Multidimensional Poverty Measurement: The Way Forward?

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Why Multidimensional Poverty?

- Missing Dimensions
  - Just low income?
- Capability Approach
  - Conceptual framework
- Data
  - More sources
- Tools
  - Unidimensional measures into multidimensional
- Demand
  - Governments and other organizations

Hypothetical Challenge

- A government would like to create an official multidimensional poverty indicator
- Desiderata
  - It must **understandable** and easy to describe
  - It must conform to a **common sense** notion of poverty
  - It must fit the **purpose** for which it is being developed
  - It must be **technically** solid
  - It must be **operationally** viable
  - It must be easily **replicable**
- What would you advise?

Not So Hypothetical

- 2006 Mexico
  - Law: must alter official poverty methods
  - Include six other dimensions
    - education, dwelling space, dwelling services, access to food, access to health services, access to social security
- 2007 Oxford
  - Alkire and Foster "Counting and Multidimensional Poverty Measurement"
- 2009 Mexico
  - Announces official methodology

Continued Interest

- 2008 Bhutan
  - Gross National Happiness Index
- 2010 Chile
  - Major conference (May)
- 2010 London
  - Release of MPI by UNDP and OPHI (July)
- 2010 Colombia
  - Major conference (July)
- 2009-2011 Washington DC
  - World Bank (several), IDB, USAID, CGD
- 2008-2011 OPHI
  - Workshops on: Missing dimensions; Weights; Country applications; Applications to governance, quality of education, corruption, fair trade, and targeting; Robustness

Our Proposal - Overview

- Identification – Dual cutoffs
  - Deprivation cutoffs
  - Poverty cutoff
- Aggregation – Adjusted FGT

- Background papers
  - Alkire and Foster "Counting and Multidimensional Poverty Measurement" forthcoming *Journal of Public Economics*
  - Alkire and Santos "Acute Multidimensional Poverty: A new Index for Developing Countries" OPHI WP 38

Review: Unidimensional Poverty

Framework	– Sen 1976 identification and aggregation
Goal	– Poverty measure P(.)
Variable	– income consumption or other aggregate
Identification	– poverty line unchanged since Rowntree
Aggregation	– Foster-Greer-Thorbecke 1984
	see also Foster, Greer, and Thorbecke 2010
	- forthcoming *Journal of Economic Inequality*

Review: Unidimensional Poverty

Example Incomes $y = (7,3,4,8)$ Poverty line $z = 5$

Deprivation vector $g^0 = (0,1,1,0)$

Headcount ratio $P_0 = \mu(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 2/5, 1/5, 0)$

Poverty gap $= P_1 = \mu(g^1) = 3/20$

Squared gap vector $g^2 = (0, 4/25, 1/25, 0)$

FGT Measure $= P_2 = \mu(g^2) = 5/100$

Decomposable across population groups WB

Policy implications Bourguignon and Fields 1990

Multidimensional Data

Matrix of achievements for n persons in d domains

Multidimensional Data

Matrix of achievements for *n* persons in *d* domains

Domains

$$y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix}$$

Persons

Multidimensional Data

Matrix of achievements for *n* persons in *d* domains

Domains

$$\mathbf{y} = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix}$$

Persons

$$\mathbf{z} \quad (13 \quad 12 \quad 3 \quad 1)$$

Cutoffs

Multidimensional Data

Matrix of achievements for *n* persons in *d* domains

Domains

$$y = \begin{bmatrix} \mathbf{13.1} & \mathbf{14} & \mathbf{4} & \mathbf{1} \\ \mathbf{15.2} & \underline{\mathbf{7}} & \mathbf{5} & \underline{\mathbf{0}} \\ \underline{\mathbf{12.5}} & \underline{\mathbf{10}} & \underline{\mathbf{1}} & \underline{\mathbf{0}} \\ \mathbf{20} & \underline{\mathbf{11}} & \mathbf{3} & \mathbf{1} \end{bmatrix}$$ Persons

$$z \quad (\mathbf{13} \quad \mathbf{12} \quad \mathbf{3} \quad \mathbf{1})$$ Cutoffs

