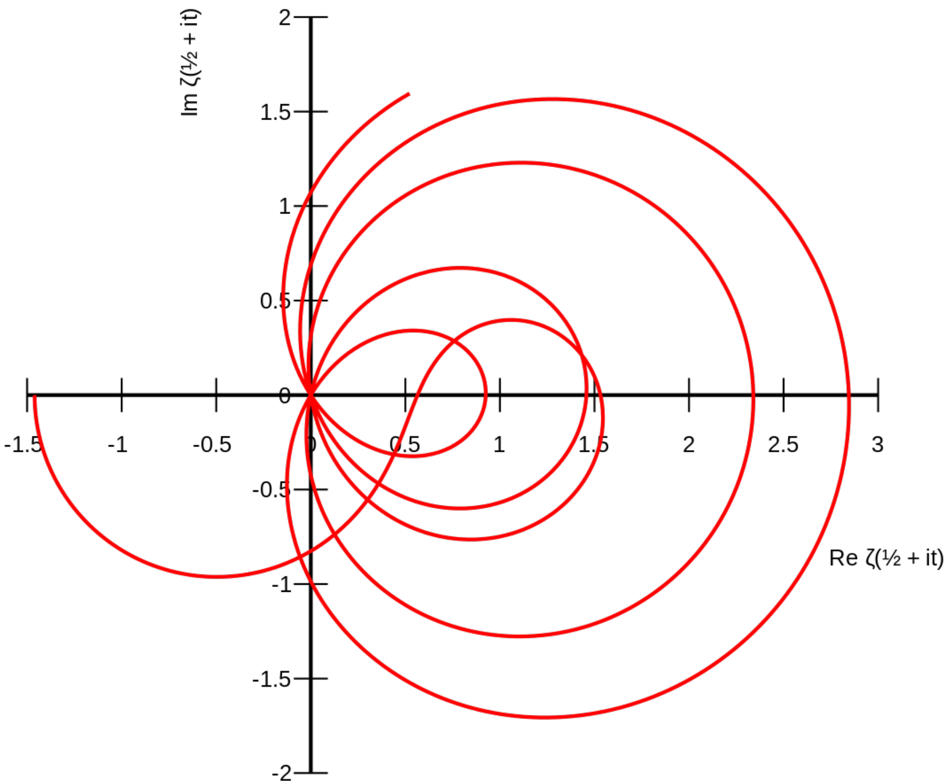
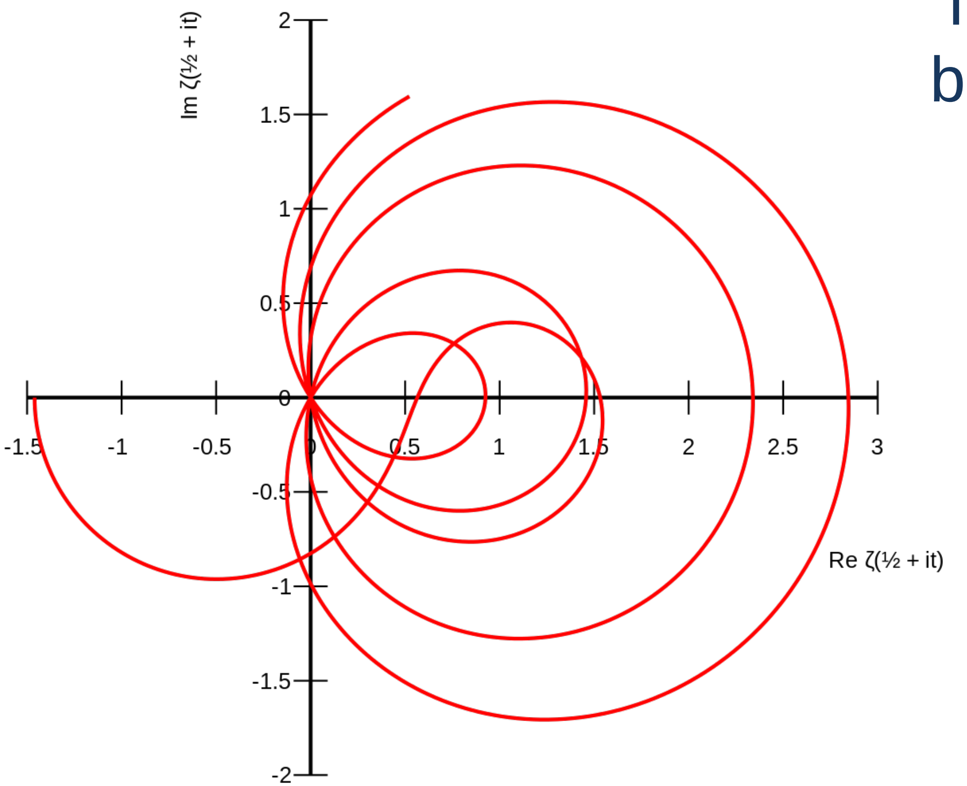
Nevertheless there are partial results and substitutes for the Riemann hypothesis that are useful.

