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Mathematics Professional Development

The ultimate goal of professional development is improving students' learning, through the mechanism of improving instruction. This brief review of research on mathematics professional development summarizes what we know about the goals and characteristics of effective mathematics professional development for teachers. We intend this review to guide educators as they plan professional development.

Over the past three decades, evidence about the nature and impact of professional development in mathematics has accumulated both from large-scale empirical studies (e.g., Desimone, Smith, & Phillips, 2007; Garet et al., 2001; Heck et al., 2008; Supovitz, Mayer, & Kahle, 2000) and from small-scale qualitative studies of planned or emergent innovations (e.g., Borko et al., 2008; Collopy, 2003; Franke et al., 1998; Sowder et al., 1998; van Es & Sherin, 2008; Warfield, Wood, & Lehman, 2005). Conceptual analyses of the mathematics knowledge for teaching and how it develops (e.g. Ball & Cohen, 1999; Borko, 2004; Hawley & Valli, 1999; Putnam & Borko, 2000; Thompson & Zeuli, 1999; Wilson & Berne, 1999) have also influenced the design of mathematics professional development. Collectively, these three bodies of research provide evidence for the following goals and features of mathematics professional development.

Core Goals of Mathematics Professional Development

In service to the long-term goal of improving students' learning through better instruction, research evidence to date suggests that mathematics professional development should promote the growth of mathematics teachers in four major areas.

1. Build teachers' mathematical knowledge and their capacity to use it in practice.

Teachers' mathematical knowledge matters and significantly predicts gains in students' achievement (Hill, Rowan & Ball, 2005; Jacobs et al., 2007). In order to enact instruction that supports students' learning, teachers need mathematical knowledge that extends beyond an understanding of mathematical procedures and concepts (Kilpatrick, Swafford, & Findell, 2001). Teachers must be able to choose appropriate mathematical tasks, judge the advantages of particular representations of a mathematical concept, help students make connections among mathematical ideas, and grasp and respond to students' mathematical arguments and solutions. A lack of mathematical content knowledge can impede teachers' abilities to notice and analyze students' mathematical thinking (Doerr & English, 2006), design actions that respond to students' understanding (Hunting & Doig, 1997), or engage in productive professional conversations (Britt, et al., 2001; Polettini, 2000). Research has found that professional development that attends to dimensions of teachers' mathematical knowledge is more effective than professional development that focuses only on pedagogy or generic teaching skills (Garet et al., 2001; Heck et al., 2008).

Research also indicates that teachers can develop their mathematical content knowledge in a number of different ways, including solving and discussing mathematics problems, studying students' mathematical thinking, collaborating with other teachers to plan or discuss instruction, analyzing instances of classroom practice, and using new curricular materials (Chazan, Ben-Chaim, & Gormas, 1998; Fernandez, 2005; Hill & Ball, 2004; Horn, 2005; Lewis, Perry, & Hurd, 2009; Remillard & Bryans, 2004). When teachers solve mathematical problems together and share solution methods, it can affect their understanding of the mathematical content and introduce new perspectives on a problem (Lachance & Confrey, 2003). Teachers can also strengthen their mathematical understanding in the process of trying to make sense of students' work or analyzing instances of classroom practice (Borko et al., 2008; Doerr & English, 2006; Grandau, 2005; Jacobs et al., 2007; Lewis, Perry, & Hurd, 2009; Peressini & Knuth, 1998; Sowder et al., 1998; Ticha & Hospesova, 2006). For example, Ticha and Hospesova (2006) report on a teacher who expanded her ability to think flexibly about subtraction by exploring a student's unexpected argument that $63 - 8 = 60 - 5$ because the difference remains the same if both 63 and 8 are reduced by 3. Improved mathematical knowledge can also help teachers connect mathematics to classroom practice as they analyze and use new curriculum materials (Cohen & Hill, 2000), investigate mathematical lessons or tasks (Lewis, Perry, & Murata, 2006) and analyze of students' mathemat-

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ical thinking (Borko et al., 2008; van Es & Sherin, 2008). Finally, professional development that focuses on improving teachers' mathematical knowledge can help them develop the confidence to teach mathematical topics that they previously avoided (Chapin, 1994).

