



# **Use of Mathematical Models in Developing Strategies for Sustainability**

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# Outline

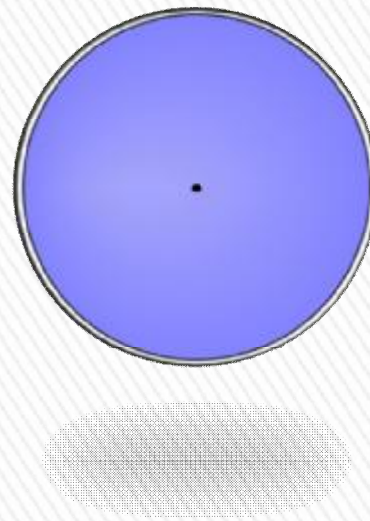
- Introduction
- Mathematical models in general
- Mathematical Models in agriculture
- Protected Agriculture -  
Controlled Environment Agriculture
- Use of mathematical in PA
- Hydraulic Grade Line Method
- Wheat Emergence Model





Mathematical  
Modeling

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# Mathematical Modeling

The art of approximation is crucial to most successful applications of theory to real problems; however, the proper level of simplification is not always obvious at the outset

John Tranquada, 1998





# Mathematical Modeling

A mathematical model is a simplified mathematical construct related to a part of reality and created for a particular purpose.

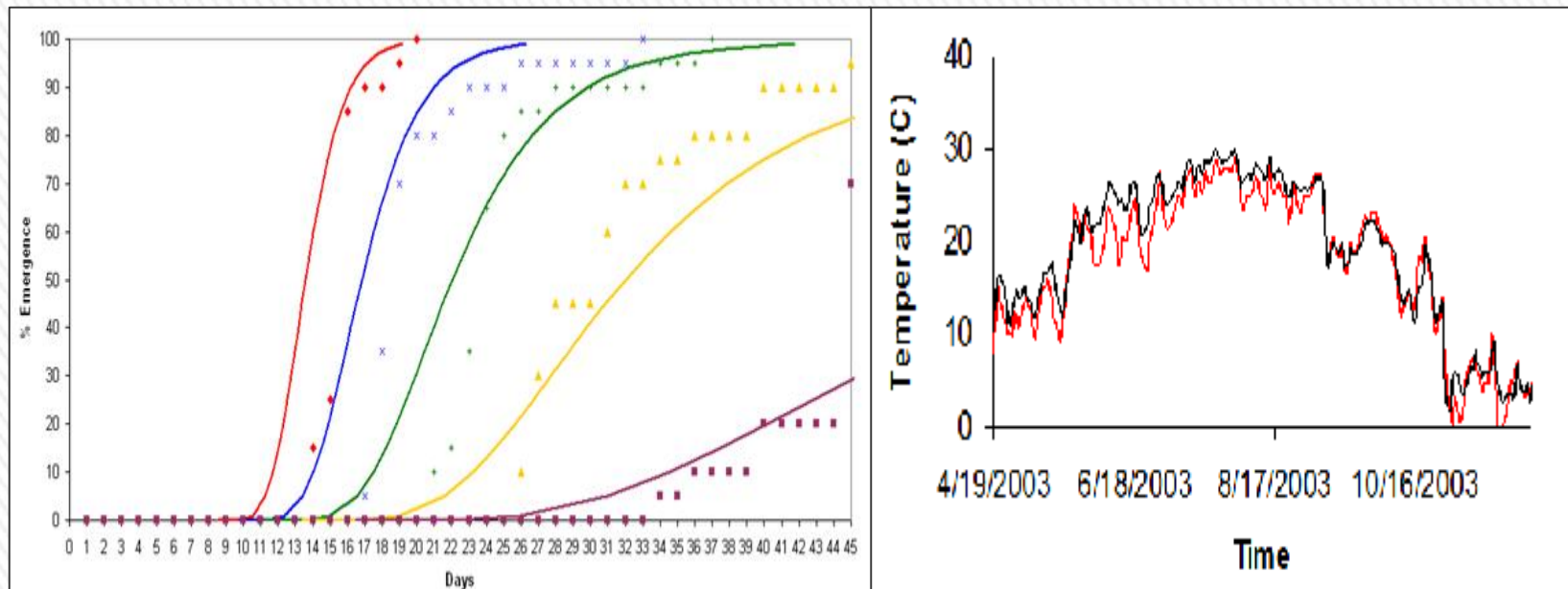
It is a simplified description of a system, built to help us better understand the operation of a real system and the interactions of its main components.

Edward A. Bender , 2000;Hamid and Abdullah, 2008



# Mathematical Modeling

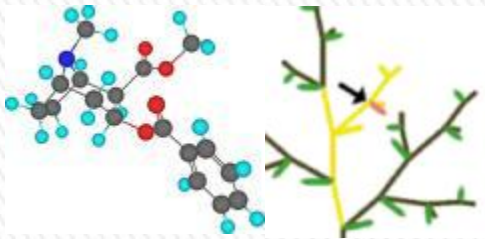
It is an/set of equation(s) which represents the behavior of a system where the variables of the model are corresponding to the observed quantities



Thornley and France, 2007; Al-Mulla, 2004



# Mathematical Modeling & Agriculture



Mathematical modeling plays an integral role in sustainable agricultural development.

