On the Probability of Occurrence of Extreme Space Weather Events

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Motivation

What is the likelihood of another Carrington event happening in my lifetime?
Nature loves Power Law Distributions

Clauset et al. (2009)
Mathematical Background

\[ p(x) = Cx^{-\alpha} \]

\[ P(x \geq x_{\text{crit}}) = \int_{x_{\text{crit}}}^{\infty} p(x') \, dx' \]

\[ P(x \geq x_{\text{crit}}) = \frac{C}{\alpha - 1} x_{\text{crit}}^{-\alpha+1} \]
\[ \alpha - 1 = N \left[ \sum_{i=1}^{N} \ln \frac{x_i}{x_{\text{min}}} \right]^{-1} \]

\[ E(x \geq x_{\text{crit}}) = NP(x \geq x_{\text{crit}}) \]

\[ P(x \geq x_{\text{crit}}, t = \Delta t) = 1 - e^{-\frac{N\Delta t}{\tau}P(x \geq x_{\text{crit}})} \]
For Bernouilli Events, with a constant probability of occurrence, the probability of occurrence is:

\[ P(x) = \frac{1}{1 + \tau} \]

Where \( \tau \) is time to event.

E.g., if event occurs every 100 years, the probability of one occurring in the next decade is: \( 1/(1+100/10) = 0.09 \) or 9%. 

Reality Check: Time to Event
Assumptions

- Quasi-time-stationarity
- Poisson process (Independent events)
- Power-law distribution through at least the size of the Carrington event
A representative selection of space weather datasets...

- Hard Solar X-ray data from BATSE
- CME speeds
- Dst
- > 30 MeV Proton Fluences
- (Others: Equatorward edge of the diffuse aurora, Auroral indices, Kp, etc.)
Hard X-Ray Data from BATSE

Peak Rate (cts/s/2000 cm²) vs Time (Years)
Hard X-Ray Data from BATSE
CME Speed
CME Speed

(a) Number of Events vs. Speed (km/s)

(b) $P(X>x)$ vs. Speed (km/s)
Probability of an Extreme ICME (based on speed)

- Assume $V_{\text{cme-crit}} \geq 5,000 \text{ km/s}$
- Slope = -3.2
- Probability of observing such an event over the next decade:
  \[ P(v>5,000 \text{ km/s}) \sim 85\% \]
CME Speed
Probability of and Extreme ICME (based on speed)

- For CMEs > 2,000 km/s,
- Slope = -6.1
- Revised probability:

\[ P (v \geq 5,000 \text{ km/s, } \Delta t=10 \text{ yrs}) \sim 12\% \]
Dst

![Graph showing Dst (nT) over time from 1970 to 2010. The graph displays a wide range of values, with periodic fluctuations.]
Dst “Events” (Dst < -100 nT)
Dst Power Laws

(a) Number of Events vs. $|D_{stl}|$ (nt)

(b) $P(X > x)$ vs. $|D_{stl}|$ (nt)
Probability of an Extreme Geomagnetic Storm (based on Dst)

- For Dst < -850 nT:
  \[ P(\text{Dst} \leq -850 \text{ nT}, \Delta t=10 \text{ yrs}) \sim 12\% \]

- For Dst < -1700 nT:
  \[ P(\text{Dst} \leq -1700 \text{ nT}, \Delta t=10 \text{ yrs}) \sim 1.5\% \]
>30 MeV Fluences
>30 MeV Fluences

(a) >30 MeV Fluence (x10^9 cm^-3)

(b) >30 MeV Fluence (x10^9 cm^-3)
Probability of an Extreme SPE (based on >30 MeV Proton Fluences)

- Fluence > $18.8 \times 10^9$ cm$^{-1}$
- Slope = -2.0

$$P(>18.8 \times 10^9 \text{ cm}^{-1}, \Delta t=10 \text{ yrs}) \sim 3\%$$
Summary

- Probability of occurrence of a Carrington-like (or worse) event is sensitive to the definition of event.

- There are a number of assumptions that may or may not hold:
  - Time stationarity
  - Poisson process
  - Power-law distribution

- Major issue that remains to be addressed is the uncertainty associated with the predictions.
Future work

- How to assess the uncertainties?
  - Jeff Love’s work (next talk)
  - Maximum likelihood methods
  - Multiple models of the distribution’s tail - Robustness
  - Estimates of parameter and model uncertainty

- Assessing the temporal variability in extreme event probabilities

- Look at other measures of extreme event behavior (GICs, Eq. edge of diffuse aurora, etc.)

- Couple probabilistic forecasts with event-based physical/empirical models
Terrorist Attacks
Terrorist Attacks: World
Terrorist Attacks: USA
Terrorist Attacks: Evolution of Probability over last 40 years

![Graph showing the probability of terrorist attacks over the years.](image-url)