2. Build teachers' capacity to notice, analyze, and respond to students' thinking.

A number of studies provide evidence that professional development can help teachers learn to notice, value, and analyze students' mathematical thinking. Professional development that helps teachers attend to students' thinking can shift teachers' focus from simply evaluating students' work as correct or incorrect to analyzing the particulars of students' thinking (Borko et al., 2008; Goldsmith & Seago, 2010; Swafford, Jones, & Thornton, 1999; van Es & Sherin, 2008). For example, elementary school teachers participating in Cognitively Guided Instruction (CGI) professional development learned to recognize increasingly sophisticated strategies among students who correctly solved addition and subtraction problems. They also learned to make principled decisions about choosing mathematics problems that would engage and extend each student's current level of reasoning (Fennema et al., 1996). Similarly, teachers participating in professional development based on CGI principles learned to recognize a variety of students' algebraic reasoning strategies and notice strengths in students' mathematical thinking that could be built on, even when students' solutions were not entirely correct (Jacobs et al., 2007).

Professional development that supports close attention to students' thinking may also help teachers recognize that they have tended either to overestimate (Schorr & Koellner-Clark, 2003) or underestimate their students' understanding (Chazan, Ben-Chaim, & Gormas, 1998; Kazemi & Franke, 2004; Lin, 2001; Warfield, Wood, & Lehman, 2005). As teachers learn to notice and analyze students' thinking, they gain a more accurate picture of the strengths and weaknesses in students' mathematical understandings (Borko et al., 2008; Jacobs et al., 2007; Kersaint & Chappell, 2001; van Es & Sherin, 2008). Teachers can then use their analyses of students' thinking to refine instruction and to respond to students' needs (Doerr & English, 2006; Kazemi & Franke, 2004; Seymour & Lehrer 2006; Sherin & Han, 2004).

3. Build teachers' productive habits of mind.

Learning to improve one's teaching practice is challenging, effortful work. An important goal of professional development is to help teachers develop the beliefs, habits, and dispositions needed to improve practice on an ongoing basis. For example, teachers' beliefs about mathematics (Borko, 2004; Drake, Spillane, & Hufferd-Ackles, 2001), curriculum (Collopy, 2003: Remillard & Bryans, 2004), and students' capacity for learning (Smylie, 1988; Warfield, Wood, & Leman, 2005) all influence what teachers learn from professional development opportunities. Likewise, teachers' dispositions and habits of mind, including habits of inquiry, curiosity, self-monitoring, attention to students' thinking, and experimentation influence teachers' learning from professional development opportunities (Allinder et al., 2000; Chazan, Ben-Chaim, & Gormas, 1998; Clarke & Hollingsworth, 2002; Edwards & Hensien, 1999; Spillane, 2000).

Professional development programs themselves shape teachers' beliefs and habits of mind in ways that influence teachers' subsequent learning from practice (Britt, Irwin, & Richie, 2001; Chapin, 1994; Chazan, Ben-Chaim, & Gormas, 1998; Jaworski, 1998; Senger, 1999; Ticha & Hospesova, 2006; van Es & Sherin, 2008; Zech et al., 2000). Hence, an important criterion for selecting a professional development program is whether it nurtures beliefs and dispositions,that result in continued learning in daily practice. For example, professional development experiences in which teachers analyze instruction, live or on videotape, may help teachers shift from a descriptive or evaluative stance toward an inquiry stance toward practice (Perry & Lewis, 2010; van Es & Sherin, 2008) and build teachers' confidence that changes in their instructional methods can improve students' learning (Perry et al., 2009). Professional learning experiences that involve learning mathematics related to teaching can build teachers' desire to learn more mathematics, perhaps by building the sense of efficacy, identity as a mathematics learner, or collegial support for learning (Polettini, 2000; Hodgen & Askew, 2007; Lewis, Perry, & Hurd, 2009). Given that professional development does not automatically build productive habits of mind, those responsible for professional development may want to directly address whether efficacious beliefs and habits of mind—such as inquiry into students' thinking, confidence that all students can make sense of mathematics, and interest in deepening one's own mathematical understanding—are developing.