McPhee, 2009; Hamid and Abdullah, 2008





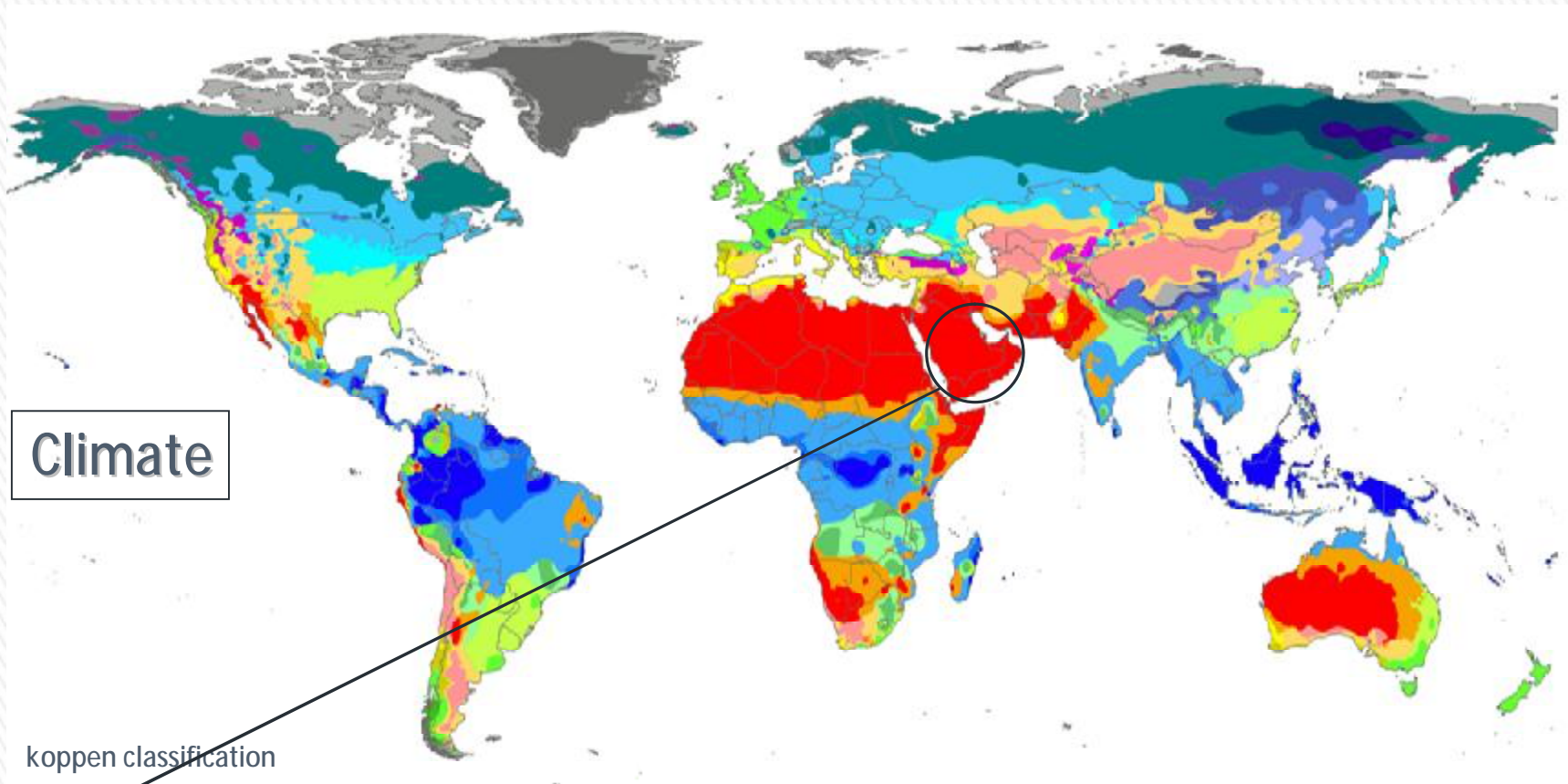


# Applications of Mathematical Modeling in Ag.

1. Protected Agriculture (PA)  
Controlled Environment Agriculture (CEA)
2. Irrigation systems
3. Wheat production



# 1. Why Protected Agriculture (PA)?



Climate

koppen classification

BWh	Csa	Cfa
BWk	Csb	Cfb
BSh	Cfc	
BSk		
Dsa	Dfa	ET
Dsb	Dfb	EF
Dsc	Dfc	

B = Dry Zones = Water supply is very scarce

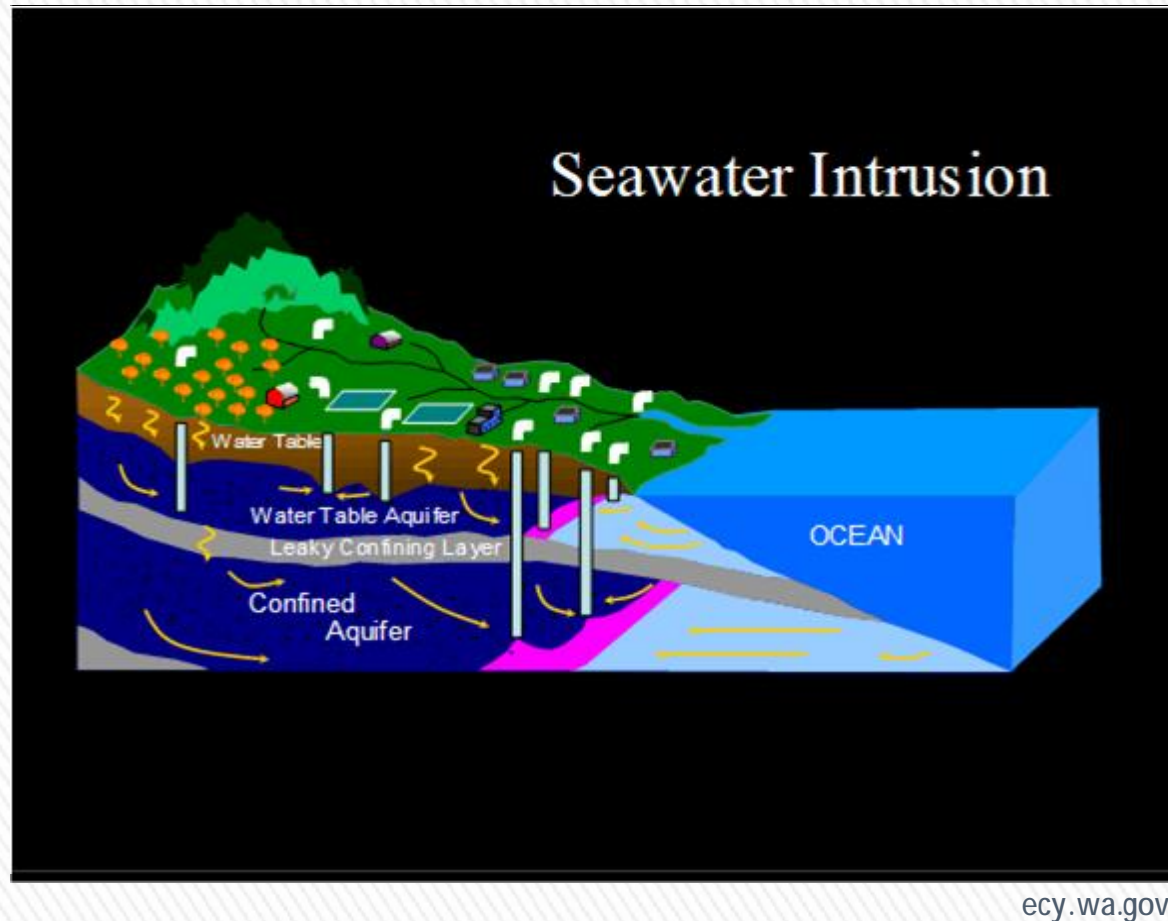
W = desert or arid =  $P_{\text{mean}} = 100 \text{ mm}$