PROGRESS ON THE RIEMANN HYPOTHESIS



PROGRESS ON THE RIEMANN HYPOTHESIS



There are algorithms that can be used to verify the Riemann hypothesis for any finite number of zeroes.

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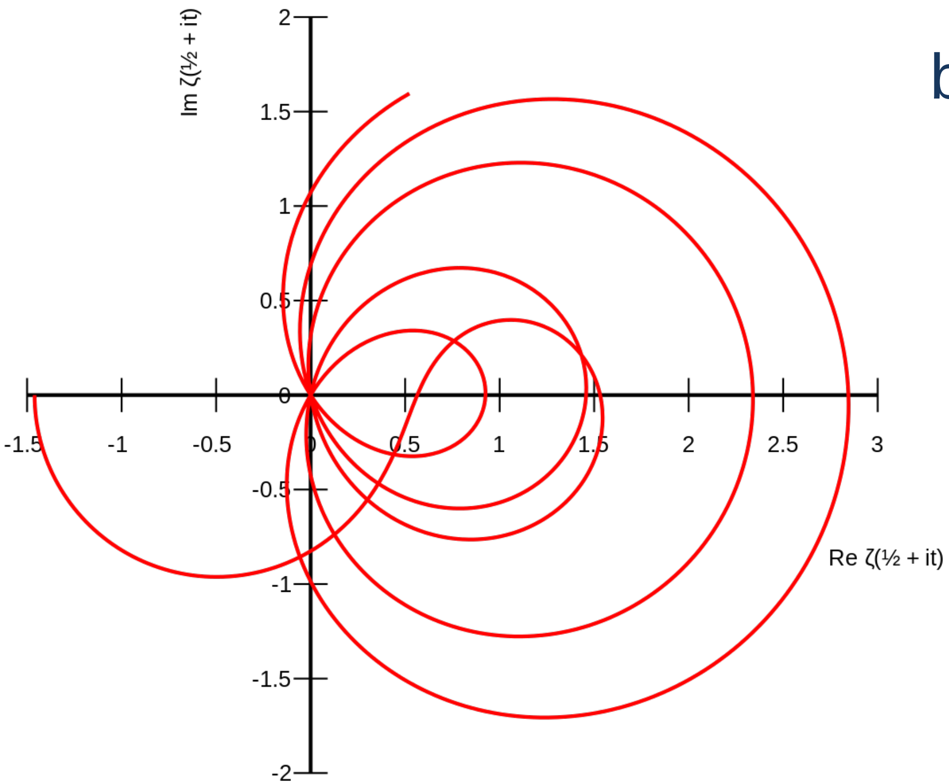
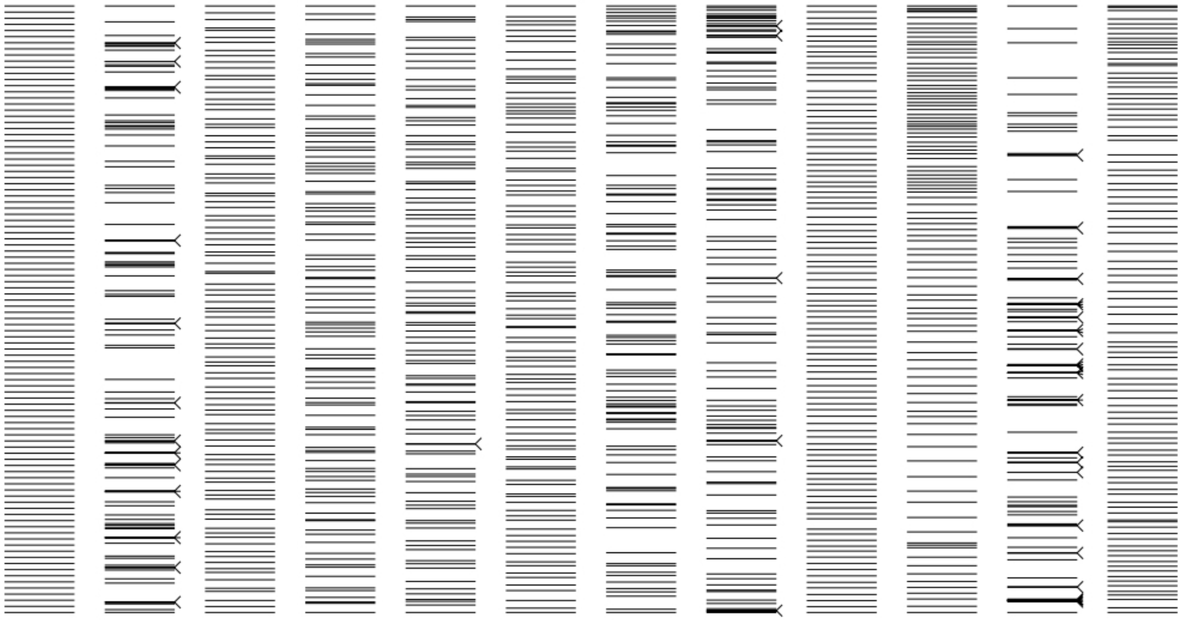