<u>These entries fall below cutoffs</u>

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

Domains

$$y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix}$$

Persons

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

Domains

$$\mathbf{g}^0 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Persons

Normalized Gap Matrix

Matrix of achievements for *n* persons in *d* domains

Domains

$$\boldsymbol{y} = \begin{bmatrix} \mathbf{13.1} & \mathbf{14} & \mathbf{4} & \mathbf{1} \\ \mathbf{15.2} & \underline{\mathbf{7}} & \mathbf{5} & \underline{\mathbf{0}} \\ \underline{\mathbf{12.5}} & \underline{\mathbf{10}} & \underline{\mathbf{1}} & \underline{\mathbf{0}} \\ \mathbf{20} & \underline{\mathbf{11}} & \mathbf{3} & \mathbf{1} \end{bmatrix}$$

Persons

$$\boldsymbol{z} \quad (\mathbf{13} \quad \mathbf{12} \quad \mathbf{3} \quad \mathbf{1})$$

Cutoffs

<u>These entries fall below cutoffs</u>

Normalized Gap Matrix

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

Domains

$$
y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \quad \text{Persons}
$$

$$
z \quad (13 \quad 12 \quad 3 \quad 1) \quad \text{Cutoffs}
$$

<u>These entries fall below cutoffs</u>

Normalized Gap Matrix

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

Domains

$$g^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix}$$

Persons

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

Domains

$$g^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix}$$

Persons

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

Domains

$$\boldsymbol{g}^2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.176} & \mathbf{0} & \mathbf{1} \\ \mathbf{0.002} & \mathbf{0.029} & \mathbf{0.449} & \mathbf{1} \\ \mathbf{0} & \mathbf{0.006} & \mathbf{0} & \mathbf{0} \end{bmatrix} \text{Persons}$$

Identification

Domains

$$g^{0} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{Persons}$$

Matrix of deprivations

Identification – Counting Deprivations

	Domains				c	
$g^0 =$	0	0	0	0	0	Persons
	0	1	0	1	2	
	1	1	1	1	4	
	0	1	0	0	1	

Identification – Counting Deprivations

Q/ Who is poor?

Domains $\qquad\qquad c$

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

Domains $\qquad c$

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Union Approach

Q/ Who is poor?

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$$
\begin{array}{c}
\text{Domains} \qquad\qquad c \\
g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ \underline{1} \end{matrix} \quad \text{Persons}
\end{array}
$$

Difficulties

Identification – Union Approach

Q/ Who is poor?

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Difficulties

Single deprivation may be due to something other than poverty (UNICEF)

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

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\begin{array}{c} \text{Domains} \\ g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}
\quad
\begin{array}{c} c \\ 0 \\ \underline{2} \\ \underline{4} \\ \underline{1} \end{array}
\quad \text{Persons}
$$

Difficulties

Single deprivation may be due to something other than poverty (UNICEF)

Union approach often predicts *very* high numbers - political constraints

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

Domains $\qquad c$

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

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\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ 2 \\ \underline{4} \\ 1 \end{matrix} \quad \text{Persons}
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Difficulties

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Persons

Difficulties

Demanding requirement (especially if d large)

Identification – Intersection Approach

Q/ Who is poor?

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\end{array}
\quad
\begin{array}{c} c \\ 0 \\ 2 \\ \underline{4} \\ 1 \end{array}
\quad \text{Persons}
$$

Difficulties

- Demanding requirement (especially if d large)
- Often identifies a very narrow slice of population

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$

Domains $\quad\quad c$

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $c_i \geq k$ **(Ex: k = 2)**

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\end{array}
\quad
\begin{array}{c}
c \\
0 \\ \underline{2} \\ \underline{4} \\ 1
\end{array}
\quad \text{Persons}
$$

Identification – Dual Cutoff Approach

Q/ Who is poor?

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$$g^0 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \begin{matrix} \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{1} \end{matrix}$$

Persons

Note

Includes both union and intersection

Identification – Dual Cutoff Approach

Q/ Who is poor?