h = hot = Summer v. hot ( $54^{\circ}\text{C}$ ), winter warm & sunny



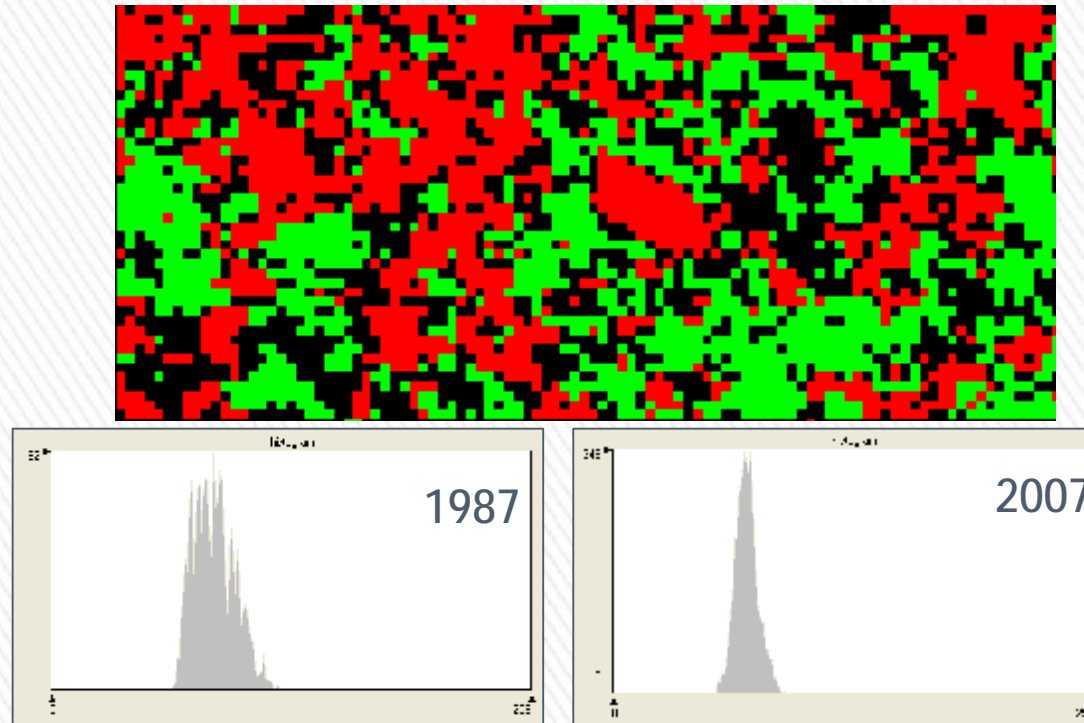
# 1. Why Protected Agriculture (PA)?

In addition to ...



# 1. Why Protected Agriculture (PA)?

Change in vegetation due to salinity between 1987 and 2007



**Red:** decrease in vegetation (32%)  
**Green:** increase in vegetation (29%)  
**Black:** unchanged

Al-Mulla, 2010



# 1. Protected Agriculture (PA)

- Therefore, application of protected agricultural structures (greenhouse/screenhouses) a very crucial in this region
  - protects crops in controlled environment.
  - provides a suitable environmental conditions to produce a specific crop at any time and any where.
  - extends the growing season
  - ensures high profitability and sustainability of an agricultural enterprise.



# 1. Protected Agriculture (PA)

- Because of that the number of greenhouses (screenhouses) in the Arabian Peninsula has increased substantially in recent years.