There are algorithms that can be used to verify the Riemann hypothesis for any finite number of zeroes.

For instance, it is known that the first ten trillion zeroes of the Riemann hypothesis lie on the critical line (Gourdon-Demichel 2004; Platt 2013).

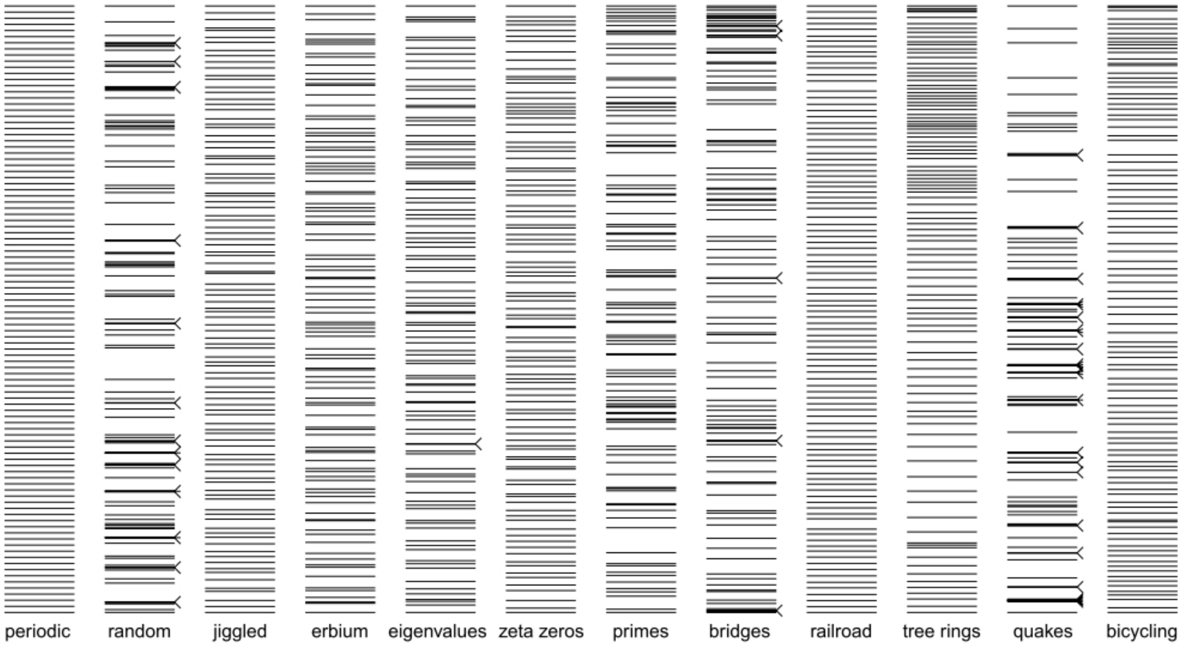
PROGRESS ON THE RIEMANN HYPOTHESIS



One-dimensional distributions each consist of 100 levels. From left to right the spectra are: a periodic array of evenly spaced lines; a random sequence; a periodic array perturbed by a slight random "jiggling" of each level; energy states of the erbium-166 nucleus, all having the same spin and parity quantum numbers; the central 100 eigenvalues of a 300-by-300 random symmetric matrix; positions of zeros of the Riemann zeta function lying just above the 10^{22}nd zero; 100 consecutive prime numbers beginning with 103,613; locations of the 100 northernmost overpasses and underpasses along Interstate 85; positions of crossties on a railroad siding; locations of growth rings from 1884 through 1983 in a fir tree on Mount Saint Helens, Washington; dates of California earthquakes with a magnitude of 5.0 or greater, 1969 to 2001; lengths of 100 consecutive bike rides.

From: "The spectrum of Riemannium", Brian Hayes, American Scientist, 2003

PROGRESS ON THE RIEMANN HYPOTHESIS