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Persons

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Includes both union and intersection

Especially useful when number of dimensions is large

Union becomes too large, intersection too small

Identification – Dual Cutoff Approach

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\end{array}
\quad
\begin{array}{c}
c \\ 0 \\ \underline{2} \\ \underline{4} \\ 1
\end{array}
\quad \text{Persons}
$$

Note

Includes both union and intersection

Especially useful when number of dimensions is large

Union becomes too large, intersection too small

Next step - *aggregate* into an overall measure of poverty

Aggregation

Domains $\quad c$

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{matrix}$$

Persons

Aggregation

Censor data of nonpoor

Domains $\quad c$

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{matrix}$$

Persons

Aggregation

Censor data of nonpoor

Domains $\qquad c(k)$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Aggregation

Censor data of nonpoor

Domains $\quad\quad$ $c(k)$

$$\mathbf{g^0(k)} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \begin{matrix} \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{0} \end{matrix}$$

Persons

Similarly for $g^1(k)$, etc

Aggregation – Headcount Ratio

	Domains				$c(k)$	
	0	**0**	**0**	**0**	**0**	
$g^0(k) =$	**0**	**1**	**0**	**1**	$\underline{\mathbf{2}}$	Persons
	1	**1**	**1**	**1**	$\underline{\mathbf{4}}$	
	0	**0**	**0**	**0**	**0**	

Aggregation – Headcount Ratio

Domains $c(k)$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Two poor persons out of four: **H = ½** **‘incidence’**

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{c} \text{Domains} \\ g^0(k) = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \end{array}
\quad
\begin{array}{c} c(k) \\ \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{0} \end{array}
\quad \text{Persons}
$$

Two poor persons out of four: **H = ½** **'incidence'**

Critique

Suppose the number of deprivations rises for person 2

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad c(k) = \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Domains (columns); Persons (rows)

Two poor persons out of four: **H = ½** **'incidence'**

Critique

Suppose the number of deprivations rises for person 2

Domains $\quad c(k)$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Two poor persons out of four: **H = ½** **‘incidence’**

No change!

Critique

Suppose the number of deprivations rises for person 2

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} c(k) \\ 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Domains (columns); Persons (rows)

Two poor persons out of four: **H = ½** **'incidence'**

No change!

Violates 'dimensional monotonicity'

Aggregation

Return to the original matrix

Domains $\quad c(k)$

$$g^{0}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Aggregation

Return to the original matrix

Domains $\quad c(k)$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Aggregation

Need to augment information

Domains $\qquad$ $c(k)$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Aggregation

Need to augment information "deprivation share"

	Domains				$c(k)$	$c(k)/d$	
$g^0(k) =$	0	0	0	0	0		Persons
	0	1	0	1	$\underline{2}$	2 / 4	
	1	1	1	1	$\underline{4}$	4 / 4	
	0	0	0	0	0		

Aggregation

Need to augment information ‘deprivation share’

‘intensity’

Domains $c(k)$ $c(k)/d$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} \quad \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix}$$

Persons

A = average intensity among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA

	Domains				$c(k)$	$c(k)/d$	
$g^0(k) =$	0	0	0	0	0		Persons
	0	1	0	1	$\underline{2}$	2 / 4	
	1	1	1	1	$\underline{4}$	4 / 4	
	0	0	0	0	0		

A = average intensity among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = **$\mu(g^0(k))$**

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Domains	$c(k)$	$c(k)/d$	
0 0 0 0	0		
0 1 0 1	$\underline{2}$	2 / 4	Persons
1 1 1 1	$\underline{4}$	4 / 4	
0 0 0 0	0		

A = average intensity among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 6/16 = .375**

	Domains				$c(k)$	$c(k)/d$	
$g^0(k) =$	**0**	**0**	**0**	**0**	**0**		
	0	**1**	**0**	**1**	**2**	**2 / 4**	Persons
	1	**1**	**1**	**1**	**4**	**4 / 4**	
	0	**0**	**0**	**0**	**0**		

A = average intensity among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 6/16 = .375**

	Domains				$c(k)$	$c(k)/d$	
	0	0	0	0	0		
	0	1	0	1	2	2 / 4	
$g^0(k) =$	1	1	1	1	4	4 / 4	Persons
	0	0	0	0	0		