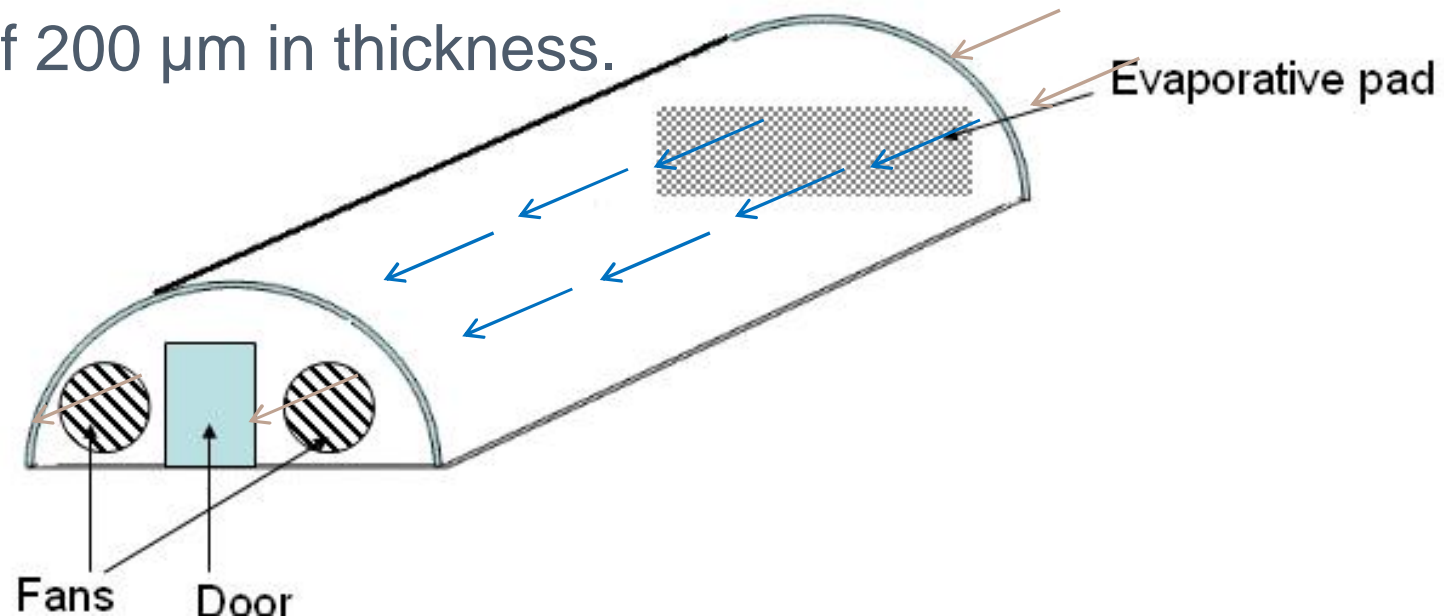


# 1. Protected Agriculture (PA)

## Greenhouses:



- The most common greenhouse type used in the region is the single-span Quonset greenhouse covered with polyethylene sheets of 200  $\mu\text{m}$  in thickness.

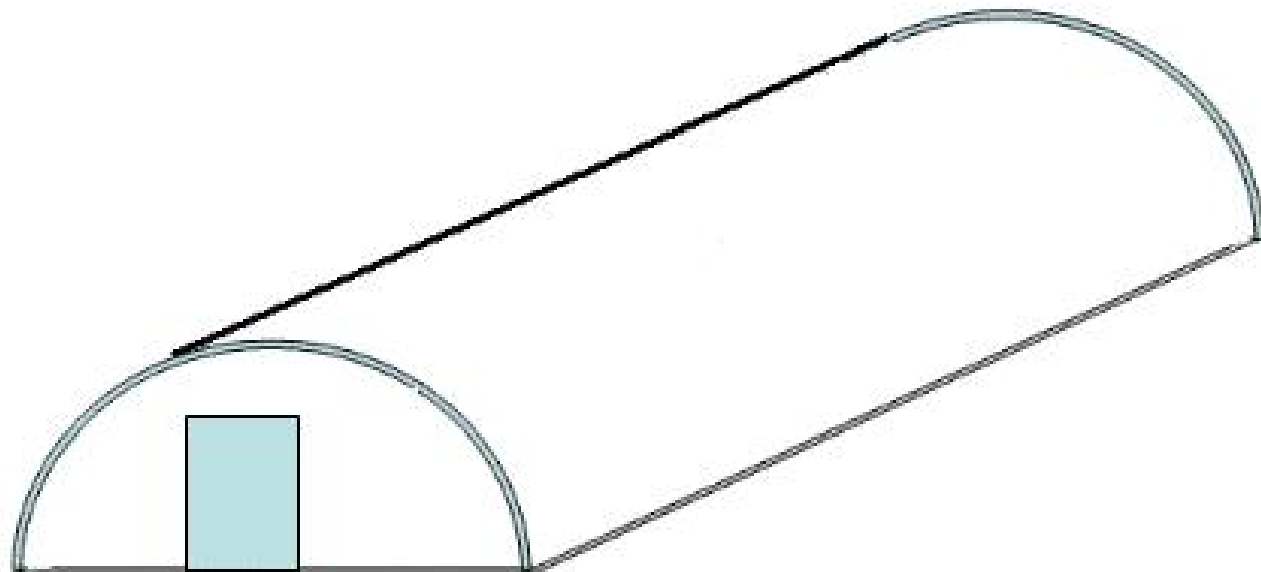


# 1. Protected Agriculture (PA)

## Screenhouses:



- Screenhouses is a 100% natural ventilation system. It is covered with white, UV, 80 micron insect proof screen opening size





# Problem

- These greenhouses are mostly imported
- The infestation of pests on crops evaluated under the current greenhouses did rise up a big concern about the design and structure of the current greenhouses.
- Although fresh water is scarce in the region, it is still used in the cooling systems of the greenhouses.