One-dimensional distributions each consist of 100 levels. From left to right the spectra are: a periodic array of evenly spaced lines; a random sequence; a periodic array perturbed by a slight random "jiggling" of each level; energy states of the erbium-166 nucleus, all having the same spin and parity quantum numbers; the central 100 eigenvalues of a 300-by-300 random symmetric matrix; positions of zeros of the Riemann zeta function lying just above the 10^{22}nd zero; 100 consecutive prime numbers beginning with 103,613; locations of the 100 northernmost overpasses and underpasses along Interstate 85; positions of crossties on a railroad siding; locations of growth rings from 1884 through 1983 in a fir tree on Mount Saint Helens, Washington; dates of California earthquakes with a magnitude of 5.0 or greater, 1969 to 2001; lengths of 100 consecutive bike rides.

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Incidentally, the statistics of these zeroes bears a striking resemblance to statistics of other sets of points, such as spectral lines of nuclei, eigenvalues of random matrices, or even the arrival times of buses in Cuernavaca, Mexico… but this is a topic for another (much longer) talk!

PROGRESS ON THE RIEMANN HYPOTHESIS

Enrico Bombieri (1940-)

Askold Ivanovich Vinogradov (1929-2005)

PROGRESS ON THE RIEMANN HYPOTHESIS

Enrico Bombieri (1940-)