4. Build collegial relationships and structures that support continued learning.

One way that professional development can support teachers' ongoing learning is by catalyzing changes in collegial relationships and structures for collegial work. Recent research has pointed to the value of collaboration for the learning of teachers. Collaboration with colleagues can spark the need for teachers to explain their practices and to articulate rationales for instructional decisions, helping teachers make tacit ideas visible and subject to shared scrutiny and develop deeper, more widely shared understandings of students' learning (Chazan, Ben-Chaim, & Gormas, 1998; Horn, 2005; Kazemi & Franke, 2004). Professional conversations can also provide teachers with the encouragement and support that is needed to begin to experiment with new approaches to teaching (Britt et al., 2001). Teachers value the kinds of professional relationships that can be built through shared inquiry into practice; such interactions with colleagues can support teachers' sense of competence as they engage in the work of changing practice (Arbaugh, 2003; Edwards & Hensien, 1999; Jaworski, 1998; Smylie, 1988).

However, collegial interactions do not always lead to professional learning. The emotional support that can come from sharing stories or observing in each other's classrooms does not necessarily lead to a focus on improving aspects of teaching (Cwikla, 2007; Manouchehri, 2001). When collegial interactions do focus on classroom instruction, teachers may experience a tension between colleagues' suggestions and their own sense of autonomy to decide whether and how to use ideas (Puchner & Taylor, 2006). Several studies suggest that of the effectiveness of collegial learning structures can be built over time (Kazemi & Franke, 2004; Lewis, Perry, & Hurd, 2009). For example, teachers in the study group that Kazemi and Franke (2004) followed were initially unaware of the details of students' problem-solving strategies and saw posing questions to understand students' ideas as unimportant, despite the facilitators' efforts to focus on students' thinking. Over time, as teachers found ways to interact with students about their strategies, and they began to share their efforts to understand students' ideas in their study group meetings. Likewise, teachers at a school-wide lesson study site showed a substantial increase in the proportion of discussion devoted to students' thinking from year one to year three of the school's adoption of lesson study (Perry & Lewis, 2010). Research reviewed in the next section illuminates why collegial structures to support learning may develop gradually over time, rather than emerge fully developed as an immediate consequence of a professional development intervention.

What professional development features support these four goals?

1. Substantial Time Investment

Several large-scale studies suggest that the duration of professional development is significantly associated with impact on teachers (Boyle, Lamprianou, & Boyle, 2005; Garet et al., 2001; Heck et al., 2008; Hill & Ball, 2004). For example, a study of summer professional development workshops ranging from 40 to 120 hours in length associated longer workshops with teachers' greater knowledge gain, although some programs were exceptions to this trend (Hill & Ball, 2004). In their evaluation of the NSF-funded Local Systemic Change initiatives, Heck and his colleagues documented a significant relationship between hours of participation and teachers' self-reported increases in investigative classroom practices, with most of the gains occurring during the first 100 hours of professional development (Heck et al., 2008). With respect to the use of instructional materials, much of the gain occurred with the first 80 hours of professional development, with an additional increase after about 180 hours. Ohio teachers participating in the State Systemic Initiative showed substantial increase in the use of inquiry-based instructional practices over the first year, after six weeks of summer professional development. These changes leveled off and were sustained over the next two years (Supovitz, Mayer, & Kahle, 2000).

Qualitative studies illuminate some of the reasons that professional learning takes time. Changes in teachers' mathematical knowledge, beliefs, dispositions, and in the collaborative structures that support learning often occur in small increments, with advances in any one of them depending on advances in the others (Kazemi & Franke, 2004). Teachers' growth is often incremental, nonlinear, and iterative, proceeding through repeated cycles of inquiry outside the classroom and experimentation inside the classroom (Clarke & Hollingsworth, 2002; Fennema et al., 1996; Jaberg, Lubinski, & Yazujian, 2002). For example, Jaberg, Lubinski, and Yazujian (2002) reported on a teacher who responded to professional development by changing her practice to elicit and respond to students' thinking more often. After making this change, she found she needed to better understand her students' thinking, which in turn convinced her that she needed to increase her own mathematical content knowledge. Similarly, studies of teachers' collaborative work suggest that increases in practice-focused collaboration and content knowledge can build incrementally on each other, as teachers' explanations of their practice lead to questions about the mathematical content (Peng, 2007). First-year results of a large-scale, randomized control study of middle school mathematics professional development indicated that professional development linked to an initial increase in teachers' activities to elicit students' thinking, but no corresponding increase occurred during the first year in either teachers' mathematical knowledge or students' achievement. These data further suggest the incremental and complex nature of changes in teachers' knowledge and practice (Garet et al., 2010).