# Objectives

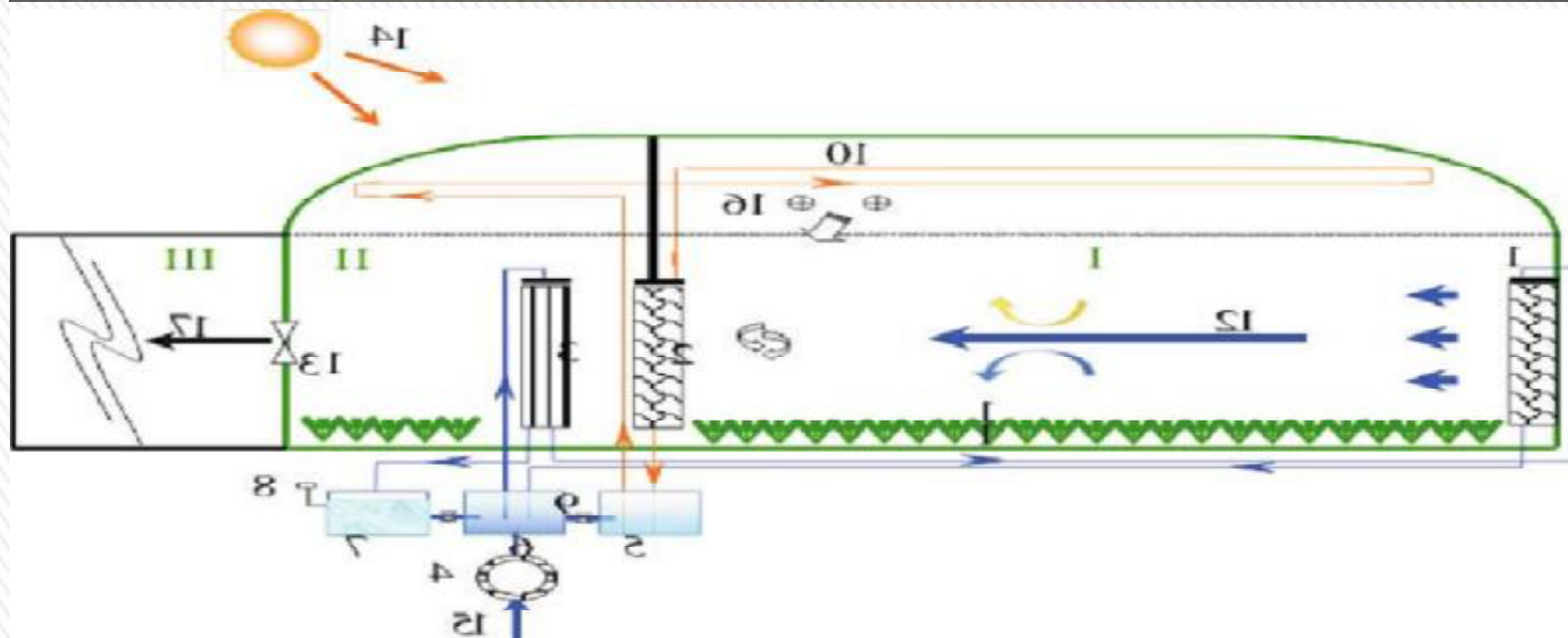
- Suitable and more profitable PA structure is needed to be designed.
- The aim of this presentation is to evaluate the application of heat transfer mathematical models in evaluating the effect of the design of modified PA structures on the inside microclimate and by which the selection of better design can be made for our region's environmental condition.



## Seawater instead of fresh water...



# Seawater instead of fresh water...



T. TAHRI, S. ABDUL-WAHAB, A. BETTAHAR, M. DOUANI, H. AL-HINAI, Y. AL-MULLA, 2009



# Mathematical Model approach

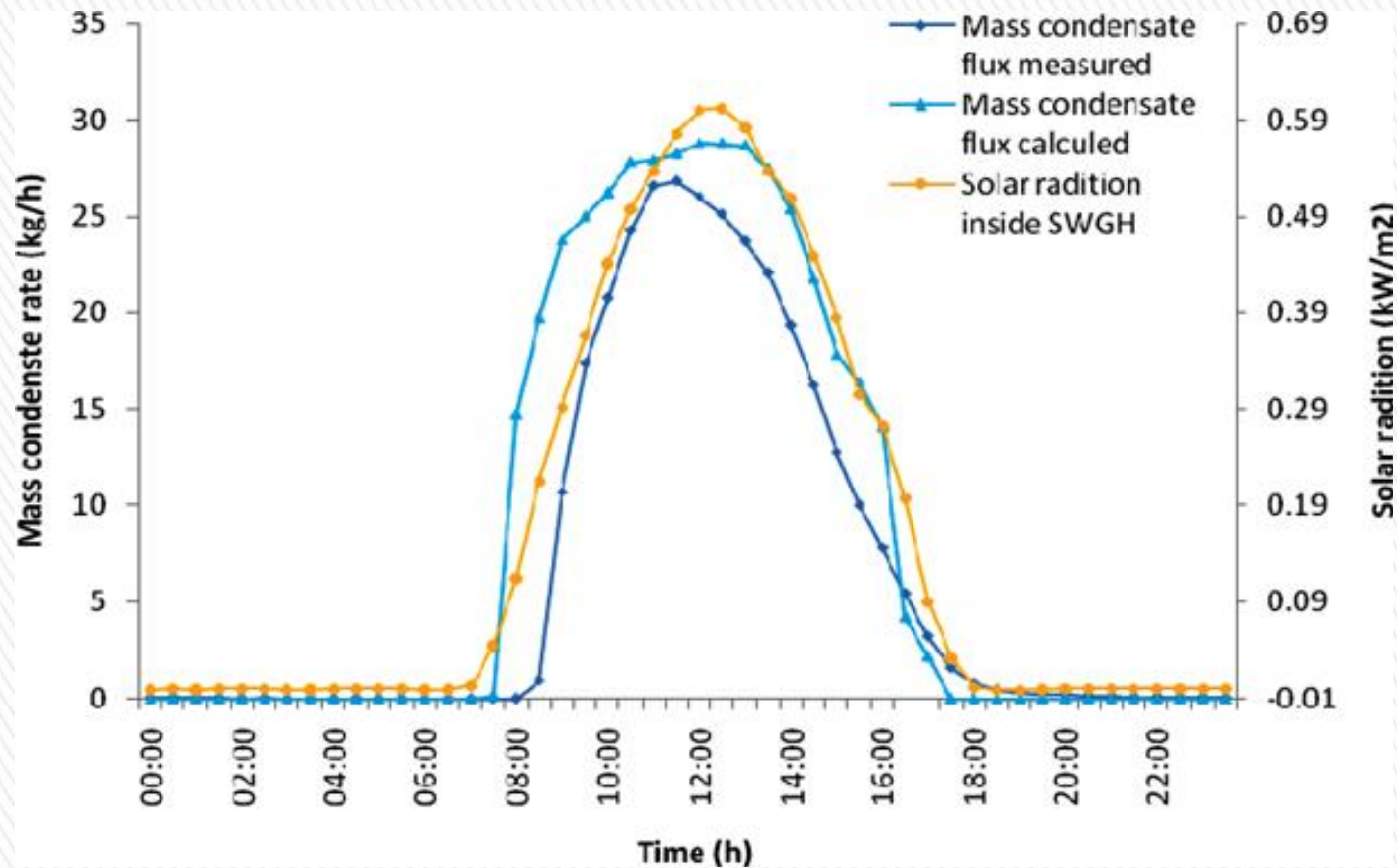
$$m_{\text{tot}} = \sum_{i=1}^{N_{\text{tot}}} \frac{h_{\text{vert}} A_{\text{out}} (T_{\text{sat}} - T_{\text{wout}})}{h_{\text{fg}} + 0.68 C_{\text{pl}} (T_{\text{sat}} - T_{\text{wout}})}$$