A = average intensity among poor = 3/4

Note: if person 2 has an additional deprivation, M_0 rises

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mu(g^0(k))$ **= 6/16 = .375**

	Domains				$c(k)$	$c(k)/d$	
$g^0(k) =$	0	0	0	0	0		
	0	1	0	1	2	2 / 4	Persons
	1	1	1	1	4	4 / 4	
	0	0	0	0	0		

A = average intensity among poor = 3/4

Note: if person 2 has an additional deprivation, M_0 rises

Satisfies dimensional monotonicity

Aggregation – Adjusted Headcount Ratio

Observations

Aggregation – Adjusted Headcount Ratio

Observations

- Uses **ordinal** data

Aggregation – Adjusted Headcount Ratio

Observations

- Uses **ordinal** data
- Similar to traditional **gap** $P_1 = HI$
  - HI = per capita poverty gap
    - = headcount H times average income gap I among poor

Aggregation – Adjusted Headcount Ratio

Observations

- Uses **ordinal** data
- Similar to traditional **gap** $P_1 = HI$
  - HI = per capita poverty gap
    - = headcount H times average income gap I among poor
  - HA = per capita deprivation
    - = headcount H times average intensity A among poor

Aggregation – Adjusted Headcount Ratio

Observations

Uses **ordinal** data

Similar to traditional **gap** $P_1 = HI$

HI = per capita poverty gap

= headcount H times average income gap I among poor

HA = per capita deprivation

= headcount H times average intensity A among poor

Decomposable across **dimensions** after identification

$M_0 = \sum_j H_j/d$

Aggregation – Adjusted Headcount Ratio

Observations

Uses **ordinal** data

Similar to traditional **gap** $P_1 = HI$

HI = per capita poverty gap

= headcount H times average income gap I among poor

HA = per capita deprivation

= headcount H times average intensity A among poor

Decomposable across **dimensions**

$M_0 = \sum_j H_j/d$

Axioms - *Characterization via freedom*

Adjusted Headcount Ratio

Note

M_0 requires only ordinal information.

Q/

What if data are cardinal?

How to incorporate information on *depth* of deprivation?

Aggregation: Adjusted Poverty Gap

Augment information of M_0 using normalized gaps

Domains

$$g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Aggregation: Adjusted Poverty Gap

Augment information of M_0 using normalized gaps

Domains

$$g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Average **gap** across all deprived dimensions of the poor:

$$G = (0.04+0.42+0.17+0.67+1+1)/6$$

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = M_1 = M_0G = HAG

Domains

$$g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Average **gap** across all deprived dimensions of the poor:

$$G = (0.04+0.42+0.17+0.67+1+1)/6$$

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0 G$ = HAG = $\mu(g^1(k))$

$$
g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Domains (columns); Persons (rows)

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Persons

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then M_1 will rise.

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = M_1 = M_0G = HAG = **$\mu(g^1(k))$**

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$$g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then M_1 will rise.

Satisfies monotonicity – reflects incidence, intensity, depth

Aggregation: Adjusted FGT

Consider the matrix of squared gaps

Domains

$$
g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Persons

Aggregation: Adjusted FGT

Consider the matrix of squared gaps

Domains

$$g^2(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Aggregation: Adjusted FGT

Adjusted FGT is M_2 = $\mu(g^2(k))$

Domains

$$g^2(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Aggregation: Adjusted FGT

Adjusted FGT is $M_2 = \mu(g^2(k))$

$$g^2(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Domains (columns); Persons (rows)

Satisfies transfer axiom

– reflects incidence, intensity, depth, severity

– focuses on most deprived

Aggregation: Adjusted FGT Family

Adjusted FGT is $M_\alpha = \mu(g^\alpha(\tau))$ for $\alpha \geq 0$

Domains

$$g^\alpha(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^\alpha & 0 & 1^\alpha \\ 0.04^\alpha & 0.17^\alpha & 0.67^\alpha & 1^\alpha \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Aggregation: Adjusted FGT Family

Adjusted FGT is $M_\alpha = \mu(g^\alpha(\tau))$ for $\alpha \geq 0$

Domains

$$g^\alpha(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^\alpha & 0 & 1^\alpha \\ 0.04^\alpha & 0.17^\alpha & 0.67^\alpha & 1^\alpha \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Satisfies numerous properties including decomposability, and dimension monotonicity, monotonicity (for $\alpha > 0$), transfer (for $\alpha > 1$).