Another example of iterative, incremental change in knowledge, beliefs, and practices comes from the CGI program, which found that using challenging mathematical tasks led teachers to expand their ideas about students' capacity to think mathematically. This expanded set of beliefs then led teachers to change classroom practices in ways that enabled teachers to increase their knowledge of students' thinking. Over time, these experiences led teachers to develop a disposition to inquire into students' thinking that, in turn, supported further development of both their classroom practices and their knowledge about students' thinking (Franke et al., 1998). In summary, these studies suggest that teaching practice is an apt term, given the repeated cycles of experimentation, reflection, and revision required to change elements of instruction. Effective professional development is more likely to look like a series of incremental changes in knowledge, beliefs, dispositions, and classroom practices that eventually lead to students' improved outcomes than a direct line from professional development to practice to students' outcomes.

2. Systemic Support

Systemic support influences the impact of professional development programs. A number of studies have reported that the nature and degree of principal support for a particular professional development program influences its impact (Desimone, Smith, & Phillips, 2007; Heck et al., 2008, Woodbury & Gess-Newsome, 2002). For example, Jaberg, Lubinski, and Aeschleman (2004) describe a number of different ways that one principal supported and encouraged the work her teachers were undertaking through their professional development, including building support among parents and other community members, making time during faculty and grade-level meetings for teachers to discuss mathematics instruction, and being flexible about assessments of student learning.

Other system factors can also make a difference. Garet et al. (2001) found that professional development was more effective when teachers perceived it to be consistent with their own goals and with state and district standards; other studies have reported that the nature and consistency between professional development and system messages about mathematics teaching and learning affect teachers' learning (Cwikla, 2007; Polettini, 2000; Scribner, 1999; Woodbury & Gess-Newsome, 2002), as does the nature of parental and community support (Tschannen-Moran & Hoy, 2007). Presumably, then, those responsible for professional development should attend to building coherent support for participating teachers. This support should come from a variety of sources, including principals, district and state officials, and parents.

3. Opportunities for active learning

For more than a decade, the literature on promising practices in mathematical professional development has advocated active involvement of teachers in inquiry and problem solving with respect to both mathematics and instruction (e.g., Putnam & Borko, 1997; Wilson & Berne, 1999). Large-scale research studies support these recommendations. For example, Garet et al. (2001) reported that professional development that offered opportunities for active learning—for example, planning lessons; observing other teachers and being observed; reviewing students' work; and making presentations, writing papers, or leading discussions—were associated with teachers' reports of increased knowledge and skill.

Concluding Remarks

The vast majority of studies about teachers' professional learning follow teachers' postprofessional development for a year or less, so evidence regarding the long-term impact of mathematics professional development on teachers' knowledge or instructional practices is limited. In fact, even in the short term, the impact of professional development may be less than is suggested by the large-scale studies, which rely on self-report. Research that includes classroom observations typically shows less use of key "reform" instructional strategies, such as eliciting students' thinking, than teachers' self-reports would lead us to expect (Fisler & Firestone, 2006). These studies reveal a tendency to adopt superficial features of reforms rather than more fundamental features (Cohen, 1990).

Despite the limitations of current research, substantial support exists for focusing mathematics professional development on the four broad goals of developing:

- teachers' mathematical knowledge and capacity to connect it to practice;
- teachers' capacity to notice, analyze, and respond to student thinking;
- the beliefs and dispositions that foster teachers' continued learning; and
- collegial relationships and learning structures that can support and sustain teachers' learning.

In addition, three features of professional development design appear to be important for supporting progress toward these goals:

- time;
- systemic support for teachers' learning; and
- opportunities for teachers' active learning.

Although research on professional development is still emerging, the goals and features that this review has identified emerge from a substantial number of studies and offer the best current guidance for practitioners.

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