$A_{\text{out}}$	area of the outer surface of wall tube, m <sup>2</sup>
$C_{pL}$	specific heat capacity of liquid water, J/kg
$h_{\text{vert}}$	corrected heat transfer coefficient for film condensation in the presence of non-condensable gas, W/m <sup>2</sup> °C
$h_{\text{fg}}$	latent heat of vaporization at saturation temperature, kJ/kg
$T_{\text{sat}}$	saturation temperature of air, K
$T_{\text{swout}}$	outlet seawater temperature in one tube of the row, °C

T. TAHRI, S. ABDUL-WAHAB, A. BETTAHAR, M. DOUANI, H. AL-HINAI, Y. AL-MULLA, 2009

M. DOUANI, T. TAHRI, S. ABDUL-WAHAB, A. BETTAHAR, H. AL-HINAI, Y. AL-MULLA, 2011

# Mathematical Model approach

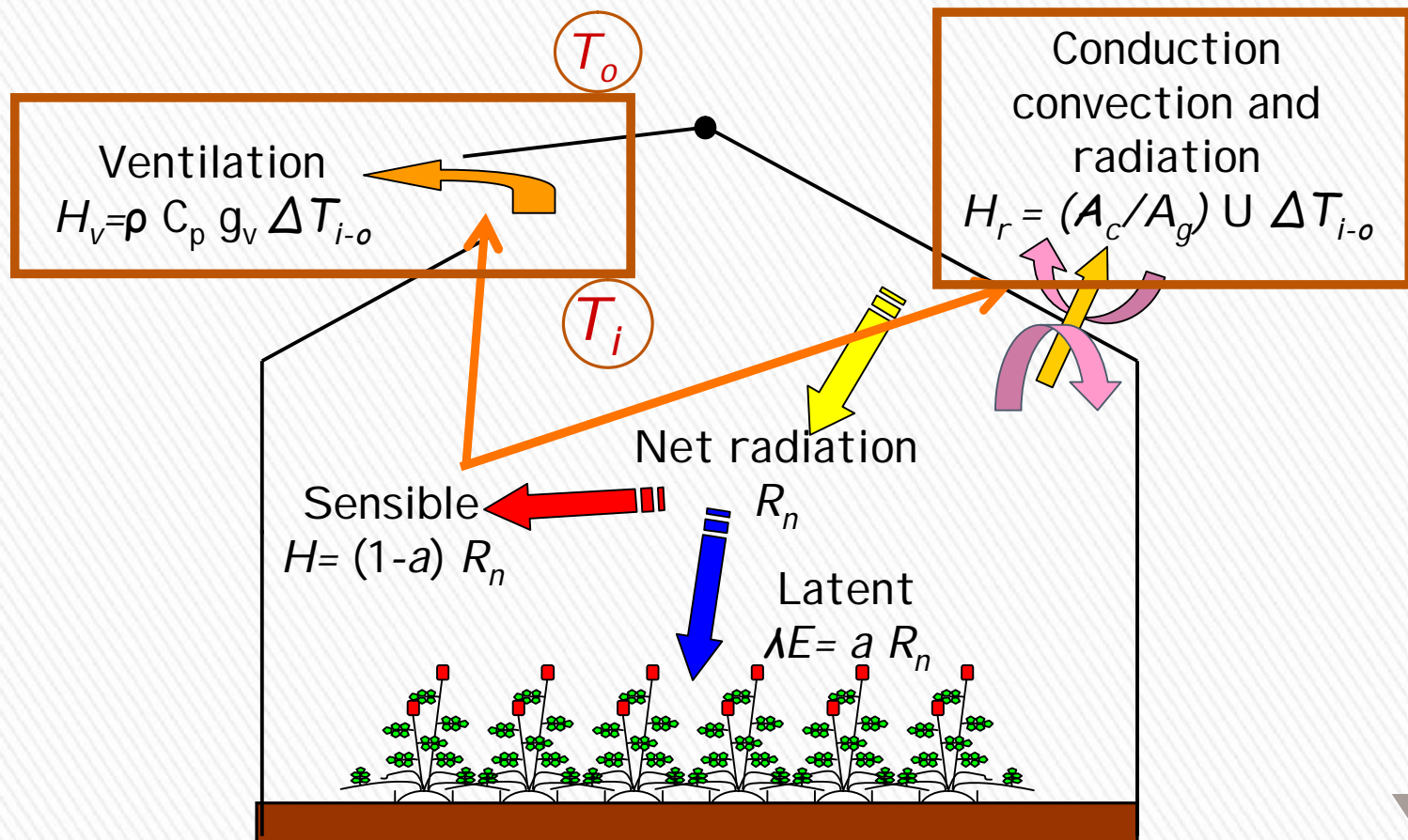


T. TAHRI, S. ABDUL-WAHAB, A. BETTAHAR, M. DOUANI, H. AL-HINAI, Y. AL-MULLA, 2009

M. DOUANI, T. TAHRI, S. ABDUL-WAHAB, A. BETTAHAR, H. AL-HINAI, Y. AL-MULLA, 2011



# Greenhouse heat energy balance



Kittas, 2007

# Greenhouse heat energy balance

$$R_n(1-a) = \left[ \left( \frac{A_c}{A_g} \right) U (T_i - T_o) \right] + \left[ \frac{rC_p N \left( \frac{V}{A_g} \right) (T_i - T_o)}{3600} \right]$$

$$T_i = T_o + \frac{(1-a)R_n}{U(A_c/A_g) + \rho C_p \left( \frac{NV/A_g}{3600} \right)}$$