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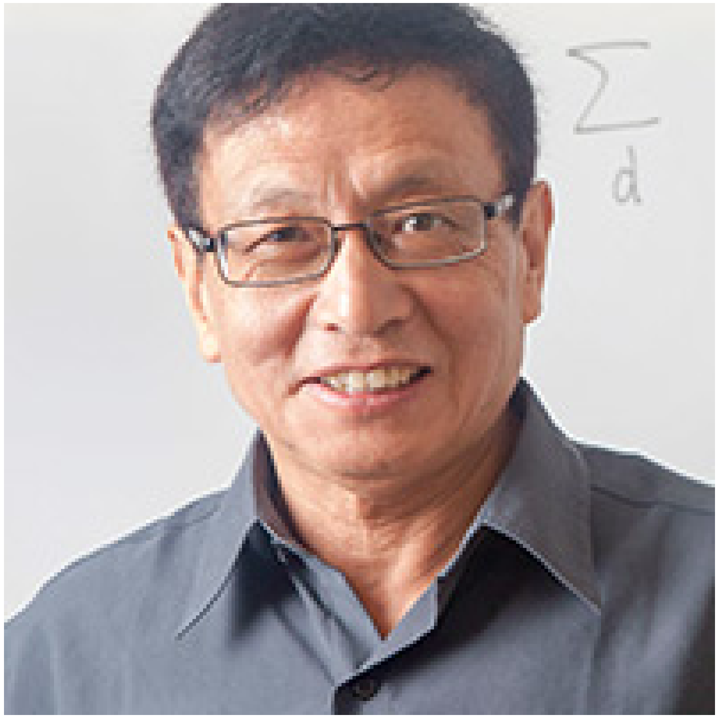
The Bombieri-Vinogradov theorem, established in 1965, gives information on the distribution of prime numbers in “most” arithmetic progressions, which is of comparable strength to what the (generalized) Riemann hypothesis gives. It has been called “the generalized Riemann hypothesis on the average”.

PROGRESS ON THE RIEMANN HYPOTHESIS



Yitang Zhang

In 2013, Yitang Zhang obtained a deep improvement of the Bombieri-Vinogradov theorem, which he then used to resolve a long-standing problem in analytic number theory:

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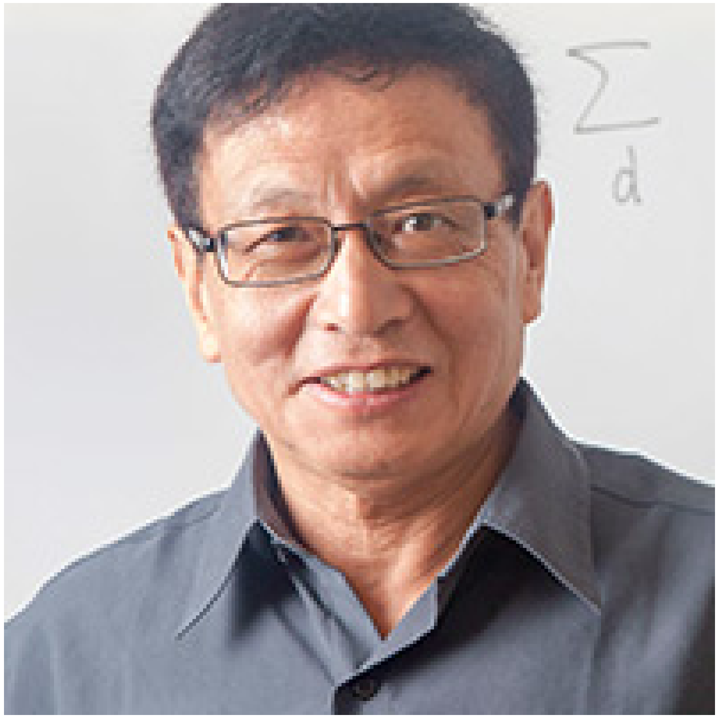