Weights

- Weighted identification
  - Weight on first dimension (say income): 2
  - Weight on other three dimensions: 2/3
    - Cutoff k = 2
      - Poor if income poor, or suffer two or more deprivations
    - Cutoff k = 2.5 (or make inequality strict)
      - Poor if income poor and suffer one or more other deprivations
        - Nolan, Brian and Christopher T. Whelan, Resources, Deprivation and Poverty, 1996
- Weighted aggregation
  - Weighted intensity – otherwise same

Caveats and Observations

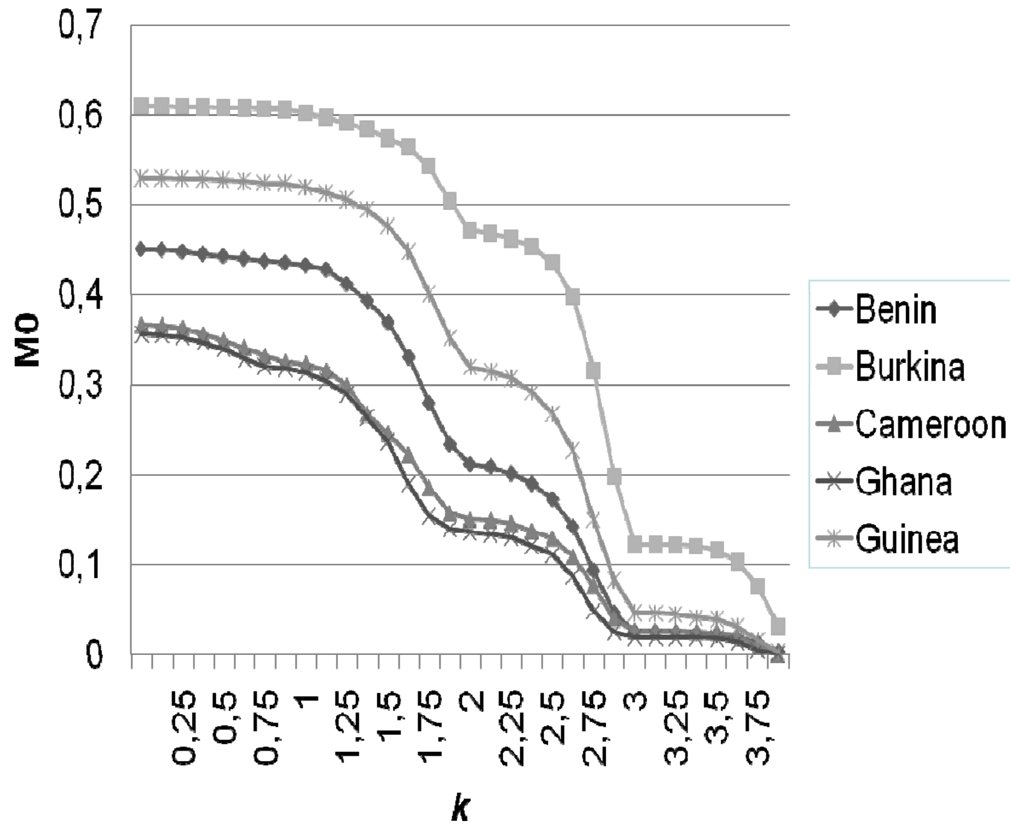
- Identification
  - No tradeoffs across dimensions
  - Fundamentally multidimensional
    - Need to set deprivation cutoffs
    - Need to set weights
    - Need to set poverty cutoff across dimension
- Aggregation
  - Neutral
    - Ignores coupling of disadvantages
    - Not substitutes, not complements
  - Discontinuities