# Screenhouse heat energy balance

$$(1-a)I A_f + (1-t_t)q_{lw} + h_g A_f (T_g - T_i) - [N_v V \rho C_p + U A_c] (T_i - T_o) = m_a C_p \frac{dT_i}{dt}$$

alpha: absorptivity of the floor, ground cover (%)

Tau: Transmissivity of the screenhouse cover material (%)

I: Global outside solar radiation (w/m2)

Af: Floor area of the screen house (m2)

Ta: Thermal transmissivity of the ground cover (%)

$q_{lw} = A_f \cdot \sigma \cdot (E_g \cdot (T_g + 273)^4 - E_a \cdot (T_o + 273)^4)$

$\sigma = 5.670 \cdot 10^{-8} \text{ W/m}^2 \text{ K}^4$

Eg: emissivity of the ground (%)

Tg: Ground Temperature (C)

Ea: apparent emissivity of the atmosphere (%)

hg: Convective heat transfer coefficient from ground to air (w/m2 K)

Ti: Inside air temperature (C)

NV: Natural ventilation rate

V: Screen house volume (m3)

qg: convection heat transfer from the ground (W)

Ac1=Ac: Area of the screenhouse cover (m2)

p: air density (kg/m3),

Cp= 1200 (J/m3/C)

$U = (q_{teff} + q_g - q_{NV}) / (A_{c1} \cdot (T_i - T_o))$

$q_{teff} = (1 - Ta) \cdot q_{lw}$

qNV: Heat of natural ventilation

To: Outside Temperature (C)

ma: mass of air (kg)

Desmarais, SC et al 1999

# Screenhouse heat energy balance

The previous equation can be solved numerically as first order differential equations using the fourth order Runge-Kutta techniques.

$$\frac{dy}{dx} = f(x, y), y(0) = y_0$$

The Runge Kutta 4<sup>th</sup> order method is given by

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

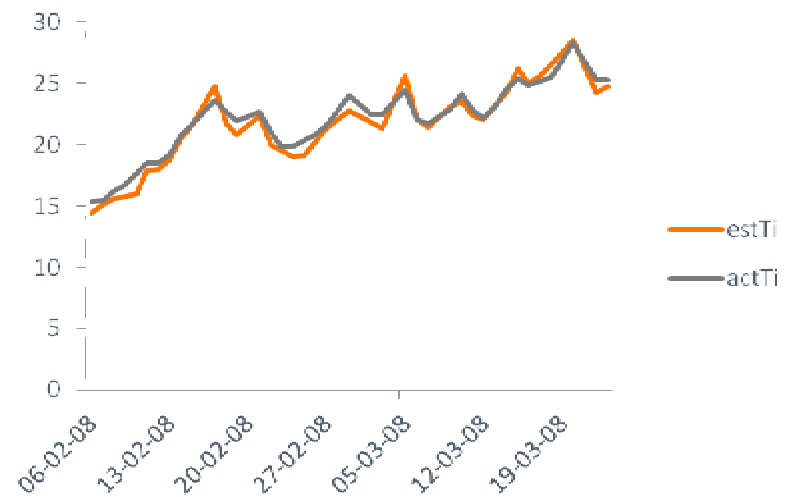
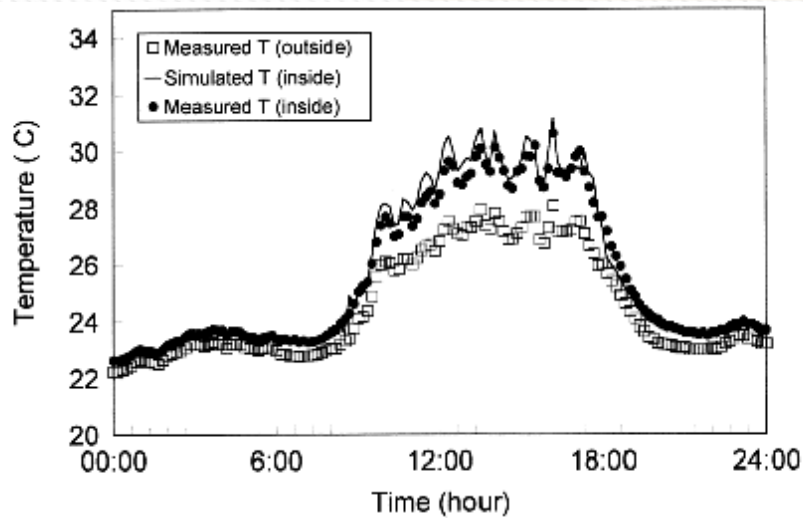
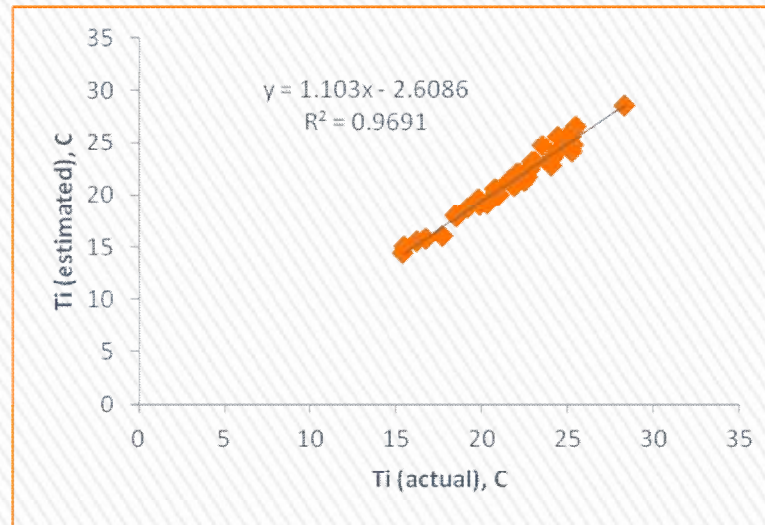
$$k_1 = f(x_i, y_i)$$

$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2h\right)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1h\right) \quad k_4 = f(x_i + h, y_i + k_3h)$$