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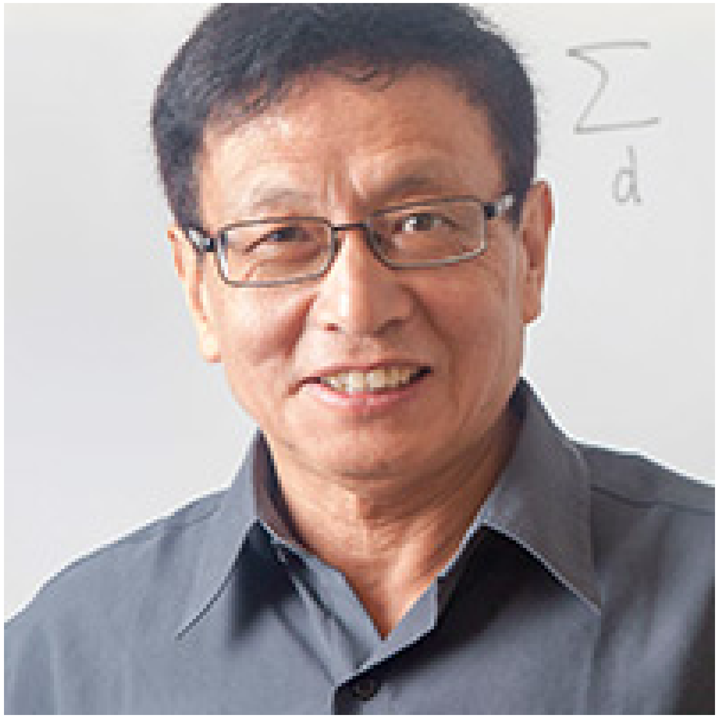

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Zhang's theorem: the gap between consecutive primes remains bounded infinitely often.

Zhang's initial bound was 70,000,000. The massively collaborative online **Polymath project** eventually reduced this bound to 246. The twin prime conjecture asserts that one can obtain the bound of 2!

PROGRESS ON THE RIEMANN HYPOTHESIS

Kaisa Matomaki

Maksym Radziwill

PROGRESS ON THE RIEMANN HYPOTHESIS

Kaisa Matomaki

Maksym Radziwill

The Riemann hypothesis can be used to study a class of functions known as multiplicative functions relating to the prime numbers. An example is the **Liouville function** $\lambda(n)$, defined to equal +1 when n is the product of an even number of primes, and -1 when n is the product of an odd number of primes.

PROGRESS ON THE RIEMANN HYPOTHESIS

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Maksym Radziwill

In 2015, Matomaki and Radziwill obtained a breakthrough theorem on multiplicative functions that gives even **more** information on their distribution than what the Riemann hypothesis was known to give! The Matomaki-Radziwill theorem has already had a major impact on the field, and its full consequences are still being explored.

PROGRESS ON THE RIEMANN HYPOTHESIS

Peter Sarnak

PROGRESS ON THE RIEMANN HYPOTHESIS

Peter Sarnak

“Right now, when we tackle problems without knowing the truth of the Riemann hypothesis, it's as if we have a screwdriver. But when we have it, it'll be more like a bulldozer.”

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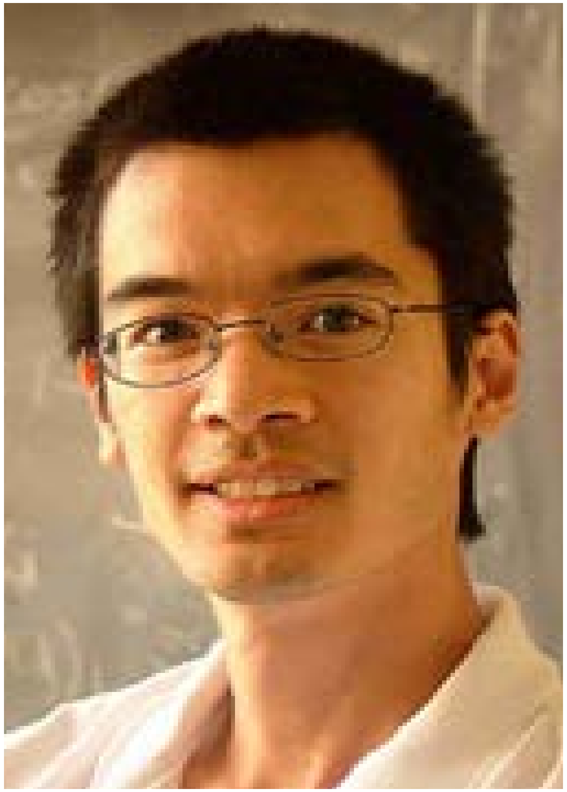
Ken Ono,
Emory University



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David Chu,
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