Sub-Sahara Africa: Robustness Across k

Burkina is *always* poorer than Guinea, regardless of whether we count as poor persons who are deprived in only *one* kind of assets (0.25) or *every* dimension (assets, health, education, and empowerment, in this example). (DHS Data used)

Batana, 2008- OPHI WP 13

Figure 3: M0 as cutoff *k* is varied in the five countries



Advantages

- Intuitive
- Transparent
- Flexible
  - MPI – Acute poverty
  - Country Specific Measures
    - Policy impact and good governance
    - Targeting
  - Accounting structure for evaluating policies
  - Participatory tool

Revisit Objectives

- Desiderata
  - It must **understandable** and easy to describe
  - It must conform to a **common sense** notion of poverty
  - It must fit the **purpose** for which it is being developed
  - It must be **technically** solid
  - It must be **operationally** viable
  - It must be easily **replicable**
- What do you think?

Thank you

Thank you

Illustration: USA

Data Source: National Health Interview Survey, 2004, *United States Department of Health and Human Services. National Center for Health Statistics* - ICPSR 4349.

Tables Generated By: Suman Seth.

Unit of Analysis: Individual.

Number of Observations: 46009.

Variables:

(1) *income* measured in poverty line increments and grouped into 15 categories

(2) self-reported *health*

(3) health *insurance*

(4) years of *schooling*.

Illustration: USA

Profile of US Poverty by Ethnic/Racial Group

Illustration: USA

Profile of US Poverty by Ethnic/Racial Group

1	2	3
Group	**Population**	**Percentage Contrib.**
Hispanic	9100	19.8%
White	29184	63.6%
African American	5742	12.5%
Others	1858	4.1%
Total	**45884**	**100.0%**

Illustration: USA

Profile of US Poverty by Ethnic/Racial Group

1 Group	2 Population	3 Percentage Contrib.	4 Income Poverty Headcount	5 Percentage Contrib.
Hispanic	9100	19.8%	0.23	37.5%
White	29184	63.6%	0.07	39.1%
African American	5742	12.5%	0.19	20.0%
Others	1858	4.1%	0.10	3.5%
Total	**45884**	**100.0%**	**0.12**	**100.0%**

Illustration: USA

Profile of US Poverty by Ethnic/Racial Group

1	4	5
Group	Income Poverty Headcount	Percentage Contrib.
Hispanic	0.23	37.5%
White	0.07	39.1%
African American	0.19	20.0%
Others	0.10	3.5%
Total	**0.12**	**100.0%**

Illustration: USA

Profile of US Poverty by Ethnic/Racial Group

1 Group	4 Income Poverty Headcount	5 Percentage Contrib.	6 *H*	7 Percentage Contrib.
Hispanic	0.23	37.5%	0.39	46.6%
White	0.07	39.1%	0.09	34.4%
African American	0.19	20.0%	0.21	16.0%
Others	0.10	3.5%	0.12	3.0%
Total	**0.12**	**100.0%**	**0.16**	**100.0%**

Illustration: USA

Profile of US Poverty by Ethnic/Racial Group

1 Group	4 Income Poverty Headcount	5 Percentage Contrib.	8 M_0	9 Percentage Contrib.
Hispanic	0.23	37.5%	0.229	47.8%
White	0.07	39.1%	0.050	33.3%
African American	0.19	20.0%	0.122	16.1%
Others	0.10	3.5%	0.067	2.8%
Total	**0.12**	**100.0%**	**0.09**	**100.0%**

Illustration: USA

1	2	3	4	5	6
Ethnicity	***H_1* Income**	***H_2* Health**	***H_3* H. Insurance**	***H_4* Schooling**	***M_0***
Hispanic	0.200	0.116	0.274	0.324	0.229
Percentage Contribution	21.8%	12.7%	30.0%	35.5%	100%
White	0.045	0.053	0.043	0.057	0.050
Percentage Contribution	22.9%	26.9%	21.5%	28.7%	100%
Black	0.142	0.112	0.095	0.138	0.122
Percentage Contribution	29.1%	23.0%	19.5%	28.4%	100%
Others	0.065	0.053	0.071	0.078	0.067
Percentage Contribution	24.2%	20.0%	26.5%	29.3%	100%