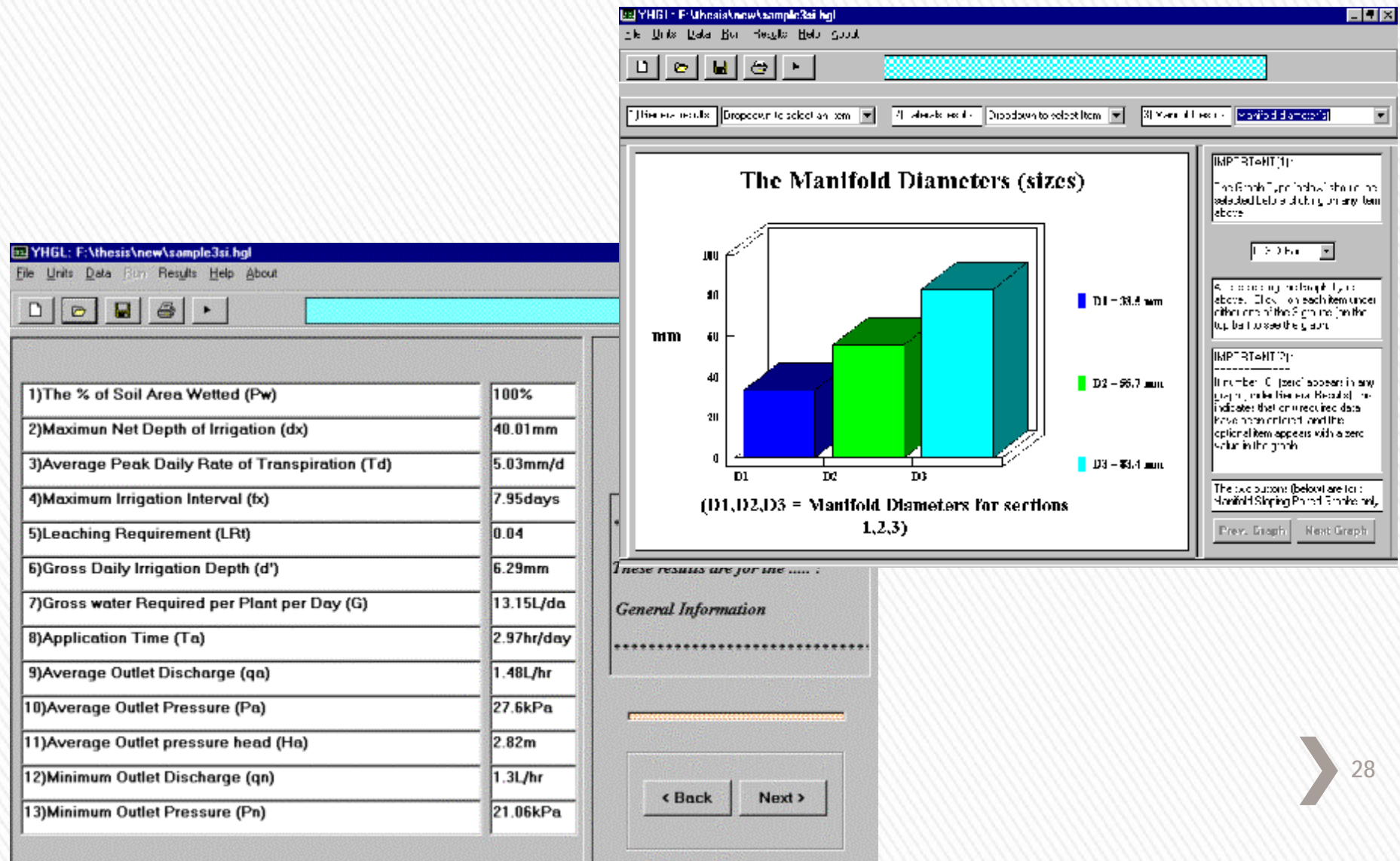


# Screenhouse heat energy balance



Desmarais, SC et al 1999; Al-Mulla, 2011

## 2. Hydraulic Grade Line Method

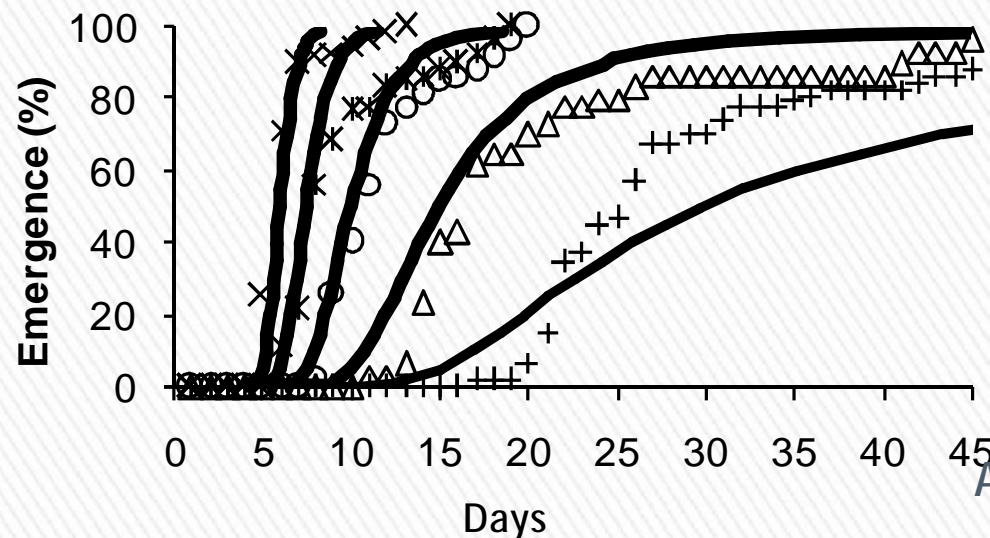




### 3. Wheat Emergence Model

$$t(f) = \frac{q_{HT}}{(y - y(f)_b)(T - T_b)}$$

$$y(f)_b = \overline{y_b} + \text{probit}(f)s_{yb}$$



Al-Mulla, 2004

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